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NPS55-82-022
NAVAL POSTGRADUATE SCHOOL Monterey, Califíonia


POSITION DETERMINATION WITH LORAN-C TRIPLETS AND THE HEWLETT-PACKARD HP-41.CV PROGRAMMABLE CALCULATOR
by

Rex H. Shudde
POSITION DETERMINATION WITH
LORAN-C TRIPLETS AND THE HEWLETT-PACKARD
HP-4ICV PROGRAMMABLE CALCULATOR
BY
September 1982

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## NAVAL POSTGRADUATE SCHOOL

## MONTEREY, CALIFORNIA

Rear Admiral J. J. Ekelund Superintendent

David A. Schrady Provost

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## CORREGTION

The coding delay for 7970W, given on page 61 of NPS55-82-022 should be 26008 microseconds.


#### Abstract

ADDENDA 1. On 9 June 1982, the Defense Mapping Agency issued upgraded WGS-72 coordinates for the Loran-C stations. This update includes positions measured to the nearest $0.001^{\prime \prime}$, the $* 7930$ Northwest Pacific Reconfiguation and the addition of the 5970 Commando Lion Chain. The upgraded coordinates are listed on the following page. 2. The HP-4lCV program may be modified to display the station coordinates to the nearest $0.001^{\prime \prime}$ by changing line 953 to FIXI.


## STATION COVERAGE

| Station | No. of Pairs |
| :---: | :---: |
| 4990 | 2 |
| 5930 | 2 |
| 5970 | 3 |
| 5990 | 3 |
| 7930 | 3 |
| 7930 | 3 |
| 7960 | 2 |
| 7970 | 4 |
| 7980 | 4 |
| 7990 | 3 |
| 8970 | 3 |
| 9940 | 3 |
| 9960 | 4 |
| 9970 | 4 |
| 9990 | 3 |

Location
Central Pacific
Canadian East Coast
Commando Lion
Canadian West Coast
North Atlantic
Northwest Pacific, Reconfigured
Gulf of Alaska
Norwegian Sea
Southeast U.S.A.
Mediterranean Sea
Great Lakes
West coast, U.S.A.
Northeast U.S.A.
Northwest Pacific
North Pacific

## LORAN-C_STATIONS

| ID | CD | MS_LAT | MS_LON | SS LAT | SSLCN |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4990x | 11000 | 16.4443950 | 169.3031200 | 20.1449160 | 155.5309706 |
| 4990Y | 29000 | 16.4443950 | 169.3031200 | 28.2341770 | 178.1730200 |
| 5930X | 11000 | 46.4827199 | 067.5537713 | 41.1511930 | 069.5839090 |
| 5930x | 25000 | 46.4827199 | 067.5537713 | 46.4632180 | 053.1028160 |
| 5970W | 11000 | 36.1105797 | -129.2027279 | 42.4437104 | -143.4309245 |
| 5970x | 31000 | 36.1105797 | -129.2027279 | 35.0223871 | -126.3226741 |
| 59702 | 42000 | 36.1105797 | -129.2027279 | 26.3624975 | -128.0856445 |
| 5990x | 11000 | 51.5758780 | 122.2202240 | 55.2620851 | 131.1519648 |
| 5990Y | 27000 | 51.5758780 | 122.2202240 | 47.0347990 | 119.4439530 |
| 5990 Z | 41000 | 51.5758780 | 122.2202240 | 50.3629731 | 127.2129043 |
| 7936W | 11000 | 59.5917270 | 045.1027470 | 64.5426580 | 023.5521750 |
| 7930x | 21000 | 59.5917270 | 045.1027470 | 62.1759640 | 007.0426538 |
| 7930 z | 43000 | 59.5917270 | 045.1627470 | 46.4632180 | 053.1028160 |
| *7930x | 11000 | 24.1707888 | -153.5853232 | 42.443710 | -143.4309245 |
| *7936Y | 30000 | 24.1707888 | -153.5853232 | 26.362497 | -128.0856445 |
| *7930 | 49000 | 24.1797888 | -153.5853232 | 09.324578 | -138.0954970 |
| 7960x | 11000 | 63.1942814 | 142.483190 | 57.2620210 | 152.2211225 |
| 7960Y | 26000 | 63.1942814 | 142.4831900 | 55.2620851 | 131.1519648 |
| 7970W | 26000 | 62.1759640 | +007.0426538 | 54.482987 | -008.1736312 |
| 7970x | 11000 | 62.1759640 | +007.0426538 | 68.3806150 | -614.2747000 |
| 7970Y | 46000 | 62.1759640 | +E07.0426538 | 64.5426580 | +823.5521750 |
| 79702 | 60000 | 62.1759640 | +007.0426538 | 70.5452610 | + 098.4358690 |
| 7986W | 11000 | 30.5938740 | 085.1009305 | 30.4333018 | 090.4943600 |
| 7980x | 23000 | 30.5938740 | 085.1009305 | 26.3155006 | 097.5000093 |
| 7980Y | 43000 | 30.5938740 | 085.1009305 | 27.0158393 | 680.0653429 |
| 7980 Z | 59000 | 30.5938740 | 085.1089305 | 34.6346081 | 077.5446654 |
| 7990x | 11000 | 38.5220587 | -016.4306159 | 35.3120787 | -612.3130245 |
| 7990Y | 29000 | 38.5220587 | -016.4306159 | 40.5820950 | -027.5201520 |
| 7990 Z | 47000 | 38.5220587 | -616.4306159 | 42.0336515 | -003.1215512 |
| 8970w | 11000 | 39.5107540 | 087.2912140 | 30.5938740 | 085.1009305 |
| 8970x | 28000 | 39.5107540 | 087.2912140 | 42.4250603 | 076.4933862 |
| 8970Y | 44000 | 39.5107546 | 087.2912140 | 48.3649844 | 094.3318469 |
| 9940W | 11000 | 39.3306621 | 118.4956370 | 47.0347990 | 119.4439530 |
| 9946X | 27000 | 39.3306621 | 118.4956370 | 38.4656990 | 122.2944529 |
| 9940x | 40000 | 39.3306621 | 118.4956370 | 35.1918180 | 114.4817435 |
| 9960W | 11000 | 42.4250603 | 076.4933862 | 46.4827199 | 067.5537713 |
| 9960X | 25000 | 42.4250603 | 076.4933862 | 41.1511930 | 069.5839090 |
| 9960 Y | 39000 | 42.4250603 | 076.4933862 | 34.0346081 | 077.5446654 |
| 9960 Z | 54000 | 42.4250603 | 076.4933862 | 39.5107540 | 087.2912140 |
| 9970W | 11000 | 24.4803597 | -141.1930303 | 24.1707888 | -153.5853232 |
| 9970x | 30000 | 24.4803597 | -141.1930303 | 42.4437104 | -143.4309245 |
| 9970Y | 55000 | 24.4803597 | -141.1930303 | 26.3624975 | -128.0856445 |
| 99702 | 75000 | 24.4803597 | -141.1930303 | 09.3245789 | -138.0954970 |
| 9990x | 11000 | 57.0912265 | +170.1506789 | 52.4944040 | -173.1648974 |
| 9990Y | 29000 | 57.0912265 | +170.1506789 | 65.1440306 | +166.5312550 |
| 9990 Z | 43000 | 57.6912265 | +170.1506789 | 57.2620210 | +152.2211225 |

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TE. SUPPLEMENTARY NOTES

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| Loran | Hyperbolic Fixing | Programmable Calculator |
| :--- | :--- | :--- |
| Loran-C | Radio Positioning | HP-41C |
| Navigation | Geodetics | HP-41CV | Position Determination Geodetic Distances Fixing Calculator


This report presents an algorithm and HP-4lCV programs for position determination with Loran-C chains. Additional computational routines include the ability to calibrate Loran station triplet data to a known benchmark and ITD's (Indicated Time Delay's), predict ITD's at given positions, compute the geodesic (similar to great circle) bearing and distance from a fix to the destination and to compute the geodesic bearing and distance from any one location to another. Utility routines allow the user to transfer station pair data between the $\mathrm{HP}-41 \mathrm{CV}$ and magnetic cards, magnetic tape and an extended

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function/memory module. $\uparrow$

## by

R. H. Shudde

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Monterey, California


September 1982

The programs in this report are for use within the Navy, and they are presented without representation or warranty of any kind.

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## ABSTRACT

This report presents an algorithm and HP-41CV programs for position determination with Loran-C chains. Additional computational routines include the ability to calibrate Loran station triplet data to a known benchmark and ITD's (Indicated Time Delay's), predict ITD's at given positions, compute the geodesic (similiar to great circle) bearing and distance from a fix to the destination and to compute the geodesic bearing and distance from any one location to another. Utility routines allow the user to transfer station pair data between the HP-41CV and magnetic cards, magnetic tape and an extended function/ memory module.

## A. Introduction

The Loran system is a radio aid to navigation which utilizes the principle of hyperbolic fixing. The locus of points for which the difference in arrival time of synchronized signals from a pair of transmitters is constant determines a hyperbolic line of positions. The intersection of two hyperbolic lines of position from two pairs of stations determines position or a hyperbolic fix. That two pairs of stations are required for a fix does not necessarily mean that there are four separate stations, for one station of one pair may be colocated with one station of the other pair forming a Leran triplet (Figure l). Triplets may be joined "end-to-end" by station colocation to form a Leran chain (Figure 2). Loran chains are common on both the East and west coasts of the North American continent.

The present day Loran-C operates at $100-k H z$ and is in use in the Atlantic, Pacific and Mediterranean areas. The computational algorithm and programs described herein can be used for position determination with Loran-C triplets. Further information on the history, development and operation of the Loran systems may be found in References 1 and 2 .

(a) Colocated Master Stations

(b) Colocated Slave Stations

(c) Colocated Master and Slave

Figure 1. Loran Triplets.


Figure 2. Loran Chain of Five Loran Triplets.

## B. User Routines

1. Computational Routines:

EI - The EXxing routine is the main program for calculating a Loran-C fix from indicated time delays.

AS - The Alternate Solution routine will allow the second Loran fix solution to be computed. This routine toggles Flag 3 so that on subsequent fixes the FI routine will calculate the alternate solution.

DN - The DestiNation routine stores the latitude and longitude of a fixed destination.

HD - Computes the Heading and Distance from the current fix to the destination stored by the DN routine.
2. Manual Mode Routines:

MI - Manual Input allows station data to be input and stored via the calculator keyboard.

ED - Echo Data is a utility routine for validating station triplet information stored in the calculator.
3. Card Reader Routines:

CS - Card Store records station data onto magnetic cards.
CB - Card Bead inputs station data from magnetic cards.
CE - Card Echo is a utility routine for validating station information stored on data cards.
4. Extended Memory Routines:

XS - XMEM Store records station data onto the extended memory module.

XB - XMEM Becall inputs station data from the extended memory module.

XD - XMEM Delete erases station data from the extended memory module.
5. Tape Cassette Routines:

TS - Tape Store records station data onto the tape cassette.
IR - Tape Recall inputs station data from the tape cassette.
TD - Tape Delete erases station data from the tape cassette.
6. Utility Routines:

DH - This routine is similiar to the HD option except that it computes the heading and distance from any origin to any destination.

PR - PR is used to RRedict the station ITD's that will be received at a given latitude and longitude.

CA - CA is the CAlibration option. Given the latitude and longitude of a known position and the indicated time delays from a Loran-C triplet, the stored station data are modified so that the FI routine (or AS) will compute the known position from the same time delays.

SW - SWitch data swaps the data of the two Loran stations stored in memory.

Note: There are no specific routines that relate to HP-41CV printer operations. However, all input and output will be recorded on a printer if one is attached.

## C. Recommended Key Assignments

It is recommended that the following HP-41CV user key assignments be made and recorded onto the program cards if the program is first prepared manually:


The EI function is placed on the ENTER key as a reminder that ENTER should not be used for data entry. To make the user key assignments on the $H P-41 C V$, refer to the ASN function in the HP-41CV Owner's Handbook and Programming Guide.

## D. User Instructions with Examples

## 1. Manual Input Routine - MI

This routine can be used to enter station data (Appendix C) to prepare data to be transferred to cards, extended memory, or tape cassette. It can also be used to enter station data manually if alternate data storage media are not available.

Example using 9940W and 9940Y. (Note: The notation <9>, for example, means that you must press the gold shift key and then the 9 key.)

## See

1. 
2. ID?
3. CODE DELAY?
4. MS LAT?
5. MS LON?
6. SS LAT?
7. SS LON?

If desired, these station parameters can now be transferred to card, extended memory or tape using the $C S, X S$, or TS routines, respectively. Otherwise, repeat the steps above with the $9940 Y$ station data. These two stations will be used in the remaining examples.

See
1.
2. ID?
3. CODE DELAY?
4. MS LAT?

Key in
$\langle 9\rangle\langle 9\rangle\langle 4\rangle\langle 0\rangle Y$
40000
39.330662

R/S

| 5. MS LON? | 118.495637 | $R / S$ |
| :--- | ---: | ---: |
| 6. SS LAT? | 35.191818 | $R / S$ |
| 7. SS LON? | 114.481743 | $R / S$ |

8. NEXT OPTION?

At this point the station data for 9940 W and $9940 Y$ are stored in the calculator. The $9940 Y$ data can be transferred to card, extended memory or tape cassette using the CS, XS or TS routines, respectively.
[Advanced User Note: The first action taken in the MI routine is to $X E Q^{\prime \prime} S^{\prime \prime}$. $S W$ is the data swap routine which exchanges to content of R16 - R25 with R26 - R35. The incoming data are then stored in the R16 - R25 registers. The CS, XS and TS routines transfer the content of R16 - R25 to card, extended memory or tape cassette, respectively. If desired, the content of these registers can be swapped once more by using the $S W$ utility routine.]

## 2. Echo Data Routine - ED

This routine allows the user to review the station data resident in the calculator.

Example: Load the station pairs 9940 W and 9940 y using either the $M I, C R, X R$ or $T R$ routines.

See Key in Press

1. XEQ"ED"
2. ID: 9940 W R/S
3. CD: 11000 R/S
4. MLT: 39.330662

R/S
5. MLN: 118.495637

R/S
6. SLT: 47.034799

R/S
7. SLN: 119.443953

R/S
8. BL: 2796.903

R/S
9. ID: 9940Y

R/S
10. CD: 40000 R/S
11. MLT: 39.330662 R/S
12. MLN: 118.495637 R/S
13. SLT: 35.191818 R/S
14. SLN: 114.481743 R/S
15. BL: 1967.302

R/S
16. NEXT OPTION?

Notation: $C D=$ coding delay, $M=$ master, $S=$ slave, $L T=$ latitude, $L N=$ longitude and $B L$ is the station pair baseline plus the secondary phase correction in microseconds.
3a. Card Store Routine - CS
With the card reader attached, station data (in Rl6 R25), which has been input using the MI, XR or TR routine, can be transferred to magnetic card using the $X E Q^{\prime \prime} C S^{"}$ command or by pressing the appropriate user defined key.
Example using 9940W.

See
Key in
Press
1.

XEQ"PD"
2. WRITE: 9940W (Pass a blank card through the card reader. Label the card track "9940W")
3. NEXT OPTION?
To proceed with the remaining examples it is recommended that you also prepare a card for the 9940 y station pair.
3b. Card Read Routine - CR
With the card reader attached, $X E Q^{\prime \prime} C R^{\prime \prime}$ or press the appropriate user defined key. This routine can be used to input the data for two station pairs, which must form a triplet.
Example using 9940W and 9940Y.

See
Key in
XEQ"CR"
1.
2. 1ST CARD (Pass the data card for 9940 W through the card reader.)
3. 2ND CARD (Pass the data card for 99440 Y through the card reader.)
4. NEXT OPTION?

## 3c. Card Echo Routine - CE

With the card reader attached, XEQ"CE" or press the appropriate user defined key. This routine is used to validate the content of data cards against the table in Appendix C.

Example using 9940W.

See
Key in
XEQ"CE"
1.
2. STA. CARD (Pass one side of a data card [9940W] through the card reader).
3. ID: 9940W R/S
4. CD: 11000 R/S
5. MLT: 39.330662

R/S
6. MLN: 118.495637

R/S
7. SLT: 47.034799

R/S
8. SLN: $119.443953 \quad R / S$
9. BL: 2796.903

R/S
10. NEXT OPTION?

Notation: CD = coding delay, $M=$ master, $S=$ slave, $L T=$ latitude, $\mathrm{LN}=$ longitude and BL is the station pair baseline plus the secondary phase correction in microseconds.

## 4a. Store Data in Extended Memory - XS

With the extended memory module in the $\mathrm{HP}-41 \mathrm{CV}$, station data (in R16 - R25), which has been input using the MI, CR, XR or $T R$ routine, can be transferred to the module using the XEQ"XS" command or by pressing the appropriate user defined key. Example using 9940W.
See Key in Press
1.

XEQ"XS"
2. NEXT OPTION?

Should the station pair already be in extended memory, the message DUP FL (duplicate file) will be displayed. If needed, the duplicate file may be erased using the $X D$ routine.

To proceed with the remaining examples it is recommended that you also store the $9940 Y$ station pair.

## 4b. Recall Data from Extended Memory - XB

With the extended memory module installed, XEQ"XR" or press the appropriate user defined key to input the data for a station pair.

Example using 9940W. (Note: The notation <9>, for example, means that you must press the gold shift key and then the 9 key.)

See
1.
2. ID?
3. NEXT OPTION?

4c. Delete Data from Extended Memory - XD
XEQ"XD" or press the appropriate user defined key to delete the specific station pair data from the extended memory module.

Example using 9940W.

## See

Key in
Press
1.

XEQ "XD"
2. ID?
$\langle 9\rangle\langle 9\rangle\langle 4\rangle\langle 0\rangle W$
$R / S$
3. NEXT OPTION?

The message $F L$ NOT FOUND will be displayed if the file you wish to delete in not in the extended memory.

Note: The extended functions/ memory module will
accommodate the data for 11 station pairs. The extended memory module will accommodate the data for an additional 22 station pairs.

5a. Store Data in the Tape_Cassette - TS
With the tape cassette attached to the $\mathrm{HP}-41 \mathrm{CV}$, station data (in R16 - R25), which has been input using the MI, $C R, X R$ or TR routine, can be transferred to the tape using the XEQ"TS" command or by pressing the appropriate user defined key.

Example using 9940W.
See
Key in
Press
1.

XEQ"TS"
2. NEXT OPTION?

Should the station pair already be in extended memory, the message DUP FL NAME (duplicate file name) will be displayed. If needed, the duplicate file may be erased using the $T D$ routine.

To proceed with the remaining examples it is recommended that you also store the $9940 Y$ station pair.

5b. Recald Data from the Tape Cassette -TB
With the tape cassette attached, $X E Q$ "TR" or press the appropriate user defined key to input the data for a station pair.

Example using 9940w. (Note: The notation <9>, for example, means that you must press the gold shift key and then the 9 key.)

See
1.
2. ID?
3. NEXT OPMION?

5c. Delete Data from the Tape Cassette - TD
XEQ"TD" or press the appropriate user defined key to delete the specific station pair data from the tape cassette. Example using 9940W.

See
1.
2. ID?
3. NEXT OPTION?

The message FL NOT FOUND will be displayed if the file to be deleted is not on the tape.

## 6. Loran-C Eixing Routines EI and AS

Given the indicated time delay (ITD) from two station pairs which form a triplet, a Loran-C fix is obtained.

Example: Load 9940 W and 9940 Y into the calculator using the MI, CR, XR or TR routine. The ITD on 9940 W is 16019 microseconds and the ITD of $9940 Y$ is 42585 microseconds. Where are you?

See
1.
2. ITD: 9940 W
3. ITD: 9940 Y
4. LAT: 39.1419
5. LON: 115.5052

Key in
XEQ"FI"
16019
R/S
42585
R/S
R/S

R/S
6. NEXT OPTION?

Since you are on a boat, you know that you cannot be in central Nevada at $39 \mathrm{dl}^{\prime \prime} \mathbf{1 9}^{\prime \prime}$ North and 115d50'52" West. Every Loran-C fix has two solutions, so in this case you must use the alternate solution.

## See

7. 
8. LAT: 35.0001
9. LON: 125.0009

Key in
XEQ"AS"
10. NEXT OPTION?

This is the proper solution at almost exactly 35 degrees North and 125 degrees West. Note that annunciator 3 (Flag 3) shows in the display indicating the alternate solution. If you should now repeat from Step 1 , you will obtain the proper
solution immediately.
The message "E: NO TRIPLET" will appear following Step 1 if the data do not comprise a valid triplet. The latitudes and longitudes of each station pair at the vertex must agree exactly. Should this error occur, use the ED routine to review the resident station data.

The message "E: ITD ERROR" will appear following Step 2 or 3 indicating that the ITD you keyed in is inconsistent with the station parameters. Press $\mathrm{R} / \mathrm{S}$ to be requeried for the ITD.

## 7a. Distance and Heading Routines DN and HD

If you know the latitude and longitude of your destination, you may key these in and then see how far your fix is from your destination and what the geodesic heading (similiar to great circle heading) is to your destination.

Example: Your destination is Moss Landing at about $36 \mathrm{~d} 48^{\prime} \mathrm{N}$ and $121 d 47^{\prime} W$. Your current fix is 35 dN and 125 dW (see the FI-SA example). First, key in your destination.
See
Key in
Press
1.
2. DEST LAT?
3. DEST LON?

XEQ"DN"
4. NEXT OPTION?

R/S
The destination is now stored in the calculator and will remain unchanged until you use either the DN or DH options. Also, the latest fix is stored and will remain unchanged until you use either the FI, AS or DH options. Now, find the distance and bearing from the latest fix (see the FI-AS example) to Moss Landing.

## See

Key in
Press
1.

XEQ"HD"
2. N.MI: 190.38

R/S
3. BRG: 54.3411

R/S
4. NEXT OPTION?

R/S
So the distance to Moss Landing is 190.38 nautical miles at a heading of 54d34'11".

## 7b. Distance and Heading Routine - DH

Given the latitude and longitude of an origin and destination, this routine will find the distance and heading from one to the other.

Example: How far, and in what direction, is Corvallis, Oregon (44d34'N, l23dl6'W) from Cupertino, California (37di9N, $122 \mathrm{de} 2^{\prime} \mathrm{W}$ )?

## See

1. 
2. ORIG LAT?
3. ORIG LON?
4. DEST LAT?
5. DEST LON?
6. N.MI: 438.32
7. BRG: 353.0259

Key in
$X E Q^{n} D H^{\prime \prime}$
37.19
122.02

R/S
44.34
123.16

R/S
8. NEXT OPTION?

Thus the distance is 438.32 nautical miles and the direction of Corvallis from Cupertino is 353d02'59".

## 8. ITD Prediction Routine - PR

As an aid to identification, this routine will allow the user to determine what ITD's should be received at a given location.

Example: Suppose that you know you are somewhere near latitude 35 North and longitude 125 West but are not sure what ITD's you should be receiving from 9940 W and 9940 Y . To determine these ITD's, proceed as follows:

See
1.
2. LAT?
3. LON?
4. $9940 \mathrm{~W}: 16019.35$
5. 9940Y: 42584.71
6. NEXT OPTION?

You should expect to receive an ITD of 16019.35 from 9940 W and an ITD of 42584.71 from 9940Y.

## 9. Calibration Routine - $C A$

This routine will allow the user to calibrate the Loran data in the calculator to a known position when the indicated time delay (ITD) is known for each station pair.

Example: Suppose you are receiving an ITD of 16308 from 9940W and 42800 from 9940 y . These ITD's would tell you that your location is 36 d $^{\prime \prime} 55^{\prime \prime} \mathrm{N}$ and l2ld47'll "W. However, you know that your position is bench marked to be at 36d47'36"N and l2ld46'58"W, and you wish to calibrate your calculator so that the ITD's of 16308 and 42800 will give you the latter fix instead of the former. Proceed as follows:

## See

1. 
2. LAT?
3. LON?
4. ITD 9940W:
5. ITD 9940Y:

Key in
XEQ"CA"
36.4736

R/S
121.4658

R/S
R/S
6. NEXT OPTION?

Entering 16308 and 42800 into the FI routine will now give you a fix at $36 \mathrm{~d} 47^{\prime} 3^{\prime \prime} \mathrm{N}$ and 12ld46'59 ${ }^{\mathrm{N}} \mathrm{W}$. The small discrepancy between this fix and the bench mark is due to assumptions made in the fixing algorithm.

Calibration is achieved by modifying the Master/Slave baseline ( $B L$ in the $C E$ and $E D$ routines). See Section $F$.

## 10. Switch Data Registers Routine - SW

The SW utility allows the user to swap the station data stored in R16 - R25 with the data stored in R26 - R35. Whenever the $M I, X R$ or $T R$ routine is used, the $S W$ routine is invoked prior to the loading of the data; the incoming data are then placed in R16 - R25. The consequence is that the first station pair data reside in $R 26$ - R35 and the second station pair data reside in R16 - R25.

One user application of $S W$ would be to change the order of the station $I D$ query in the $F I$ routine (this also affects the order of determination of the solution and alternate solution). Another user application would be to output both resident station pairs to card, extended memory or tape cassette using the CS, XS or $T S$ routines, respectively. To accomplish this, first use the CS, XS, or TS routine; then XEQ"SW" (note that the ID of the station data in R16 - R25 appears in the display instead of the NEXT OPTION? prompt): and finally use the CS, XS or TS routine once more.

## 1la. Recording the Loran-C Program onto Magnetic_Cards

(1) Attach the card reader to the HP-41CV.
(2) Place the calculator in the USER mode.
(3) Press the PRGM key to place the calculator in the program mode.
(4) Pass one side of a blank magnetic card through the card reader. Then follow the display prompts until nine program cards (17 tracks) have been recorded.
(5) Press the PRGM key once more to leave the program mode.
(6) To record a status card, XEQ"WSTS". Then pass a blank card through the card reader following the display prompts.

11b. Recording the Loran-C Program onto Magnetic Tape
(1) Attach the tape cassette to the HP-4lCV.
(2) Place the calculator in the USER mode.
(3) Press the alpha key, key in the word LORANC, press the alpha key once more, and then XEQ"WRTP".
(4) To record the program status, press the alpha key, key in the word STATUS, press the alpha key once more, and then XEQ"WRTS".
12a. Loading the Loran-C Program from Magnetic Cards(1) Attach the card reader to the HP-41CV.(2) Clear program memory: Turn the calculator off, then,while pressing the left arrow (erase) key down, turnthe calculator on.
(3) Place the calculator in the USER mode.
(4) Read in the STATUS card. The status card will set thecalculator to SIZE 42.
(5) Read in the nine program cards (17 tracks).
12b. Loading the Loran-C program from Tape Cassette
(1) Attach the tape cassette to the HP-41CV.
(2) Clear program memory. (See Step 2 above).
(3) Place the calculator in the USER mode.
(4) Press the alpha key, key in the word STATUS, press thealpha key once more, and then XEQ"READS". The statusfile will set the calculator to SIZE 42.(5) press the alpha key, key in the word LORANC, press thealpha key once more, and then XEQ"READP".

## E. The Loran-C Fixing Algorithm

The principles of Loran lines of position (LOP's) and fixing are adequately covered in Reference 1 and will not be repeated here.

The basic Loran-C equation [Ref. 4] can be written as

$$
\begin{equation*}
\operatorname{ITD}=\left[T_{S}+p\left(T_{S}\right)\right]-\left[T_{M}+p\left(T_{M}\right)\right]+\left[T_{B}+p\left(T_{B}\right)\right]+\delta \tag{1}
\end{equation*}
$$

where
ITD is the "indicated time difference" in microseconds,
$\mathrm{T}_{\mathrm{M}}$ is the distance, in microseconds, from the master to the receiver,
$T_{S}$ is the distance, in microseconds, from the slave to the receiver,
$T_{B}$ is the distance, in microseconds, between the master and the slave,
$\delta$ is the assigned station pair coding delay, in microseconds, and
$p(T)$ is the secondary phase correction, in microseconds, for a surface seawater path of length $T$.

The quantity

$$
\Delta t=T_{B}+p\left(T_{B}\right)+\delta
$$

is a constant for each master/slave pair. The quantity $T_{B}$ is computed from the positions of the master and slave using the reverse solution algorithm (Section H) at the time of manual data input (Routine MI).

The following World Geodetic System 1972 (WGS 72) values have been adopted for Loran-C navigation [Ref. 4]:

$$
\begin{aligned}
& v_{0}= 299792458 \text { meters/second is the velocity of light } \\
& \text { in free space, } \\
& \eta= 1.000338 \text { is the index of refraction of the } \\
& \text { surface of the earth for standard atmosphere } \\
& \text { and } 100 \mathrm{kHz} \text { electromagnetic waves, } \\
& a_{e}=6378135.000 \text { meters is the equatorial radius of } \\
& \text { the earth }
\end{aligned}
$$

and
$f=1 / 298.26$ is the flattening factor (l-b/ae, where
$b$ is the polar radius) of the earth.

Accurate formulas for computing the secondary phase correction $p(T)$ are contained in Reference 4 , but for use in the HP-41CV, the form

$$
\begin{equation*}
p(T)=a_{0} / T+a_{1}+a_{2} T \tag{2}
\end{equation*}
$$

is used, where $T$ is in microseconds and

1. For $T \geq 537 \mu \mathrm{sec}$ :
$a_{0}=129$,
$a_{1}=-0.408$, and
$a_{2}=0.0006458$.
2. For $T<537 \mu \mathrm{sec}:$
$a_{0}=2.74$,
$a_{1}=-0.011$, and
$a_{2}=0.00033$.

On the surface of a sphere, a hyperbolic line of position (LOP) can be represented by the equation [Ref. 1, page 175]

$$
\begin{equation*}
\tan r=\frac{\cos 2 a-\cos 2 c}{\sin 2 c \cos \omega+\zeta \sin 2 a} \tag{3}
\end{equation*}
$$

where the origin of the coordinate system is at the prime focus of the spherical hyperbola, $2 c$ is the spherical arc joining the foci, 2 a is a constant for any one hyperbola, and $r$ and $\omega$ are the spherical coordinates of a point on the hyperbola. If the base line of the coordinate system is the arc joining the foci the $\omega$ is the spherical polar angle from the baseline to a point $P$ on the spherical hyperbola and $r$ is spherical polar distance (or arc) from the prime focus to $P$. Using the Loran system we take $\zeta=+1$ if the prime focus is at a master station and $\zeta=-1$ if the prime focus is at a slave station.

If we let $v=v_{0} / \eta$ be the velocity of 100 kHz electromagnetic radiation at the earth's surface then, for a spherical earth, we can relate the parameters in Equations 1 and 3 as follows:

$$
2 c=v T_{B} / a_{e}
$$

and

$$
2 a=v\left(T_{S}-T_{M}\right) / a_{e}
$$

Using the spherical approximation for now, we see that 2 c is known for any Loran pair. The "indicated time delay" ITD is measured by the receiver at point $P$, and to determine $a$ hyperbolic line of position we must determine 2 a , but $\mathrm{T}_{\mathrm{S}}-\mathrm{T}_{\mathrm{M}}$ cannot be computed from Equations 1 and 2. If a were zero
in Equation 2 , then it would be possible to determine $T_{S}-T_{M}$ uniquely. As an approximation we use the following parameters in Equation 2:
and

$$
\begin{aligned}
& a_{0}=0 \\
& a_{1}=-0.321 \\
& a_{2}=0.000635
\end{aligned}
$$

These values have been obtained by setting $a_{0}=0$ and determining $a_{1}$ and $a_{2}$ by linear regression of the $T>537$ values over the interval of $1000<T<8000$. This approximation is quite good (within $0.03 \mu \mathrm{~s}$ ) for distances up to $10,000 \mathrm{microseconds}$ where small changes in the LOP's can cause large position errors. D.t short distances the error increases from $0.05 \mu s$ at $1000 \mu s$ to $0.58 \mu s$ at $10 \mu s ;$ although these errors are large for small distances, the LOP's are not as sensitive to these changes as they would be at large distances. When this approximation is substituted into Equation 1, we obtain

$$
\left[T_{S}+a_{1}+a_{2} T_{S}\right]-\left[T_{M}+a_{1}+a_{2} T_{M}\right]=I T D-\Delta t
$$

or

$$
\begin{equation*}
T_{S}-T_{M}=(I T D-\Delta t) /\left(1+a_{2}\right) \tag{4}
\end{equation*}
$$

and hence $2 a=v\left(T_{S}-T_{M}\right) / a_{e}$ is determined for use in the spherical approximation.

Consider a Loran-C triplet with the master stations colocated. Let $\xi_{1}$ and $\xi_{2}$ denote the azimuth angles of slave $1\left(S_{1}\right)$ and slave $2\left(S_{2}\right)$, respectively, measured from North toward the East from the master stations (M) (see Figure 3).

Further, let $\alpha$ and $r$ be the azimuth and spherical polar arc (distance) of the receiver (R) from $M$. For this geometry, Equation 3 can be written in the form

$$
\begin{equation*}
\tan r_{i}=\frac{B_{i}}{C_{i} \cos \left(\alpha-\xi_{i}\right)+A_{i}}, \tag{5}
\end{equation*}
$$

where

$$
\begin{aligned}
& A_{i}=\zeta_{i} \sin 2 a_{i} \\
& B_{i}=\cos 2 a_{i}-\cos 2 c_{i}
\end{aligned}
$$

and

$$
c_{i}=\sin 2 c_{i}
$$

for the $i^{\text {th }}$ Loran pair, $i=1,2$. Since $r_{1}=r_{2}=r$, we can eliminate tan $r$ between the two equations. The resulting equation can be rewritten as

$$
\begin{equation*}
C \cos \alpha+S \sin \alpha=K \tag{6}
\end{equation*}
$$

where

$$
\begin{aligned}
& C=B_{1} C_{2} \cos \xi_{2}-B_{2} C_{1} \cos \xi_{1}, \\
& S=B_{1} C_{2} \sin \xi_{2}-B_{2} C_{1} \sin \xi_{1},
\end{aligned}
$$

and

$$
K=B_{2} A_{1}-B_{1} A_{2}
$$

If we define $\rho>0$ and $\gamma$ by the equations

$$
\begin{equation*}
\rho \cos \gamma=C \tag{7}
\end{equation*}
$$

and $\quad \rho \sin \gamma=S$,
then

$$
\rho=\sqrt{c^{2}+s^{2}}
$$

and

$$
\gamma=\operatorname{qatn}(S, C)
$$



Figure 3. Geometry of a Loran Triplet and a Receiver.

Here the function $q a t n(y, x)$ is the arctangent of $y / x$ adjusted for the proper quadrant according to the signs of $x$ and $y$. A compact form of this function is

$$
\operatorname{qatn}(y, x)=\tan ^{-1} \frac{y}{x+10^{-9} t(x=0 ?)}+\pi t(x<0 ?)
$$

where

$$
t(z)=1
$$

when $z$ is true
and

$$
t(z)=0
$$

$$
\text { when } z \text { is false. }
$$

When convenient we will use the notation qatn ( $y / x$ ) interchangeably with qatn $(y, x)$. Now we can substitute Eq. (7) into Eq. (6) and solve for

$$
\begin{equation*}
\alpha=\gamma \pm \cos ^{-1}(K / \rho) \tag{8}
\end{equation*}
$$

to obtain the azimuth angle $\alpha$ of the two points of intersection of the spherical hyperbolic LOP's. Finally we can obtain a value for $r$ by substituting each $\alpha$ into Eq. (5) for either $i=1$ or $i=2$. We find that

$$
\begin{equation*}
r=\operatorname{qatn}\left[\frac{B_{i}}{C_{i} \cos \left(\alpha-\xi_{i}\right)+\bar{A}_{i}}\right] \text { for } i=1 \text { or } 2 \tag{9}
\end{equation*}
$$

The distance and azimuth from $M$ or the triplet vertex can be converted into the latitude and longitude of the two possible positions of $r$.

The fixing algorithm then uses $\alpha$ and $r$ in the direct solution algorithm of spheroidal geodesy (Section G).

## ־. The Calibration and ITD Prediction Algorithms

Calibration can be achieved when an ITD is measured at a known bench marked position. From the bench marked position and the known master and slave positions, the quantities $T_{M}+p\left(T_{M}\right)$ and $T_{S}+p\left(T_{S}\right)$ can be computed using the reverse solution algorithm (Section $H$ ) and the accurate secondary phase correction formula (Eq. 2). Equation 1 can then be solved for $T_{B}+P\left(T_{B}\right)$ to obtain a modified baseline. This modified baseline is stored and then used instead of the true baseline in subsequent computations. The affect on the accuracy of fixes using this modified baseline with positions far removed from the bench mark has not studied.

The ITD prediction algorithm is a direct application of Equation 1. A known position, together with the known master and slave positions, is used to compute the quantities $T_{M}+p\left(T_{M}\right)$ and $T_{S}+p\left(T_{S}\right)$. When these values, along with a computed or calibrated baseline, $T_{B}+P\left(T_{B}\right)$, are substituted into Equation 1 , a predicted ITD is obtained.

## G. The Direct Solution Algorithm

This direct solution algorithm is a modification of the second order in flattening (f) algorithm given by Thomas
[Ref. 5, pp. 7-8]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qatn function is defined in a previous section. East longitudes and South latitudes are negative. We are given the point $P_{1}\left(\phi_{1}, \lambda_{1}\right)$ on the spheroid, where $\phi_{1}, \lambda_{1}$ are the geodetic latitude and longitude (geographic coordinates); the forward azimuth $\alpha_{12}$ and distance $S$ to a second point $P_{2}\left(\phi_{2}, \lambda_{2}\right)$; and from these we are to find the geographic coordinates $\phi_{2}, \lambda_{2}$ and the back azimuth $\alpha_{21}$. The given quantities are $\phi_{1}, \lambda_{1}, \alpha_{12}$ and $S$. No assumptions about the relative location of $P_{1}$ and $P_{2}$ are required. The modified direct solution algorithm is:

$$
\begin{aligned}
\theta_{1} & =\tan ^{-1}\left[(1-f) \tan \theta_{1}\right], M=-\sin \alpha_{12} \cos \theta_{1}, \\
C_{1} & =f M, C_{2}=f\left(1-M^{2}\right) / 4, \\
D & =\left(1-C_{2}\right)\left(1-C_{2}-C_{1} M\right), P=C_{2}\left[1+(1 / 2) C_{1} M\right] / D, \\
N & =\cos \theta_{1} \cos \alpha_{12}, \sigma_{1}=q \operatorname{atn}\left(N, \sin \theta_{1}\right), \\
d & =S /\left(a_{e} D\right), u=2\left(\sigma_{1}-d\right), W=1-2 P \cos u, \\
V & =\cos (u+d), X=C_{2}^{2} \sin d \cos d\left(2 V^{2}-1\right), \\
Y & =2 P V W \sin d, \Delta \sigma=d+X-Y, \\
K & =\left[M^{2}+\left(N \cos -\Delta \sigma \sin \theta_{1} \sin \Delta \sigma\right)^{2}\right]^{1 / 2}, \\
\theta_{2} & =\tan ^{-1}\left[\left(\sin \theta_{1} \cos \Delta \sigma+N \sin \Delta \sigma\right) / K\right], \\
\Delta n & =q \operatorname{qatn}\left(-\sin \Delta \sigma \sin \alpha_{12}, \cos \theta_{1} \cos \Delta \sigma-\sin \theta_{1} \sin \Delta \sigma \cos \alpha_{12}\right), \\
H & =C_{1}\left(1-C_{2}\right) \Delta \sigma-C_{1} C_{2} \sin \Delta \sigma \cos \left(2 \sigma_{1}-\Delta \sigma\right), \\
\lambda_{2} & =\lambda_{1}+\Delta n-H,
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{21} & =\operatorname{qatn}\left[-M,-\left(N \cos \Delta \sigma-\sin \theta_{1} \sin \Delta \sigma\right)\right] \\
\phi_{2} & =\tan ^{-1}\left[\tan \theta_{2} /(1-f)\right]
\end{aligned}
$$

Details of the modifications made to Thomas' algorithm are contained in Reference 3. The algorithm above has been further modified so that Eastern longitudes, rather than Western longitudes, are negative.

## H. The Reverse Solution Algorithm

This reverse solution algorithm is a modification of the second order in flattening (f) algorithm given by Thomas [Ref. 5, pp. 8-10]. Thomas' notation has been followed as closely as possible for ease of comparison of the algorithms. The qatn function is defined in a previous section. East longitudes ( $\lambda$ ) and South latitudes ( $\phi$ ) are negative. We are given the points $P_{1}\left(\phi_{1}, \lambda_{1}\right), P_{2}\left(\phi_{2}, \lambda_{2}\right)$ on the spheroid and are to find the distance $S$ between the points and the forward and back azimuths, $\alpha_{12}$ and $\alpha_{21}$. Given quantities are $\phi_{1}, \lambda_{1}, \phi_{2}$ and $\lambda_{2}$. No assumptions about the relative location of $P_{1}$ and $P_{2}$ are required. The modified reverse solution algorithm is:

$$
\begin{aligned}
\theta_{i} & =\tan ^{-1}\left[(1-f) \tan \phi_{i}\right], i=1,2, \\
\Delta \lambda & =\lambda_{2}-\lambda_{1}, \Delta \theta_{m}=\left(\theta_{2}-\theta_{1}\right) / 2, \theta_{m}=\left(\theta_{1}+\theta_{2}\right) / 2, \\
H & =\cos ^{2} \Delta \theta_{m}-\sin ^{2} \theta_{m}, L=\sin ^{2} \Delta \theta_{m}+H \sin ^{2}(\Delta \lambda / 2), \\
d & =2 \sin ^{-1} \sqrt{L}, U=2 \sin ^{2} \theta_{m} \cos ^{2} \Delta \theta_{m} /(1-L), \\
V & =2 \sin ^{2} \Delta \theta_{m} \cos ^{2} \theta_{m} / L, X=U+V, Y=U-V, \\
T & =d / \sin d, D=4 T^{2}, E=2 \cos d, A=D E, \\
C & =T-(A-E) / 2, n_{1}=X(A+C X), \\
B & =2 D, n_{2}=Y(B+E Y), n_{3}=D X Y, \\
\delta_{2} d & =f^{2}\left(n_{1}-n_{2}+n_{3}\right) / 64, \delta_{1} d=f(T X-Y) / 4, \\
S / a & =\left(T-\delta_{1} d+\delta_{2} d\right) \sin d, M=32 T-(20 T-A) X-(B+4) Y, \\
F & =2 Y-E(4-X), G=f T / 2+f^{2} M / 64, Q=-(F G \tan \Delta \lambda) / 4, \\
\Delta \lambda_{m}^{\prime} & =(\Delta \lambda+Q) / 2,
\end{aligned}
$$

$$
\begin{aligned}
t_{1} & =\text { qatn }\left(\sin \Delta \theta_{m} \cos \Delta \lambda_{m}^{\prime}, \cos \theta_{m} \sin \Delta \lambda_{m}^{\prime}\right) \\
t_{2} & =\text { qatn }\left(-\cos \Delta \theta_{m} \cos \Delta \lambda_{m}^{\prime}, \sin \theta_{m} \sin \Delta \lambda_{m}^{\prime}\right) \\
\alpha_{12} & =t_{1}+t_{2}, \alpha_{21}=t_{1}-t_{2}
\end{aligned}
$$

Details of the modifications made to Thomas' algorithm are contained in Reference 3. The algorithm above has been further modified so that Eastern longitudes, rather than Western longitudes, are negative.

## I. Program_Accuracy

The direct and reverse solution algorithms are equivalent to the second order flattening algorithms given by Thomas (Ref. 5); the parameters of the WGS 1972 spheroid are used. The reverse solution algorithm reproduces the baselines provided by the Defense Mapping Agency for all 40 Loran-C stations to within 0.15 meters (the average deviation is -0.031 meters, DMA minus HP-41CV, with a standard deviation of 0.037 meters) and to within 0.01 microseconds, including the secondary phase correction for an all seawater path. The reverse solution algorithm is also used to generate predicted ITD's; these are presumed to be within 0.01 microseconds, also.

The fixing algorithm uses the direct solution algorithm with the azimuth and distance of the fix from the vertex of the Loran-C triplet computed from Equations 8 and 9 as inputs. Equations 8 and 9 are based upon spherical geometry and include an approximation to the secondary phase correction for an all seawater path. The largest source of error is the assumption that the azimuth and distance to the fix are accurately represented by this spherical approximation. This approximation has not been rigorously tested, however it is possible to use the reverse solution algorithm to predict the ITD's that will be received at a given position and then to enter these ITD's into the fixing algorithm to determine how accurately the fixing algorithm reproduces the original position. The distance between the fix and the original position can be determined using the HD algorithm. Tables 1,2 and 3 were producted in this manner. Similiar tables with different station
pairs are given in References 3, 6 and 7. It is felt that the results are accurate enough to not warrant the inclusion of an iterative improvement routine. Of the samples in Tables 1,2 and 3, the largest error, 0.22 n.mi., is the first entry in Table 1. From the chart LCNC-2, it estimated that the angle of intersection of the two hyperbolic lines of position is about 5 degrees and so an error of 0.22 n.mi. should not be unexpected.

Table 1. Colocated Master Stations

| Position |  | Predicted ITD's |  | HP-41CV Fix |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lat | Long | 9940 W | 9940X | Lat (N) | Long (W) | n.mi |
| $31^{\circ}$ | $123^{\circ}$ | 16413.28 | 27570.93 | $30^{\circ} 59^{\prime} 47^{\prime \prime}$ | $123^{\circ} 00^{\prime} 03^{\prime \prime}$ | 0.22 |
| $37^{\circ}$ | $126^{\circ}$ | 15610.11 | 27020.50 | $36^{\circ} 59^{\prime} 53^{\prime \prime}$ | $126^{\circ} 00^{\prime} 09^{\prime \prime}$ | 0.17 |
| $42^{\circ}$ | $129^{\circ}$ | 13881.78 | 27285.58 | $42^{\circ} 00^{\prime} 00^{\prime \prime}$ | $129^{\circ} 00^{\prime} 00^{\prime \prime}$ | 0 |
| $44^{\circ}$ | $132^{\circ}$ | 13180.89 | 27371.19 | $44^{\circ} 00^{\prime} 00^{\prime \prime}$ | $132^{\circ} 00^{\prime} 01^{\prime \prime}$ | 0.01 |
| $48^{\circ}$ | $135^{\circ}$ | 12301.25 | 27552.06 | $48^{\circ} 00^{\prime} 01^{\prime \prime}$ | $135^{\circ} 00^{\prime} 03^{\prime \prime}$ | 0.03 |
| $50^{\circ}$ | $138^{\circ}$ | 12068.67 | 27584.22 | $50^{\circ} 00^{\prime} 01^{\prime \prime}$ | $138^{\circ} 00^{\prime} 04^{\prime \prime}$ | 0.05 |

Table 2. Colocated Slave Stations

| Position |  | Predicted ITD's |  | HP-41CV Fix |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lat | Long | 9940W | 5990Y | Lat (N) | Long (W) | n.mi |
| $31^{\circ}$ | $123^{\circ}$ | 16413.28 | 27177.18 | $31^{\circ} 00^{\prime} 03^{\prime \prime}$ | $122^{\circ} 59^{\prime} 58^{\prime \prime}$ | 0.06 |
| $37^{\circ}$ | $126^{\circ}$ | 15610.11 | 27403.20 | $37^{\circ} 00^{\prime} 01^{\prime \prime}$ | $125^{\circ} 59^{\prime} 59^{\prime \prime}$ | 0.01 |
| $42^{\circ}$ | $129^{\circ}$ | 13881.78 | 27955.45 | $42^{\circ} 00^{\prime} 00^{\prime \prime}$ | $129^{\circ} 00^{\prime} 00^{\prime \prime}$ | 0 |
| $44^{\circ}$ | $132^{\circ}$ | 13180.89 | 28512.90 | $44^{\circ} 00^{\prime} 00^{\prime \prime}$ | $132^{\circ} 00^{\prime} 00^{\prime \prime}$ | 0 |
| $48^{\circ}$ | $135^{\circ}$ | 12301.25 | 29413.61 | $48^{\circ} 00^{\prime} 00^{\prime \prime}$ | $134^{\circ} 59^{\prime} 59^{\prime \prime}$ | 0.01 |
| $50^{\circ}$ | $138^{\circ}$ | 12068.67 | 29816. 84 | $50^{\circ} 00^{\prime} 70^{\prime \prime}$ | $137^{\circ} 59^{\prime} 59^{\prime \prime}$ | 0.02 |

Table 3. Colocated Master and Slave Stations

| Position |  | Predicted ITD's |  | HP-41CV Fix |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Lat | Long | 5930Y | 9960W | Lat (N) | Long (W) | n.mi |
| $44^{\circ}$ | $63^{\circ}$ | 29864.46 | 11685.15 | $44^{\circ} 00^{\prime} 00^{\prime \prime}$ | $63^{\circ} 00^{\prime} 00^{\prime \prime}$ | 0 |
| $41^{\circ}$ | $66^{\circ}$ | 30585.61 | 12946.91 | $41^{\circ} 00^{\prime} 00^{\prime \prime}$ | $66^{\circ} 00^{\prime} 00^{\prime \prime}$ | 0 |
| $39^{\circ}$ | $69^{\circ}$ | 31020.46 | 14111.31 | $39^{\circ} 00^{\prime} 00^{\prime \prime}$ | $69^{\circ} 00^{\prime} 00^{\prime \prime}$ | 0 |
| $35^{\circ}$ | $72^{\circ}$ | 31064.57 | 15139.48 | $35^{\circ} 00^{\prime} 00^{\prime \prime}$ | $72^{\circ} 00^{\prime} 00^{\prime \prime}$ | 0 |
| $30^{\circ}$ | $75^{\circ}$ | 31040.82 | 15610.46 | 2959'59' | $75^{\circ} 00^{\prime} 01^{\prime \prime}$ | 0.02 |
| $26^{\circ}$ | $78^{\circ}$ | 31106.20 | 15858.46 | $25^{\circ} 59^{\prime} 57^{\prime \prime}$ | $78^{\circ} 00^{\prime} 02^{\prime \prime}$ | 0.05 |

## J. References

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5. Paul D. Thomas, "Spheroidal Geodesics, Reference Systems, and Local Geometry", PS-138, U.S. Naval Oceanographic Office, Washington, D.C., January 1970.
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## Appendix A: Program Storage Allocations, Flag Usage and Program Listing

## Program Size: 42

## Registers:

R00 - Rl3: Scratch storage
Rl4: Flattening

| 2nd Sta. Fair | Variable | 1st Sta. Pair |
| :---: | :---: | :---: |
| R15: | 2a |  |
| R16: | ID | : R26 |
| R17: | $\mathrm{T}_{\mathrm{B}}+\mathrm{p}\left(\mathrm{T}_{\mathrm{B}}\right)$ | : R27 |
| R18: | 2c | : R28 |
| R19: | $C D$ | : R29 |
| R20: | $\theta$ master | : R30 |
| R21: | $\lambda$ master | :R31 |
| R22 : | $\alpha$ master-slave | :R32 |
| R23: | $\theta$ slave | :R33 |
| R24: | $\lambda$ slave | :R34 |
| R25: | $\alpha$ slave-master | :R35 |
|  | 2a | : P36 |
| R37: | $\theta \mathrm{fix}$ |  |
| R38: | $\lambda \mathrm{fix}$ |  |
| R39 : | $\alpha \mathrm{fix}$-dest |  |
| R40: | $\theta$ dest |  |
| R41: | $\lambda$ dest |  |

Flag Usage:
F'00: 1. Requery erroneous ITD at LBL06
2. Echo two data sets at LBL ED
F01: $\quad \mathrm{PR}$ and CA interlock
F03: Set for 2nd solution
F05: Set if slave at vertex of 2 nd station pair
F06: Set if slave at vertex of lst station pair
F07: Set for XMEM functions, clear for tape functions
F08: $\quad$ DN and DH interlock
F09: Set if DMS conversion required
F10: 1. SW and AS interlock
2. Input interlock in FI vertex check
F14: PR and CA loop control1




DIRECT SOLUTION
$\theta_{1,}, \lambda_{1}, \alpha_{1} 2$ and $S / a_{e}$ are in
the $T, Z, Y$ and $X$ registers

$$
\begin{gathered}
N \\
\underset{O}{E} \\
\underset{C}{c} \\
1
\end{gathered}
$$

$$
{ }_{r}^{r-1}
$$

$\square$ $\Sigma$
$4 \quad 0$

$$
e^{N}
$$

$\square$

かた
DIRECT SOLUTION
$\theta_{1}, \lambda_{1}, \alpha_{1} 2$ and $S / a_{e}$ are in
the $T, Z, Y$ and $X$ registers


0
0
0
0
0
$\Sigma \overrightarrow{C l}^{\boldsymbol{E}}$
$e^{N}$

| $\frac{6}{6}$ |  |
| :---: | :---: |
|  |  |
| 0 |  |
|  |  |
|  |  |
|  |  |










|  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  の日～NM＋ <br>  <br>  |  |  |  |  |  |  |  |  |







## Appendix B: STATION COVERAGE

| Station | No.of Pairs | Location |
| :--- | :--- | :--- |
| 4990 | 2 | Central Pacific |
| 5930 | 2 | East Coast, Canada |
| 5990 | 3 | West Coast, Canada |
| 7930 | 3 | North Atlantic |
| 7960 | 4 | Gulf of Alaska |
| 7970 | 4 | Norwegian Sea |
| 7980 | 3 | Mediterranean Sea |
| 7990 | 3 | Great Lakes |
| 8970 | 4 | Nest coast, U.S.A. |
| 9940 | 4 | Northeast U.S.A. |
| 9960 | 3 | North Pacific |
| 9970 |  |  |

The following list contains the pertinent parameters for each Loran-C station pair. This list was compiled from the data in Reference 4. Each column contains the following information:

1. The Loran-C station pair designator.
2. The coding delay.
3. The master station latitude.
4. The master station longitude.
5. The slave station latitude.
6. The slave station longitude.

In this list, positive longitudes are West, negative longitudes are East, positive latitudes are North and negative latitudes are South. In columns 3 through 6 the latitudes and longitudes appear to be in decimal form, but the actual format is DDD.MMSSFF (which is compatible with the HP-4lCV D.MS or H.MS input mode) where

```
DDD designates degrees,
    MM designates minutes,
    SS designates seconds, and
    FF designates hundredths of seconds.
```

| ID | 0 D | Ns |  | QS Lita | SS LON |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4990 X | 11000 | 10 | 二.4 - 20 | 20.144916 | 155.530970 |
| 4990 Y | 29000 | 16.44 | 169.33126 | 26.234177 | 178.173020 |
| 5930 X | $11000^{\circ}$ | 4 | 17 | 41.151143 | 069.583909 |
| 5930 Y | 25000 | $46.53-6$ | 067.55 .377 | 46.463218 | 053.102816 |
| 5990 X | 11000 | $51.574 \%$ | $\pm-2.220224$ | 55.262085 | 131.151965 |
| $5990 Y$ | 27000 | 51 | $\cdots 2.80 \cdot \ldots 4$ | 47.034799 | 119.443953 |
| 59902 | 41000 | 51 | $\therefore 2.22624$ | 50.362972 | 127.212935 |
| 7930 W | 11000 |  |  | 14.542658 | 023.552175 |
| 7930 X | 21000 | 69 |  | 62. 275968 | 007.042671 |
| 7930 z | 4300 C | - |  | 46.463218 | 053.102816 |
| 7960 X | 11006 | $\ell$ |  | 勺. 2 E2:21 | 152.221122 |
| 7960 Y | 26006 | 53.3 |  | 5.262085 | 131.151965 |
| 7970W | 36000 | 62: . ${ }^{\text {a }}$ | - 64 | -4.402980 | -008.173633 |
| 7970 X | 11000 | $\epsilon$ |  | 58.380615 | -014.274700 |
| $7970 Y$ | 46000 | t |  | 6.4.542658 | +023.552175 |
| 7970 Z | 60000 | 6.... $\%$ | + 20.4 | 70.545261 | +008.435869 |
| 7980 W | 11090 | $3 . \quad \cdots$ |  | 36.435302 | 090.494360 |
| 7980X | 23000 | 3 |  | $\therefore 5.315501$ | 097.500009 |
| 7980 Y | 43000 | 30 | 095. 20.930 | $\because 7.015849$ | 080.065352 |
| 7980 Z | 59000 | 3 mb | C8:. 019930 | 34.034604 | 077.544676 |
| 7990 X | 11000 | 38.522 | -016.430596 | 35.312088 | -012.312996 |
| 7990 Y | 29000 | 35. $322^{\prime \prime}$ | -626.430096 | 40.582095 | -027.520152 |
| 7990 Z | 47000 |  | -016.429596 | 42.033649 | -003.121590 |
| 8970 W | 11000 | 39.5:-54 | 987.291214 | 30.593874 | 085.100930 |
| 8970 X | 28000 | $30.50-4$ | 887.291214 | 42.425060 | 076.493386 |
| 8970 Y | 44000 | 4 | 187.231214 | 48.364984 | 094.331847 |
| 9940W | 11000 | 35. | 158.40637 | 47.034799 | 119.443953 |
| 9940 X | 270000 | 39.32x53 | 2-49563\% | 38.465699 | 122.294453 |
| 9940 Y | 40000 | $39.3966=$ | 118.495637 | 35.191818 | 114.481743 |
| 9960W | 11000 | 42.42:06.9 | ?76.493386 | 46.482720 | 067.553771 |
| 9960 X | 25000 | 42.42560 | 47,6,493386 | 41.151193 | 069.583909 |
| $9960 Y$ | 39000 | 42.1256.0 | 876.493386 | 34.034604 | 077.544676 |
| 9960 z | 54000 | 42 | $\therefore-49586$ | 39.510754 | 087.291214 |
| 9970W | 11000 | 24.48 | -15i.15208 | -4.37077 | -153.58515 |
| 9970 X | 30000 | 24.460 | 41.9.40 | $\because .443710$ | -143.430906 |
| 9970 Y | 55000 | 24.454 | - $\therefore 1.10090$ | 26.362499 | -128.085621 |
| 99702 | 75000 | $24.46{ }^{\text {a }}$ | -14?.19290 | 0.324566 | -138.895523 |
| 9990 X | 11000 | 57.090206 | +170.14593i | 52.494505 | -173.105231 |
| 9990 Y | 29000 | $57.00^{\circ} \%$ | +29.1430: | 65.141912 | +166.531447 |
| 9990 Z | 43000 | 57.090988 | +.70.145981 | 57.262021 | +152.221122 |

[^0]
## Appendix D; COLOCATED STATIONS

| 5930 MASTER -9960 W |  |
| :--- | :--- |
| 5930 X | -9960 X |
| 5930 Y | -7930 Z |
| 5990 X | -7960 Y |
| 5990 Y | -9940 W |
| 7930 W | -7970 Y |
| 7930 X | -7970 MASTER |
| 7960 X | -9990 Z |
| 7980 Z | -9960 Y |
| 7980 MASTER -8970 W |  |
| 8970 X | -9960 MASTER |
| 8970 MASTER -9960 Z |  |

Slave stations are denoted with a letter suffix. Master stations are so designated.
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