READINGS AND LEARNING OUTCOME STATEMENTS

READINGS

The following material is a review of the Alternative Asset Valuation and Fixed Income principles designed to address the learning outcome statements set forth by CFA Institute.

STUDY SESSION 13

Reading Assignments

Alternative Asset Valuation and Fixed Income, CFA Program Curriculum, Volume 5, Level 2 (CFA Institute, 2011)

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Reading Assignments

Alternative Asset Valuation and Fixed Income, CFA Program Curriculum, Volume 5, Level 2 (CFA Institute, 2011)

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Reading Assignments

Alternative Asset Valuation and Fixed Income, CFA Program Curriculum, Volume 5, Level 2 (CFA Institute, 2011)

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Learning Outcome Statements (LOS)

The CFA Institute Learning Outcome Statements are listed below. These are repeated in each topic review; however, the order may have been changed in order to get a better fit with the flow of the review.

Study Session 13

The topical coverage corresponds with the following CFA Institute assigned reading:

47. Investment Analysis
The candidate should be able to:

a. illustrate, for each type of real property investment, the main value determinants, investment characteristics, principal risks, and most likely investors. (page 9)
b. evaluate a real estate investment using net present value (NPV) and internal rate of return (IRR) from the perspective of an equity investor. (page 18)
c. calculate the after-tax cash flow and the after-tax equity reversion from real estate properties. (page 14)
d. explain the potential problems associated with using IRR as a measurement tool in real estate investments. (page 20)

The topical coverage corresponds with the following CFA Institute assigned reading:

48. Income Property Analysis and Appraisal
The candidate should be able to:

a. explain the relation between a real estate capitalization rate and a discount rate. (page 27)
b. determine the capitalization rate by the market-extraction method, band-of-investment method, and built-up method, and justify each method's use in capitalization rate determination. (page 28)
c. estimate the market value of a real estate investment using the direct income capitalization approach and the gross income multiplier technique. (page 31)
d. contrast the limitations of the direct capitalization approach to those of the gross income multiplier technique. (page 32)

The topical coverage corresponds with the following CFA Institute assigned reading:

49. Private Equity Valuation
The candidate should be able to:

a. explain the sources of value creation in private equity. (page 41)
b. explain how private equity firms align their interests with those of the managers of portfolio companies. (page 42)
c. distinguish between the characteristics of buyout and venture capital investments. (page 43)
d. discuss the valuation issues in buyout and venture capital transactions. (page 47)
e. explain alternative exit routes in private equity and their impact on value. (page 51)
f. explain private equity fund structures, terms, valuation, and due diligence in the context of an analysis of private equity fund returns. (page 52)
g. explain the risks and costs of investing in private equity. (page 57)
h. interpret and compare financial performance of private equity funds from the perspective of an investor. (page 59)
i. calculate management fees, carried interest, net asset value, distributed to paid in (DPI), residual value to paid in (RVPI), and total value to paid in (TVPI) of a private equity fund. (page 62)

j. calculate pre-money valuation, post-money valuation, ownership fraction, and price per share applying the venture capital method 1) with single and multiple financing rounds and 2) in terms of IRR. (page 64)

k. demonstrate alternative methods to account for risk in venture capital. (page 69)

The topical coverage corresponds with the following CFA Institute assigned reading:

50. Investing in Commodities
The candidate should be able to:

a. explain why commodity futures such as gold have limited “contango,” whereas others such as oil often have natural “backwardation,” and indicate why these conditions might be less prevalent in the future. (page 87)

b. discuss how “roll yield” in a commodity futures position can be positive (negative). (page 89)

c. discuss the argument that commodity futures are not an asset class. (page 89)

d. demonstrate how the geometric return of an actively managed commodity basket can be positive, whereas the underlying average commodity has a geometric return near zero. (page 91)

e. discuss why investing in commodities offers diversification opportunities during periods of economic fluctuation in the short run and inflation in the long run. (page 92)

The topical coverage corresponds with the following CFA Institute assigned reading:

51. Evaluating the Performance of Your Hedge Funds
The candidate should be able to:

a. discuss how the characteristics of hedge funds affect traditional methods of performance measurements. (page 99)

b. compare and contrast the use of market indices, hedge fund indices, and positive risk-free rates to evaluate hedge fund performance. (page 100)

The topical coverage corresponds with the following CFA Institute assigned reading:

52. Buyers Beware: Evaluating and Managing the Many Facets of the Risks of Hedge Funds
The candidate should be able to:

a. discuss common types of investment risks for hedge funds. (page 109)

b. evaluate maximum drawdown and value-at-risk for measuring risks of hedge funds. (page 112)

The topical coverage corresponds with the following CFA Institute assigned reading:

53. General Principles of Credit Analysis
The candidate should be able to:

a. distinguish among default risk, credit spread risk, and downgrade risk. (page 122)

b. explain and analyze the key components of credit analysis. (page 123)

c. calculate and interpret the key financial ratios used by credit analysts. (page 126)

d. evaluate the credit quality of an issuer of a corporate bond, given such data as key financial ratios for the issuer and the industry. (page 126)

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c. analyze why and how cash flow from operations is used to assess the ability of an issuer to service its debt obligations and to assess the financial flexibility of a company. (page 129)
f. explain and interpret the typical elements of the corporate structure and debt structure of a high-yield issuer and the effect of these elements on the risk position of the lender. (page 131)
g. discuss the factors considered by rating agencies in rating asset-backed securities. (page 132)
h. explain how the credit worthiness of municipal bonds is assessed, and contrast the analysis of tax-backed debt with the analysis of revenue obligations. (page 134)
i. discuss the key considerations used by Standard & Poor's in assigning sovereign ratings, and describe why two ratings are assigned to each national government. (page 135)
j. contrast the credit analysis required for corporate bonds to that required for 1) asset-backed securities, 2) municipal securities, and 3) sovereign debt. (page 136)

The topical coverage corresponds with the following CFA Institute assigned reading:

54. The Liquidity Conundrum
The candidate should be able to:

a. contrast the concept of liquidity as “appetite for risk” with the more traditional view that liquidity is created by the central bank. (page 150)
b. describe how Minsky’s “financial instability hypothesis” predicts a mortgage market crisis as debt creation journeys from conservative hedging activities to more speculative activities, and finally to a Ponzi scheme phase. (page 152)
c. explain how subprime mortgage borrowers are granted a free at-the-money call option on the value of their property. (page 153)

The topical coverage corresponds with the following CFA Institute assigned reading:

55. Term Structure and Volatility of Interest Rates
The candidate should be able to:

a. illustrate and explain parallel and nonparallel shifts in the yield curve, a yield curve twist, and a change in the curvature of the yield curve (i.e., a butterfly shift). (page 160)
b. describe the factors that drive U.S. Treasury security returns, and evaluate the importance of each factor. (page 161)
c. explain the various universes of Treasury securities that are used to construct the theoretical spot rate curve, and evaluate their advantages and disadvantages. (page 163)
d. explain the swap rate curve (LIBOR curve), and discuss why market participants have used the swap rate curve rather than a government bond yield curve as a benchmark. (page 165)
e. illustrate the theories of the term structure of interest rates (i.e., pure expectations, liquidity, and preferred habitat), and discuss the implications of each for the shape of the yield curve. (page 166)
f. compute and interpret the yield curve risk of a security or a portfolio by using key rate duration. (page 171)
g. compute and interpret yield volatility, distinguish between historical yield volatility and implied yield volatility, and explain how yield volatility is forecasted. (page 172)
The topical coverage corresponds with the following CFA Institute assigned reading:

56. Valuing Bonds with Embedded Options
The candidate should be able to:

a. evaluate, using relative value analysis, whether a security is undervalued or overvalued. (page 186)
b. evaluate the importance of benchmark interest rates in interpreting spread measures. (page 191)
c. illustrate the backward induction valuation methodology within the binomial interest rate tree framework. (page 192)
d. compute the value of a callable bond from an interest rate tree. (page 192)
e. illustrate the relations among the values of a callable (putable) bond, the corresponding option-free bond, and the embedded option. (page 193)
f. explain the effect of volatility on the arbitrage-free value of an option. (page 194)
g. interpret an option-adjusted spread with respect to a nominal spread and to benchmark interest rates. (page 196)
h. illustrate how effective duration and effective convexity are calculated using the binomial model. (page 198)
i. calculate the value of a putable bond, using an interest rate tree. (page 200)
j. describe and evaluate a convertible bond and its various component values. (page 201)
k. compare and contrast the risk-return characteristics of a convertible bond with the risk-return characteristics of ownership of the underlying common stock. (page 206)

STUDY SESSION 15

The topical coverage corresponds with the following CFA Institute assigned reading:

57. Mortgage-Backed Sector of the Bond Market
The candidate should be able to:

a. describe a mortgage loan, and illustrate the cash flow characteristics of a fixed-rate, level payment, and fully amortized mortgage loan. (page 220)
b. illustrate the investment characteristics, payment characteristics, and risks of mortgage passthrough securities. (page 222)
c. calculate the prepayment amount for a month, given the single monthly mortality rate. (page 226)
d. compare and contrast the conditional prepayment rate (CPR) with the Public Securities Association (PSA) prepayment benchmark. (page 224)
e. explain why the average life of a mortgage-backed security is more relevant than the security’s maturity. (page 228)
f. explain the factors that affect prepayments and the types of prepayment risks. (page 227)
g. illustrate how a collateralized mortgage obligation (CMO) is created and how it provides a better matching of assets and liabilities for institutional investors. (page 228)
h. distinguish among the sequential pay tranche, the accrual tranche, the planned amortization class tranche, and the support tranche in a CMO. (page 229)
i. evaluate the risk characteristics and the relative performance of each type of CMO tranche, given changes in the interest rate environment. (page 235)
j. explain the investment characteristics of stripped mortgage-backed securities.  
   (page 236)

k. compare and contrast agency and nonagency mortgage-backed securities.  
   (page 238)

l. distinguish credit risk analysis of commercial mortgage-backed securities  
   (CMBS) from credit risk analysis of residential nonagency mortgage-backed  
   securities. (page 239)

m. describe the basic structure of a CMBS, and illustrate the ways in which a  
   CMBS investor may realize call protection at the loan level and by means of the  
   CMBS structure. (page 240)

The topical coverage corresponds with the following CFA Institute assigned reading:

58. Asset-Backed Sector of the Bond Market
The candidate should be able to:
   a. illustrate the basic structural features of and parties to a securitization  
      transaction. (page 250)
   b. explain and contrast prepayment tranching and credit tranching. (page 251)
   c. distinguish between the payment structure and collateral structure of a  
      securitization backed by amortizing assets and non-amortizing assets. (page 252)
   d. distinguish among the various types of external and internal credit  
      enhancements. (page 253)
   e. describe the cash flow and prepayment characteristics for securities backed by  
      home equity loans, manufactured housing loans, automobile loans, student  
      loans, SBA loans, and credit card receivables. (page 256)
   f. describe collateralized debt obligations (CDOs), including cash and synthetic  
      CDOs. (page 262)
   g. distinguish among the primary motivations for creating a collateralized debt  
      obligation (arbitrage and balance sheet transactions). (page 264)

The topical coverage corresponds with the following CFA Institute assigned reading:

59. Valuing Mortgage-Backed and Asset-Backed Securities
The candidate should be able to:
   a. illustrate the computation, use, and limitations of the cash flow yield, nominal  
      spread, and zero-volatility spread for a mortgage-backed security and an asset-  
      backed security. (page 275)
   b. describe the Monte Carlo simulation model for valuing a mortgage-backed  
      security. (page 277)
   c. describe path dependency in pass-through securities and the implications for  
      valuation models. (page 277)
   d. illustrate how the option-adjusted spread is computed using the Monte Carlo  
      simulation model and how this spread measure is interpreted. (page 278)
   e. evaluate a mortgage-backed security using option-adjusted spread analysis.  
      (page 281)
   f. discuss why effective durations reported by various dealers and vendors may  
      differ. (page 282)
   g. analyze the interest rate risk of a security given the security’s effective duration  
      and effective convexity. (page 282)
   h. explain other measures of duration used by practitioners in the mortgage-  
      backed market (e.g., cash flow duration, coupon curve duration, and empirical  
      duration), and describe the limitations of these duration measures. (page 284)
   i. determine whether the nominal spread, zero-volatility spread, or option-adjusted  
      spread should be used to evaluate a specific fixed income security. (page 285)
The following is a review of the Alternative Asset Valuation principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**INVESTMENT ANALYSIS**

**Study Session 13**

**EXAM FOCUS**

Probably the most important things for you to take away from this review are the risk and return characteristics of the different types of real estate investments and the type of investor that is most likely to be interested in each. Also be prepared to calculate the after tax cash flows from a real estate project (including the equity reversion after tax) and apply the NPV methodology.

**LOS 47.a: Illustrate, for each type of real property investment, the main value determinants, investment characteristics, principal risks, and most likely investors.**

Types of real property investments are discussed in the following paragraphs as they relate to this LOS. This discussion is then summarized in Figure 1.

**Raw Land**

- **Main value determinants.** The investment return from raw land is from appreciation, which is a function of supply and demand. While the total supply of raw land is limited, the supply of urban land may be increased via the addition of roads and utility services to otherwise undeveloped raw land. The demand for a specific parcel of raw land is a function of its relative location in a given community. The proximity of raw land to roads and travel patterns is directly related to demand, and therefore value. Zoning and planning also affect raw land values.

- **Investment characteristics.** Raw land is a passive and typically illiquid investment. It is difficult to leverage raw land investments to any great extent due to their relatively low loan-to-value ratio. Raw land investing provides no depreciation for tax purposes, and because it generates no income, expenses are capitalized and returns are subject only to capital gains taxes.

- **Principal risks.** Because raw land generates no ongoing income, the cost of carrying it (e.g., maintenance, taxes) must be paid from other income. For this reason, an investment in raw land is sometimes referred to as an “alligator” (i.e., it must be fed periodically, but it doesn’t give anything back until it’s sold). If an investor suffers a loss of income from other sources, raw land may have to be sold at a distressed sale price. Also, the nonconstant rate of appreciation of raw land adds to the uncertainty of the investment.

- **Most likely type of investor.** Speculators invest in raw land in hopes of short-term capital gains. Developers invest in raw land to support long-term operating needs. Portfolios with long-term investment horizons also invest in raw land as a store of value.
Residential Rentals (Apartments)

- **Main value determinants.** Residential rental property value is primarily a function of the number of rental units in the property and the income the property provides. Population growth, location, convenience, and prestige also contribute to the value of residential rental property.

- **Investment characteristics.** Residential rental property (apartments) requires continuous attention. Because they are understood by a broad range of investors, apartments are a relatively liquid type of real estate investment. The returns from apartments come in the form of both periodic income and value appreciation, and are subject to both ordinary income and capital gains taxes. Apartment returns are highly leveraged as loan-to-value ratios of 90% or more are not uncommon. Apartments provide tax depreciation, and because apartment leases are normally adjusted at least annually, they provide a good hedge against inflation.

- **Principal risks.** There may be significant risk associated with the start-up phase of a new apartment investment because demand can never be known with certainty. There is also risk associated with obtaining quality property management. This risk increases as the number of rental units being managed increases. Competition from single family homes as an alternative to renting may also be a risk of apartment investments.

- **Most likely type of investor.** Apartments are attractive for investors who can afford the relatively large initial equity outlay. That is, even with substantial mortgages, the initial equity requirement can be large. Investors who desire a tax shelter (depreciation, interest) are especially attracted to residential rental property investments.

Office Buildings

- **Main value determinants.** Office building values are a function of the economic health of the business community in which the property is located, the convenience of the location, the compatibility of the tenant mix, and the property's perceived status.

- **Investment characteristics.** As with residential rentals, office buildings are relatively liquid and provide returns in the form of current income and value appreciation. With multiple tenants, office buildings require relatively active management. Office buildings are depreciable, and returns are subject to both ordinary and capital gains taxation. Moderate leverage is possible, the extent of which is often dependent on the quality of the tenants.

- **Principal risks.** Investment risks that are mostly under the control of the owner of office buildings include the start-up risks associated with new property, the enrollment of high-quality management on an on-going basis, and the threat of the property becoming obsolete. Risks that are not under the control of the owners include shifts in the location of business activity and the development of competing properties.

- **Most likely type of investor.** Office building investors are typically wealthy, high-income individuals or firms with the capital resources required by the relatively high initial equity investment. However, public and private entities that own and operate office buildings have been formed, thus enabling investors with moderate wealth to earn the returns of office building investments.
Warehouses

• **Main value determinants.** The value of a warehouse is directly related to the level of industrial and commercial activity and the warehouse's ability to support changing material-handling processes. The value of a warehouse is often a function of the ease with which it allows movement (transportation convenience) within a community.

• **Investment characteristics.** Warehouse investments are very passive, moderately liquid, and accommodate a modest degree of leverage. Relative to apartments and office buildings, warehouse investment returns tend to be more from periodic income than property appreciation.

• **Principal risks.** Because warehouses are relatively cheap to construct, warehouse space tends to be prone to oversupply. Also, warehouses may become obsolete if they are not designed to accommodate changes in material handling procedures.

• **Most likely type of investor.** Investors that desire high cash flow, a tax shelter, and minimal management involvement are attracted to warehouse investments.

Community Shopping Centers

• **Main value determinants.** Shopping center values are highly dependent on the population and income level of the shoppers in the market area. A convenient location with adequate parking is an important value determinant, as well as the suitability of the tenant mix to the demands of the shoppers in the relevant market area. Favorable lease terms are also an important value determinant.

• **Investment characteristics.** The establishment and maintenance of shopping centers requires a relatively high level of active management. Shopping center investments offer relatively low liquidity, moderate leverage, and depreciation for tax purposes. Like other commercial real estate investments, shopping center investment returns come in the form of both periodic income and capital appreciation.

• **Principal risks.** The main risks for shopping center investing are those associated with obtaining a “good” tenant mix at start up and maintaining professional management with a service orientation. Vacancies, difficult lease negotiations, obsolescence, and the development of competing commercial properties are also risks associated with shopping centers.

• **Most likely type of investor.** Investors with the relatively large initial equity investment requirement, who can use the tax shelter, are attracted to shopping center investments.

Hotels and Motels

• **Main value determinants.** The level of tourist and business travel in the area of a hotel or motel is directly related to demand and consequent value for these types of real estate investments. The ability of hotels to host conventions and business meetings contributes to the value of hotel investments.

• **Investment characteristics.** Hotels and motels are active investments offering tax depreciation along with average to poor liquidity and leverage. Returns come from income and capital gains, so they are subject to ordinary income and capital gains taxes.

• **Principal risks.** Sufficient size to capitalize on economies of scale and obtaining and retaining competent management are the major risks of hotel and motel investments. The development of competing businesses and obsolescence are also major risk factors.
- **Most likely type of investor.** Hotel and motel investments require sizeable equity outlays. Therefore, they are limited to wealthy investors and real estate investment trusts (REITs). Relatively small properties may be suitable for investors who are willing to manage the property themselves.

**Figure 1: Real Estate Investment Characteristics**

<table>
<thead>
<tr>
<th>Investment</th>
<th>Factors that Affect Valuation</th>
<th>Principal Characteristics</th>
<th>Risk</th>
<th>Typical Investor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw land</td>
<td>• Supply/demand&lt;br&gt;• Location&lt;br&gt;• Planning and zoning</td>
<td>• Passive investment&lt;br&gt;• Illiquid&lt;br&gt;• Low leverage&lt;br&gt;• Return from value appreciation only&lt;br&gt;• No tax depreciation&lt;br&gt;• Capital gains tax exposure&lt;br&gt;• Capitalized expenses</td>
<td>Cost of carry*&lt;br&gt;• Unstable appreciation</td>
<td>Speculators/developers&lt;br&gt;• Estates and long term horizon portfolios</td>
</tr>
<tr>
<td>Apartments</td>
<td>• Population growth&lt;br&gt;• Income growth&lt;br&gt;• Location</td>
<td>• Moderately active&lt;br&gt;• Medium liquidity&lt;br&gt;• High leverage&lt;br&gt;• Return from income plus appreciation&lt;br&gt;• Tax depreciation&lt;br&gt;• Ordinary and capital gains tax exposure&lt;br&gt;• Inflation hedge</td>
<td>Start up for new construction&lt;br&gt;• Hiring effective management for large investments</td>
<td>High income in need of tax shelter&lt;br&gt;• Anyone with sufficient initial equity requirement</td>
</tr>
<tr>
<td>Office buildings</td>
<td>• Local economic expansion&lt;br&gt;• Location&lt;br&gt;• Tenant mix&lt;br&gt;• Favorable status</td>
<td>• Active if more than one tenant&lt;br&gt;• Medium liquidity&lt;br&gt;• Moderate leverage&lt;br&gt;• Return from income plus appreciation&lt;br&gt;• Tax depreciation&lt;br&gt;• Ordinary and capital gains tax exposure</td>
<td>Start up for new construction&lt;br&gt;• Hiring effective management for high service needs&lt;br&gt;• Competition&lt;br&gt;• Obsolescence&lt;br&gt;• Business activity location shifts</td>
<td>High income in need of tax shelter&lt;br&gt;• Anyone with sufficient initial equity requirement if professional management is employed</td>
</tr>
<tr>
<td>Warehouses</td>
<td>• Commercial/industrial activity&lt;br&gt;• Location&lt;br&gt;• Design for material handling change</td>
<td>• Passive&lt;br&gt;• Medium liquidity&lt;br&gt;• Medium leverage&lt;br&gt;• Return mostly from periodic income&lt;br&gt;• Tax depreciation&lt;br&gt;• Mostly ordinary income tax exposure</td>
<td>Oversupply&lt;br&gt;• Obsolescence when material handling procedures change</td>
<td>Retirees with desire for high cash flow and little management involvement&lt;br&gt;• Anyone in need of tax shelter with sufficient initial equity requirement</td>
</tr>
</tbody>
</table>
### Figure 1: Real Estate Investment Characteristics\(^1\) (Continued)

| Shopping centers | • Community growth  
|                  | • Population and income  
|                  | • Location  
|                  | • Adequate parking  
|                  | • Suitable tenant mix  
|                  | • Lease terms  
|                  | • Moderately active  
|                  | • Low liquidity  
|                  | • Medium leverage  
|                  | • Return from income plus appreciation  
|                  | • Tax depreciation  
|                  | • Ordinary and capital gains tax exposure  
|                  | • Establishing proper tenant mix at startup  
|                  | • Service-focused management needed  
|                  | • High vacancy rate  
|                  | • Competition  
|                  | • Obsolescence  
|                  | • High wealth to make large equity outlay  
|                  | • Anyone in need of tax shelter with sufficient initial equity requirement  

| Hotels/motels | • Location  
|              | • Demand by business and tourists  
|              | • Facility and service mix  
|              | • Active  
|              | • Medium/low liquidity  
|              | • Medium/low leverage  
|              | • Return from income plus appreciation  
|              | • Tax depreciation  
|              | • Ordinary and capital gains tax exposure  
|              | • Maintaining sufficient size  
|              | • Competent management  
|              | • Competition  
|              | • Anyone in need of tax shelter with sufficient initial equity  
|              | • Owner/managers for smaller properties  

1. Based on Figure 1 on pages 10-11, Alternative Asset Valuation and Fixed Income, CFA Program Curriculum, Volume 5, Level 2 (CFA Institute, 2011).

### Valuing Real Estate Investments

You are about to see two new concepts: recapture of depreciation and equity reversion. At the sale of a depreciable asset, the amount of depreciation beyond the actual decline in the asset’s value must be *recaptured* for the purpose of calculating the gain on the sale and any associated tax bill. If the asset actually appreciates in value, all depreciation must be recaptured. Also, at the sale of the asset, we must measure the *equity reversion*, which is the net equity returned to the investors after expenses and repayment of debt.

We will use the following comprehensive example to illustrate the process for estimating cash flows from a real estate investment (LOS 47.c) and the evaluation of the project using net present value and internal rate of return (LOS 47.b).
Comprehensive Example: Royal Arms Apartments

Consider the following real estate investment data for Royal Arms Apartments:

- Purchase price = $577,500.
- Net operating income (NOI) in year 1 = $70,400.
- Net operating income growth rate = 6% per year.
- Tax depreciation = $18,000 per year.
- Initial equity requirement = 25% of purchase price.
- Leverage: 75% of the purchase price is provided via a 30-year loan at a fixed rate of 6%, compounded monthly, which corresponds to a monthly payment of $2,596.80.
- Equity investors’ marginal income tax rate = 36%.
- Equity investors’ capital gains tax rate = 20%.
- Recaptured depreciation tax rate = 25%.
- After-tax required return on equity capital = 12%.
- Investment horizon = four years.
- End-of-year-4 market value = $855,716.
- Cost of sale at end of year 4 = $60,000.

LOS 47.c: Calculate the after-tax cash flow and the after-tax equity reversion from real estate properties.

The inputs needed to evaluate real estate investments are the cash flows after taxes (CFAT) for each year in the investment holding period and the equity reversion after taxes (ERAT) associated with the sale of the property. The procedure for computing these inputs for the Royal Arms Apartments investment opportunity is described in the following three-step process.

Step 1: Computing taxes payable. Prior to computing CFAT, it is necessary to determine the taxes that must be paid on the property’s earnings. The general formula for computing income taxes payable is:

\[
\text{taxes} = (\text{net operating income (NOI)} - \text{depreciation} - \text{interest}) \times \text{tax rate}
\]

where:
- tax rate = equity investors’ marginal income tax rate

The calculations of income taxes payable for each of the four years in the Royal Arms investment are shown in Figure 2.
Study Session 13

Cross-Reference to CFA Institute Assigned Reading #47 – Investment Analysis

Figure 2: Computation of Income Taxes Payable for Royal Arms Apartments*

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOI (growth = 6%)</td>
<td>$70,400</td>
<td>$74,624</td>
<td>$79,101</td>
<td>$83,848</td>
</tr>
<tr>
<td>Less depreciation</td>
<td>(18,000)</td>
<td>(18,000)</td>
<td>(18,000)</td>
<td>(18,000)</td>
</tr>
<tr>
<td>Less interest**</td>
<td>(25,843)</td>
<td>(25,515)</td>
<td>(25,166)</td>
<td>(24,797)</td>
</tr>
<tr>
<td>Taxable income</td>
<td>26,557</td>
<td>31,109</td>
<td>35,935</td>
<td>41,051</td>
</tr>
<tr>
<td>× income tax rate</td>
<td>×0.36</td>
<td>×0.36</td>
<td>×0.36</td>
<td>×0.36</td>
</tr>
<tr>
<td>Income taxes payable</td>
<td>$9,561</td>
<td>$11,199</td>
<td>$12,937</td>
<td>$14,778</td>
</tr>
</tbody>
</table>

* All values are rounded to the nearest whole dollar.
** Interest expense has been determined from a loan amortization schedule. Because you are not asked to construct an amortization table, this would either be given on the exam, or the loan would be interest-only.

Step 2: Computing cash flow after taxes (CFAT). Given the values for income taxes payable, we can now compute CFAT for each year using the procedure demonstrated in Figure 3.

Figure 3: Computation of Cash Flow After Taxes for Royal Arms Apartments*

<table>
<thead>
<tr>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOI</td>
<td>$70,400</td>
<td>$74,624</td>
<td>$79,101</td>
<td>$83,848</td>
</tr>
<tr>
<td>Less debt service**</td>
<td>($31,162)</td>
<td>($31,162)</td>
<td>($31,162)</td>
<td>($31,162)</td>
</tr>
<tr>
<td>Pretax cash flow</td>
<td>$39,238</td>
<td>$43,462</td>
<td>$47,939</td>
<td>$52,686</td>
</tr>
<tr>
<td>Less taxes payable</td>
<td>(9,561)</td>
<td>(11,199)</td>
<td>(12,937)</td>
<td>(14,778)</td>
</tr>
<tr>
<td>CFAT</td>
<td>$29,677</td>
<td>$32,263</td>
<td>$35,002</td>
<td>$37,908</td>
</tr>
</tbody>
</table>

* All values are rounded to the nearest whole dollar.
** Annual debt service = monthly payment × 12 = $2,596.80 × 12 = $31,161.60

Step 3: Computing equity reversion after taxes (ERAT). The equity reversion after taxes is computed using the general formula:

\[ \text{ERAT} = \text{selling price} - \text{selling costs} - \text{mortgage balance} - \text{taxes on sale} \]

Recaptured depreciation. Recaptured depreciation represents depreciation that was taken in anticipation of a decline in the value of an asset, which ultimately did not materialize. If the asset actually appreciates in value, all depreciation must be recaptured and taxed, and the appreciation in the asset’s value is then taxed as a capital gain. If the asset has declined in value, but by less than the total depreciation taken, recaptured depreciation equals net selling price less book value. Before continuing with the Royal Arms example, let’s work through an example where the property declines in value.
Example: Recaptured depreciation (selling price < original cost)

Calculate recaptured depreciation and any taxes on the sale:

- Purchase price = $500,000
- Accumulated depreciation = $150,000
- Selling price after three years = $450,000
- Selling expenses = 10% of sales price
- Tax rate on recaptured depreciation = 32%
- Tax rate on capital gains = 28%

**Answer:**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>$450,000</td>
</tr>
<tr>
<td>Less selling expenses @ 10% of sales price</td>
<td>(45,000)</td>
</tr>
<tr>
<td>Net selling price</td>
<td>$405,000</td>
</tr>
<tr>
<td>Purchase price</td>
<td>$500,000</td>
</tr>
<tr>
<td>Less accumulated depreciation</td>
<td>($150,000)</td>
</tr>
<tr>
<td>Adjusted basis (book value)</td>
<td>$350,000</td>
</tr>
<tr>
<td>Realized gain on sale</td>
<td>$55,000</td>
</tr>
<tr>
<td>Less recaptured depreciation (net selling price – book value)</td>
<td>$55,000*</td>
</tr>
<tr>
<td>Taxable gain on sale</td>
<td>$0</td>
</tr>
<tr>
<td>Tax on recaptured depreciation ($55,000 x 0.32)</td>
<td>$17,600</td>
</tr>
</tbody>
</table>

*When the net selling price is less than the purchase price, the realized gain on the sale will equal recaptured depreciation and the taxable gain will equal zero, but taxes are payable on the recaptured depreciation.*

**Important notes:**

- Net selling price < original cost ⇒ Recaptured depreciation < accumulated depreciation
- Net selling price ≥ original cost ⇒ Recaptured depreciation = accumulated depreciation

**Back to our Royal Arms Apartments example:**

The procedure for computing the taxes due from the sale of Royal Arms Apartments at the end of year 4 is illustrated in Figure 4. Once we have the taxes due on the sale of Royal Arms, ERAT can be computed as demonstrated in Figure 5.
Figure 4: Computation of Taxes Due on Sale of Royal Arms Apartments

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selling price</td>
<td>$855,716</td>
</tr>
<tr>
<td>Less cost of sale</td>
<td>(60,000)</td>
</tr>
<tr>
<td>Net selling price</td>
<td>$795,716</td>
</tr>
<tr>
<td>Less adjusted cost basis</td>
<td></td>
</tr>
<tr>
<td>Purchase price</td>
<td>$577,500</td>
</tr>
<tr>
<td>Less accumulated depreciation ($18,000 × 4)*</td>
<td>(72,000)</td>
</tr>
<tr>
<td>Realized gain on property sale</td>
<td>$290,216</td>
</tr>
<tr>
<td>Less recaptured depreciation</td>
<td></td>
</tr>
<tr>
<td>Long-term capital gain on property sale</td>
<td>$218,216</td>
</tr>
<tr>
<td>Tax on recaptured depreciation (0.25 × $72,000)</td>
<td>18,000</td>
</tr>
<tr>
<td>Tax on long-term capital gain (0.20 × $218,216)</td>
<td>43,643</td>
</tr>
<tr>
<td>Total tax due on property sale</td>
<td>$61,643</td>
</tr>
</tbody>
</table>

*In this case, the asset appreciated in value, so all depreciation is recaptured.

Figure 5: Computation of Equity Reversion After Taxes

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net selling price</td>
<td>$795,716</td>
</tr>
<tr>
<td>Less outstanding mortgage balance (from amortization)*</td>
<td>(409,799)</td>
</tr>
<tr>
<td>Before-tax sales proceeds</td>
<td>$385,917</td>
</tr>
<tr>
<td>Less taxes due on property sale</td>
<td>(61,643)</td>
</tr>
<tr>
<td>Equity reversion after taxes (ERAT)</td>
<td>$324,274</td>
</tr>
</tbody>
</table>

* Outstanding loan balance equals the face value of the loan less loan service payments reduced by interest paid.

WARM-UP: REVIEW OF NPV AND IRR FROM LEVEL 1

Net present value (NPV) methodology. The NPV of a real estate investment may be expressed as:

\[ NPV = \text{net present value of investment cash flows} - \text{equity investment} \]
This relationship may be expressed as:

\[
\text{NPV} = \frac{\text{CFAT}_1}{(1+i_{at})^1} + \frac{\text{CFAT}_2}{(1+i_{at})^2} + \ldots + \frac{\text{CFAT}_n}{(1+i_{at})^n} + \frac{\text{ERAT}}{(1+i_{at})^n} - \text{EI}
\]

where:
- \(\text{CFAT}_t\) = cash flow after taxes for period \(t\)
- \(\text{ERAT}\) = equity reversion after taxes (net equity at sale)
- \(\text{EI}\) = initial equity investment in the property
- \(i_{at}\) = risk-adjusted after-tax required return

NPV decision rule: Undertake an investment only if its NPV is equal to or greater than zero (i.e., \(\text{NPV} \geq 0\)). If \(\text{NPV} \geq 0\), the property’s expected return is equal to or greater than required return. If the \(\text{NPV} < 0\), the expected return is less than the required return.

Internal rate of return (IRR) methodology. The IRR for a real estate investment is the discount rate that makes the present value of a property’s cash flow equal to the amount of the equity investment (i.e., the IRR is the discount rate that makes the NPV of the real estate investment equal zero). The IRR is the investment’s expected return.

This relationship may be expressed as:

\[
\text{NPV} = 0 = \frac{\text{CFAT}_1}{(1+\text{IRR})^1} + \frac{\text{CFAT}_2}{(1+\text{IRR})^2} + \ldots + \frac{\text{CFAT}_n}{(1+\text{IRR})^n} + \frac{\text{ERAT}}{(1+\text{IRR})^n} - \text{EI}
\]

The IRR decision rule is to undertake an investment if its IRR is equal to or greater than a specified required return or hurdle rate. (Note: if the IRR equals the hurdle rate, the project has a zero NPV.) The hurdle rate used with real estate investments is \(i_{at}\), which we have defined as the after-tax required return on the investment property.

Los 47.b: Evaluate a real estate investment using net present value (NPV) and internal rate of return (IRR) from the perspective of an equity investor.

We now have the inputs necessary to evaluate the Royal Arms Investment using the NPV and IRR methodologies. A summary of the relevant cash flows for the NPV and IRR methodologies is presented in Figure 6. Note that because 25% of the purchase price represented the equity investment, while the other 75% was financed with debt, the equity investment is equal to 25% of the purchase price, or \(577,500 \times 0.25 = 144,375\).
Figure 6: Relevant Cash Flows for Royal Arms Apartments

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>EI</td>
<td>$144,375</td>
<td>$29,677</td>
<td>$32,263</td>
<td>$35,002</td>
<td>$37,908</td>
</tr>
<tr>
<td>CFAT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$324,274</td>
</tr>
<tr>
<td>ERAT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NPV and IRR evaluations.** Using the cash flows represented in Figure 6, we can compute the NPV for the Royal Arms Apartments investment as:

\[
\text{NPV} = \frac{\text{CFAT}_1}{(1 + i_{at})^1} + \frac{\text{CFAT}_2}{(1 + i_{at})^2} + \frac{\text{CFAT}_3}{(1 + i_{at})^3} + \frac{\text{CFAT}_4}{(1 + i_{at})^4} + \frac{\text{ERAT}}{(1 + i_{at})^4} - \text{EI}
\]

\[
= \frac{29,677}{1 + 0.12} + \frac{32,263}{(1 + 0.12)^2} + \frac{35,002}{(1 + 0.12)^3} + \frac{37,908}{(1 + 0.12)^4} + \frac{324,274}{(1 + 0.12)^4} - 144,375
\]

\[
= 162,929
\]

Using the TI BA II PLUS:

\[
[\text{CF}] [2^n] [\text{CLR WORK}]
144,375 [+/-] [ENTER] [\downarrow]
29,677 [ENTER] [\downarrow] [\downarrow]
32,263 [ENTER] [\downarrow] [\downarrow]
35,002 [ENTER] [\downarrow] [\downarrow]
362,182* [ENTER] [\downarrow] (*\text{CF}_4 = 37,908 + 324,274 = 362,182)
[\text{NPV}] [12] [ENTER] [\downarrow]
[\text{CPT}] = 162,929
[\text{IRR}] [\text{CPT}] = 40.095\% 
\]

Since the NPV for Royal Arms Apartments is greater than zero and the IRR is greater than the hurdle rate of 12%, the investment is acceptable.

**WARM-UP: THE MULTIPLE IRR PROBLEM**

Be on the lookout for negative net cash flows! If the net cash flow after taxes is negative in one of the years after the inception of the investment (time 0), just enter that cash flow as a negative number and proceed as usual in calculating the NPV. The IRR, however, will be undefined, as it will typically have more than one value.
Example: A project with Multiple IRRs
Calculate NPV and IRR for the following real estate investment:

Cost = $60,000
CFAT$_1$ = $100,000
CFAT$_2$ = $100,000
CFAT$_3$ = -$160,000
Required return = 10%

Answer:

On the TI BA II PLUS:

```
[CF] [2^nd] [CLR WORK]
60,000 [+/-] [ENTER] [j]
100,000 [ENTER] [j] [j]
100,000 [ENTER] [j] [j]
160,000 [+/-] [ENTER] [j]
[NPV] [10] [ENTER] [j]
[CPT] = -$6,657
[IRR] [CPT] = 19.61
```

However, because the cash flow is negative in year 3, this project has another IRR of 74.69%. You can check this by entering 74.69 as the required return and calculating NPV: it turns out to be approximately zero, which means 74.69% is another IRR for this project. The project should be rejected because the NPV at the required return of 10% is less than zero.

LOS 47.d: Explain the potential problems associated with using IRR as a measurement tool in real estate investments.

**Multiple IRRs.** When the cash flows from a project change signs during the life of the investment, the IRR calculation may result in multiple solutions. This is common when an investment requires a large expense, such as a major repair or renovation, at some point during its useful life (planned holding period). In such cases, use the NPV methodology and accept the project if the NPV is greater than zero.

**Ranking conflicts.** When ranking mutually exclusive projects (e.g., only one of a set of possible investments may be accepted), NPV and IRR may yield different decisions. This may occur when: (1) there is a relatively large difference in the size of the projects being evaluated and/or (2) the pattern or timing of the cash flows for the projects is significantly different.

When conflict exists between the NPV and IRR decision recommendations for mutually exclusive real estate investments, the project with the highest positive NPV should be accepted.
**Key Concepts**

**LOS 47.a**

**Raw Land**

*Value Determinants:* Supply/demand, location, planning, and zoning.

*Principal Characteristics:* Passive, illiquid, low leverage, return from value appreciation, no tax depreciation, capital gains tax exposure, capitalized expenses.

*Risk:* Cost of carry, unstable appreciation.

*Typical Investor:* Speculators/developers, estates, and longterm horizon portfolios.

**Residential Rentals (Apartments)**

*Value Determinants:* Population growth, income growth, location.

*Principal Characteristics:* Moderately active, medium liquidity, high leverage, return from income plus appreciation, tax depreciation, ordinary and capital gains tax exposure, inflation hedge.

*Risk:* Start up for new construction, hiring effective management for large investments.

*Typical Investor:* High income in need of tax shelter, anyone with sufficient initial equity requirement.

**Office Buildings**

*Value Determinants:* Economic expansion, location, tenant mix, favorable status.

*Principal Characteristics:* Active, medium liquidity, moderate leverage, return from income plus appreciation, tax depreciation, ordinary and capital gains tax exposure.

*Risk:* New construction, good management, competition, obsolescence, business activity location shifts.

*Typical Investor:* High income in need of tax shelter, anyone with sufficient initial equity requirement (typically need to employ professional management).

**Warehouses**

*Value Determinants:* Commercial/industrial activity, location, designed to accommodate changing material handling processes.

*Principal Characteristics:* Passive, medium liquidity, medium leverage, periodic income, tax depreciation, ordinary income tax exposure.

*Risk:* Oversupply, obsolescence if material handling procedures change.

*Typical Investor:* Retirees with desire for high cash flow, anyone in need of tax shelter with sufficient initial equity requirement.

**Shopping Centers**

*Value Determinants:* Community growth, population and income, location, adequate parking, suitable tenant mix, lease terms.

*Principal Characteristics:* Moderately active, low liquidity, medium leverage, return from income plus appreciation, tax depreciation, ordinary and capital gains tax exposure.

*Risk:* Proper tenant mix, good management needed, high vacancy rate, competition, obsolescence.

*Typical Investor:* High wealth to make large equity outlay, anyone in need of tax shelter.
Hotels/Motels
*Value Determinants:* Location, demand by business and tourists, facility and service mix.
*Principal Characteristics:* Active, medium/low liquidity, medium/low leverage, return from income plus appreciation, tax depreciation, ordinary and capital gains tax exposure.
*Risk:* Sufficient size, competent management, competition.
*Typical Investor:* Anyone in need of tax shelter with sufficient initial equity requirement, owner/managers for smaller properties.

**LOS 47.b**
The net present value (NPV) decision rule is to accept an investment if its NPV $\geq 0$.

The internal rate of return (IRR) decision rule is to accept an investment if its IRR $\geq$ the investor's required rate of return or some other stated hurdle rate.

**LOS 47.c**
Cash flow after taxes (CFAT) = NOI – debt service – taxes payable.

Equity reversion after taxes (ERAT) = net selling price – mortgage balance – taxes.

Recaptured depreciation represents depreciation that was taken in anticipation of a decline in the value of an asset that ultimately did not materialize.

**LOS 47.d**
IRR may have multiple values when an investment’s cash flows change sign more than once during the investment horizon.

IRR and NPV may give conflicting results for mutually exclusive projects of different scale and/or projects with different cash flow timing patterns. If there is a conflict, select the investment with the higher positive NPV.
Use the following information to answer Questions 1 through 3.

Assume you are considering investing in an apartment building with the following estimated financial characteristics:

- Net operating income (NOI$_1$) = $50,000.
- Net operating income growth rate = 5% per year.
- Tax depreciation = $12,000 per year.
- Annual interest expense = $15,000.
- Annual debt service = $18,000.
- Equity investors marginal income tax rate = 36%.
- Investment horizon = three years.

1. The year 1 cash flow after taxes is closest to:
   A. $23,720.
   B. $25,320.
   C. $27,336.

2. Taxes due in year 2 are closest to:
   A. $8,280.
   B. $9,180.
   C. $10,314.

3. The year 3 cash flow after taxes is closest to:
   A. $25,320.
   B. $27,000.
   C. $28,650.

4. Which of the following types of real estate is normally considered to be the least liquid?
   A. Raw land.
   B. Warehouses.
   C. Shopping centers.

5. Which of the following types of real estate is the most passive investment?
   A. Shopping centers.
   B. Warehouses.
   C. Office buildings.

6. Which of the following real estate returns are mostly, if not exclusively, subject to capital gains taxes?
   A. Raw land.
   B. Warehouses.
   C. Office buildings.
7. Multiple internal rates of return are most likely to occur under which of the following conditions?
   A. The cash flows from an investment reverse signs.
   B. The investments being compared are of significantly different scale.
   C. The timing of the cash flows from the investments being compared are significantly different.

**CHALLENGE PROBLEMS**

Use the following information to answer Questions 8 through 10.

Consider a real estate investment with an initial cost of $450,000 that was sold after five years at a price of $750,000. Costs associated with the sale were $50,000, and the tax depreciation in each year was $20,000. At the time of the sale, the outstanding mortgage balance will be $340,000. The tax rate on recaptured depreciation is 28%, and the long-term capital gains tax rate is 15%.

8. The amount of taxes payable on the sale attributed to the recapture of depreciation is closest to:
   A. $5,600.
   B. $20,000.
   C. $28,000.

9. Some of the sales proceeds may be taxed at the long-term capital gains tax rate. This amount is closest to:
   A. $100,000.
   B. $250,000.
   C. $350,000.

10. The equity reversion after taxes from this investment is closest to:
    A. $294,500.
    B. $315,500.
    C. $360,000.
ANSWERS – CONCEPT CHECKERS

Computations for Questions 1, 2, and 3

Taxes Payable Computation:

<table>
<thead>
<tr>
<th>Year</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOI (g = 5%)</td>
<td>50,000</td>
<td>52,500</td>
</tr>
<tr>
<td>Less depreciation</td>
<td>(12,000)</td>
<td>(12,000)</td>
</tr>
<tr>
<td>Less interest</td>
<td>(15,000)</td>
<td>(15,000)</td>
</tr>
<tr>
<td>Taxable income</td>
<td>23,000</td>
<td>25,500</td>
</tr>
<tr>
<td>Times tax rate</td>
<td>×0.36</td>
<td>×0.36</td>
</tr>
<tr>
<td>Income taxes payable</td>
<td>$8,280</td>
<td>$9,180</td>
</tr>
</tbody>
</table>

CFAT Computation:

<table>
<thead>
<tr>
<th>Year</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOI (g = 5%)</td>
<td>$50,000</td>
<td>$52,500</td>
</tr>
<tr>
<td>Less debt service</td>
<td>(18,000)</td>
<td>(18,000)</td>
</tr>
<tr>
<td>Before tax cash flow</td>
<td>$32,000</td>
<td>$34,500</td>
</tr>
<tr>
<td>Less taxes payable</td>
<td>(8,280)</td>
<td>(9,180)</td>
</tr>
<tr>
<td>CFAT</td>
<td>$23,720</td>
<td>$25,320</td>
</tr>
</tbody>
</table>

1. A CFAT₁ = $23,720
2. B Year 2 income taxes payable = $9,180
3. B CFAT₃ = $27,000
4. A Raw land is the least liquid real estate investment among the choices provided, followed by shopping centers and hotels. Apartments, warehouses, and office buildings provide moderate liquidity.
5. B Warehouses are considered the most passive investment among the choices provided, mostly because of the long-term leases typically associated with warehouse space.
6. A Since raw land generally does not produce any periodic income, the returns are taxed as capital gains when the investment is sold.
7. A Multiple IRR solutions may be produced when an investment’s cash flows change sign during the life of an investment. This typically happens when large cash outlays are required for capital improvements or maintenance during a real estate investment’s life.
8. C  Recaptured depreciation = 5 \times 20,000 = 100,000

   Tax on recaptured depreciation = 100,000 \times 0.28 = 28,000

9. B  Realized gain on sale = net selling price – adjusted basis

   Net selling price = sales price – cost of sale = 750,000 – 50,000 = 700,000

   Adjusted basis = cost – accumulated depreciation = 450,000 – 100,000 = 350,000

   Realized gain = 700,000 – 350,000 = 350,000

   Tax on realized gain = tax on recaptured depreciation + tax on long-term capital gain

   Since recaptured depreciation is 100,000, the amount of the sales proceeds subject to long-term capital gains taxes is 350,000 – 100,000 = 250,000.

10. A  Equity reversion after taxes (ERAT) = net selling price – mortgage balance – taxes

   Net selling price = 750,000 – 50,000 = 700,000

   Mortgage balance = 340,000 (given)

   Taxes = tax on recaptured depreciation + long-term capital gains tax

   Tax on recaptured depreciation = 100,000 \times 0.28 = 28,000 (see #8)

   Long-term capital gains tax = 250,000 \times 0.15 = 37,500 (see #9)

   Total taxes due on sale = 28,000 + 37,500 = 65,500

   ERAT = 700,000 – 340,000 – 65,500 = 294,500
The following is a review of the Alternative Asset Valuation principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**INCOME PROPERTY ANALYSIS AND APPRAISAL**

**Study Session 13**

**EXAM FOCUS**

The main points to take away from this new material for Level 2 on real estate appraisal are the direct income capitalization approach (an application of the Gordon growth model to real estate) and the gross income multiplier approach (a version of relative valuation) for real estate appraisals. For the exam, know the three different methods used to estimate a capitalization rate and when each of these is appropriate.

**LOS 48.a: Explain the relation between a real estate capitalization rate and a discount rate.**

The easiest way to distinguish between real estate discount rates and capitalization rates is to consider the following valuation model:

\[
MV_0 = \frac{NOI_1}{r - g} = \frac{NOI_1}{R_0}
\]

where:
- \(MV_0\) = current market value
- \(NOI_1\) = the net operating income expected from a real estate investment
- \(r\) = the rate that equity investors require for similar real estate investments
- \(g\) = the growth rate of NOI (assumed to be constant)
- \(R_0 = r - g\) = the market capitalization rate

*Professor’s Note: This equation should look familiar to you—it’s just a modified version of the constant growth dividend discount model, also known as the Gordon growth model, from Study Session 11. This is the direct income capitalization approach that will be applied in LOS 48.c.*

The discount rate \((r)\) is the required rate of return on the real estate investment, which is determined in much the same way as any investment (i.e., relative to the uncertainty of cash flows).

The capitalization rate \((R_0 = r - g)\) is the required rate of return less the expected growth in NOI (increase or decrease in value). In times of increasing inflation, capitalization rates may decline and value estimates rise as NOI and \(g\) increase. Of course, overall interest rates directly affect the required return, \(r\), of any investment, including real estate. Generally, as interest rates increase, capitalization rates increase and values decline. The increase in value from increased inflation, therefore, is dependent
upon the overall effects of inflation on interest rates compared to its effects on NOI and growth in NOI.

Professor’s Note: The growth in NOI, \( g \), in the direct income capitalization approach, is analogous to the constant growth rate in the constant growth dividend valuation model. In that case, the growth rate is the constant growth in dividends. It can also be construed as the growth (change) in value (i.e., capital gain or loss), associated with an investment.

LOS 48.b: Determine the capitalization rate by the market-extraction method, band-of-investment method, and built-up method, and justify each method’s use in capitalization rate determination.

The most prevalent techniques used to determine capitalization rates are (1) the market extraction method, (2) the band-of-investment method, and (3) the built-up method.

Market Extraction Method. The market extraction method for estimating capitalization rates is considered the most accurate of the three techniques because it uses comparable properties. Assuming that comparable properties can be found, the market extraction method is relatively simple to use because all that is required is the NOI and selling price for each comparable property. The formula used to compute a market extraction capitalization rate for a property is:

\[
R_0 (\text{ME}) = \frac{\text{NOI}}{\text{MV}}
\]

Professor’s Note: In the curriculum source readings, the authors are somewhat casual in their treatment of NOI in the various valuation equations. The market extraction technique, for example, rearranges the equation for the direct income capitalization technique to solve for the capitalization rate. As such, NOI in the equation should be the NOI expected next year (\( \text{NOI}_t \)). Be careful on the exam to use what is given. If an NOI figure is given without a growth rate, use it. Do not assume some arbitrary rate of growth.

Example: Market extraction method

Assume you are estimating the value of a property that is similar to two other properties. These other properties have NOI of $650,000 and $756,000, respectively, and recently sold for $6,500,000 and $8,400,000, respectively. Estimate a reasonable capitalization rate to be used when valuing the subject property using the market extraction method.
Answer:

For the two comparable properties, the capitalization rates are computed as:

- **Property 1:**
  \[
  R_0 = \frac{\text{NOI}}{\text{MV}} = \frac{\$650,000}{\$6,500,000} = 0.10 = 10.0\%
  \]

- **Property 2:**
  \[
  R_0 = \frac{\text{NOI}}{\text{MV}} = \frac{\$756,000}{\$8,400,000} = 0.09 = 9.0\%
  \]

**Ave. Cap Rate =** 10.0% + 9.0% = 9.5%

A reasonable capitalization rate to use for the subject property is the average of the capitalization rates for the comparable properties, or 9.5%.

**Justification.** Although the market extraction method is probably the best and most accurate technique to use, it is totally dependent upon the appraiser’s ability to identify comparable properties. Of course, the appraiser must also be able to accurately estimate the NOI from the properties and, if selling prices are not current, adjust them to arrive at reasonable current selling prices.

**Band-of-Investment Method (BOI).** BOI utilizes a weighted average cost of capital as an estimate of the market capitalization rate. It is appropriate for properties that utilize both debt and equity financing.

In the BOI method, we adjust the capitalization rate by adding a *sinking fund factor.*

**Example: Band-of-investment method**

Assume you are estimating the value of a property that is financed 60% with a 15-year first mortgage and 40% with equity capital. The interest rate on the mortgage is 7% with monthly payments. The required *cash on cash* return on equity capital is 14%.

Compute the market capitalization rate.

*Professor’s Note: The cash on cash return to equity holders is also referred to as the “equity dividend rate.”*

**Answer:**

The capitalization rate to be used under the band-of-investment method, \( R_0(BOI) \), is the weighted average cost of the individual capital components:

\[
\text{Annual mortgage cost} = (\text{mortgage weight} \times \text{mortgage cost}) + (\text{equity weight} \times \text{equity cost})
\]

The annual mortgage cost is the annual interest rate plus a *sinking fund factor.* The sinking fund factor in this case is the future value interest factor of an annuity of $1 at 7% per year compounded monthly for 15 years (the parameters of the loan). Using your financial calculator, it can be calculated as:

\[N = 15 \times 12; \ I/Y = 7 \ / 12; \ PV = 0; \ FV = -1; \ CPT \rightarrow PMT = 0.00316 \times 12 = \$0.0379\]
Professor’s Note: Even though the sinking fund factor is calculated as a payment, it is used as though it were a rate of interest. It might help to think of the sinking fund factor as the annuity required to pay off each dollar of the mortgage. In this case, paying 3.79 cents per year will pay off one dollar of the mortgage. That amount (3.79 cents) is then treated as though it were a rate of interest. (Since we are working with one dollar, cents and percentages are the same.) Also, note that when using the financial calculator, we needed to enter the FV as a negative number.

The sinking fund factor is then added to the stated interest cost of the mortgage. Adding 3.79% to the 7% mortgage interest rate gives us a total mortgage cost of 10.79%.

We now compute the capitalization rate as the weighted average of the mortgage and equity costs:

\[ R_{0(BOI)} = (0.6 \times 10.79\%) + (0.4 \times 14\%) = 12.07\% \]

Note that the capitalization rate of 12.07% derived for the property in this example is only applicable for properties that are financed with a 60/40 debt-equity mix, having a (total) mortgage cost of 10.79% and a required return on equity capital of 14%.

**Justification.** The band-of-investments method is justified only for properties that are financed with the same debt/equity mix and have the same total mortgage cost.

**Built-Up Method.** The built-up method for estimating the capitalization rate starts with an adjusted risk-free rate and adds premiums:

\[ R_{0(BU)} = \text{Pure interest rate} + \text{Liquidity premium} + \text{Recapture premium} + \text{Risk premium} \]

- or -

\[ R_{0(BU)} = \text{pure rate} + \text{liquidity premium} + \text{recapture premium} + \text{risk premium} \]
Example: Built-up method

Assume you have determined that a real estate investment will provide a 1.5% appreciation-adjusted return on investment, and has a 3% liquidity premium and a 1% risk premium. Further, assume that the prevailing rate on government bonds, net of real estate tax savings, is 5.5%. Compute the capitalization rate using the built-up method.

Answer:

Inserting the values provided in the built-up capitalization rate formula above gives us:

\[
R_{0(BU)} = \text{pure rate} + \text{liquidity premium} + \text{recapture premium} + \text{risk premium} = 5.5 + 3.0 + 1.5 + 1.0 = 11.0\%
\]

Justification. The built-up method is useful when comparables are not available and the appraiser must use a subjective, macro-factor approach. It can also be used to back out the various components of the capitalization rate.

**LOS 48.c:** Estimate the market value of a real estate investment using the direct income capitalization approach and the gross income multiplier technique.

**Direct Income Capitalization Approach.** Under the direct income capitalization approach, market value is estimated using the formula:

\[
\text{estimated market value}_0 = \frac{\text{net operating income}_1}{\text{capitalization rate}} \Rightarrow MV_0 = \frac{\text{NOI}_1}{R_0}
\]

The capitalization rate is estimated using one of the three methods discussed in the previous LOS.

Note: It is assumed that NOI grows at a constant rate (of \(g\)).

Example: Valuation using the direct income capitalization approach

Assume that net operating income (NOI\(_1\)) for an office building is expected to be $175,000 and the capitalization rate is 8%. Compute an estimated market value for this property using the direct capitalization approach.

Answer:

The estimated market value is:

\[
MV = \frac{\text{NOI}_1}{R_0} = \frac{175,000}{0.08} = 2,187,500
\]
Gross Income Multiplier Technique. Under the gross income multiplier technique, market value (MV) is estimated as a multiple of a subject property’s estimated gross income. That is:

\[ MV = \text{gross income} \times \text{gross income multiplier} \]

The gross income multiplier, \( M \), for a subject property is derived on the basis of observed multipliers for comparable properties using the following relationship:

\[ \text{gross income multiplier (M)} = \frac{\text{sales price}}{\text{gross income}} \]

Normally, annual gross income is used with the gross income multiplier technique. However, for one- to four-family residential rental properties, monthly estimates are sometimes used.

**Example: Valuation using the gross income multiplier technique**

Assume you are considering the purchase of an urban office building with an estimated gross annual income of $2,500,000. Further assume that the average gross income multiplier of several comparable urban office buildings is 2.7 times. Compute the value of the subject property using the gross income multiplier technique.

**Answer:**

\[ MV = \text{gross income} \times \text{income multiplier} = 2,500,000 \times 2.7 = 6,750,000 \]

LOS 48.d: Contrast the limitations of the direct capitalization approach to those of the gross income multiplier technique.

**Limitations of the direct capitalization approach:**

- **Selecting the appropriate capitalization rate.** It is difficult to estimate a capitalization rate that accurately reflects investors’ behavior, particularly when market data are unavailable or lacking in quality.
- **Application to income-producing property.** Income capitalization is only applicable to properties that generate monetary income, not to owner-occupied properties that provide other benefits or amenities.

**Limitations of the gross income multiplier approach:**

- **Discontinuous pricing.** Sales of some types of income-generating properties occur infrequently, which may result in the need to estimate the income multiplier with limited (or noncurrent) information.
- **Lack of information.** Rental income may not be available.
- **Gross rent versus NOI.** The use of gross rents may distort multipliers and, consequently, appraised values. This is because gross rent, versus NOI, does not account for differences in building-to-land ratios or differences in the ages of buildings among otherwise comparable properties.
• *Distorted selling prices.* Sales prices may be affected by poor maintenance, zoning, or high taxes, while rents may not be as affected. This will render the gross income multiplier inaccurate unless comparables are exposed to these same factors.

• *Unique or non-income-producing properties.* The income multiplier approach is not useful for unique properties or properties that produce benefits instead of monetary income.
KEY CONCEPTS

LOS 48.a
The relationship between a capitalization rate and a discount rate is evident in the market value formula:

\[ MV = \frac{NOI}{r-g} = \frac{NOI}{R_0} \]

(direct income capitalization technique)

where:

- NOI = the net operating income
- \( g \) = the constant growth rate of NOI
- \( r \) = the required rate of return on equity, also known as the discount rate
- \( R_0 = r - g \) = the market capitalization rate

LOS 48.b
There are three methods used to estimate the market capitalization rate, \( R_0 \):

- **Market extraction:**
  \[ R_0^{(ME)} = \frac{NOI}{MV} \]
  Useful for income generating property when data is available.

- **Band-of-investment:**
  \[ R_0^{(BOI)} = \text{weighted mortgage cost} + \text{weighted equity cost} \]
  Useful for properties financed with debt and equity.

- **Built-up:**
  \[ R_0^{(BU)} = \text{pure rate} + \text{liquidity premium} + \text{recapture premium} + \text{risk premium} \]
  Can be used to separate various components of the capitalization rate.

LOS 48.c
Direct income capitalization approach:

\[ \text{market value (MV)} = \frac{\text{net operating income}}{\text{capitalization rate}} = \frac{NOI}{R_0} \]

Gross income multiplier technique: \( MV = \text{gross income} \times \text{income multiplier} \)

\[ \text{Gross income multiplier (M)} = \frac{\text{sales price}}{\text{gross income}} \]
LOS 48.d
Limitations to the direct capitalization approach:
• Difficult to select appropriate capitalization rate without adequate data.
• Only applicable for monetary income-generating properties.

Limitations to the gross income multiplier approach:
• Sales prices (for comparables) may not be current.
• Rental income may not be available.
• Gross rents may be inaccurate when building-to-land ratios and building ages are different.
• Sale prices may be affected by factors that render the gross income multiplier inaccurate unless comparables are exposed to these same factors.
• Not useful for unique properties or properties that produce benefits instead of income.
CONCEPT CHECKERS

Use the following information to answer Questions 1 through 4.

Suppose you have collected the following information for properties A, B, and C.

<table>
<thead>
<tr>
<th>Property</th>
<th>Gross income</th>
<th>Net operating income</th>
<th>Sales price</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$250,000</td>
<td>$100,000</td>
<td>$1,250,000</td>
</tr>
<tr>
<td>B</td>
<td>$250,000</td>
<td>$120,000</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>C</td>
<td>$250,000</td>
<td>$150,000</td>
<td>$1,500,000</td>
</tr>
</tbody>
</table>

1. A reasonable estimate for the capitalization rate for another comparable property is closest to:
   A. 8.0%.
   B. 10.0%.
   C. 12.0%.

2. Assume that a comparable property, Property D, has an annual net operating income equal to $130,000. The direct capitalization approach provides a market value for Property D that is closest to:
   A. $1,200,000.
   B. $1,300,000.
   C. $1,500,000.

3. A reasonable estimate for the annual gross income multiplier for comparable Property D is:
   A. 4 times.
   B. 5 times.
   C. 6 times.

4. Assume that a comparable property, Property D, has gross annual income equal to $280,000. The gross income multiplier approach provides a market value for Property D that is closest to:
   A. $1,200,000.
   B. $1,400,000.
   C. $1,600,000.

5. Which of the following relationships most accurately represents the relationship between a discount rate and a capitalization rate for real estate?
   A. The capitalization rate is the appropriate discount rate less NOI growth.
   B. The appropriate discount rate is the capitalization rate less NOI growth.
   C. The capitalization rate is the present value of the appropriate discount rate.
6. Assume you have determined that a real estate investment will provide a 2.5% appreciation adjusted return of investment, has a 2% liquidity premium, and has a 1% risk premium. Further, assume that the prevailing rate on government bonds, net of real estate tax savings, is 5.25%. The capitalization rate using the built-up technique is closest to:
   A. 9.75%.
   B. 10.00%.
   C. 10.75%.

7. When estimating a capitalization rate, which of the following methods is most appropriate for a real estate investment that is financed with both debt and equity?
   A. Built-up method.
   B. Market extraction method.
   C. Band-of-investments method.

8. Assume that a property is financed with 45% debt and 55% equity. The total mortgage cost is 9% and the cost of equity financing is 12%. The band-of-investments method yields a capitalization rate closest to:
   A. 10.35%.
   B. 10.65%.
   C. 11.55%.

9. Consider a 10-year, 8% amortizing loan that has a face value of $5,000,000. Assuming monthly compounding and an 8% amortization rate, the sinking fund factor for this loan is closest to:
   A. 2.04%.
   B. 3.46%.
   C. 6.56%.

10. Consider a 12-year, 9% amortizing loan that has a face value of $5,000,000. Assuming monthly compounding, the total mortgage cost to the borrower for this loan is closest to:
    A. 8.55%.
    B. 13.66%.
    C. 14.76%.
1. B \[ R_0 = \frac{\text{NOI}}{\text{MV}} \]

\[ R_0(A) = \frac{\text{NOI}_A}{\text{MV}_A} = \frac{100,000}{1,250,000} = 8.0\% \]

\[ R_0(B) = \frac{\text{NOI}_B}{\text{MV}_B} = \frac{120,000}{1,000,000} = 12.0\% \]

\[ R_0(C) = \frac{\text{NOI}_C}{\text{MV}_C} = \frac{150,000}{1,500,000} = 10.0\% \]

Estimated capitalization rate: \( R_0(D) = \frac{8 + 10 + 12}{3} = 10\% \)

2. B See preceding problem for computation of capitalization rate, \( R_0(D) \).

\[ \text{MV}_D = \frac{\text{NOI}_D}{R_0(D)} = \frac{130,000}{0.10} = $1,300,000 \]

3. B Gross income multiplier (M) = \( \frac{\text{sales price}}{\text{gross income}} \)

\[ M_A = \frac{1,250,000}{250,000} = 5 \times \]

\[ M_B = \frac{1,000,000}{250,000} = 4 \times \]

\[ M_C = \frac{1,500,000}{250,000} = 6 \times \]

\[ M_D = \frac{5 + 4 + 6}{3} = 5 \times \]

4. B Gross income multiplier technique: \( \text{MV} = \text{gross income} \times \text{income multiplier} \)

\[ \text{MV}_D = $280,000 \times 5 = $1,400,000 \]

(See preceding problem for computation of multiplier.)

5. A The capitalization rate, \( R_0 \), is the discount rate (required rate of return on equity, \( r \)) net of the constant growth rate in net operating income, \( g \) (i.e., \( R_0 = r - g \)).

6. C \( R_0(BU) = \) pure rate + liquidity premium + recapture premium + risk premium = 5.25 + 2.00 + 2.50 + 1.00 = 10.75\%
7. C The band-of-investments method recognizes the relative costs of debt and equity. Under this method, the capitalization rate, $R_{BOI}$, is represented as:

$$R_{BOI} = (\text{mortgage weight} \times \text{mortgage cost}) + (\text{equity weight} \times \text{equity cost})$$

8. B The band-of-investments method recognizes the relative costs of debt and equity. Under this method, the capitalization rate, $R_{BOI}$, is represented as:

$$R_{BOI} = (\text{mortgage weight} \times \text{mortgage cost}) + (\text{equity weight} \times \text{equity cost})$$

In this case, the capitalization rate is $(0.45)(0.09) + (0.55)(0.12) = 0.1065 = 10.65\%$.

9. C The sinking fund factor is simply the future value interest factor for an annuity at the specified rate over a specified term. For the loan in this problem, the sinking fund may be calculated with a financial calculator as follows: $N = 10 \times 12; I/Y = 8 / 12; PV = 0; FV = -1; CPT \rightarrow PMT \times 12 = 0.0055 \times 12 = 0.0656$, or $6.56\%$.

10. B The total mortgage cost on this $9\%$ mortgage is the annual interest rate plus the annual sinking fund factor. For the loan in this problem, the sinking fund may be calculated with a financial calculator as follows: $N = 12 \times 12; I/Y = 9 / 12; PV = 0; FV = -1; CPT \rightarrow PMT \times 12 = 0.0039 \times 12 = 0.0466$, or $4.66\%$. Given this annual sinking fund factor, we have a total mortgage cost equal to $9 + 4.66 = 13.66\%$. 

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PRIVATE EQUITY VALUATION

EXAM FOCUS

This topic has a great deal of testable material, both conceptual and quantitative. For the exam, know the three sources of value creation in private equity. Know that, relative to buyout firms, venture capital concerns firms that are immature and generally more risky. Understand that the drivers of return for buyouts are earnings growth, the increase in multiple upon exit, and the reduction in the debt; whereas for venture capital firms, it is the pre-money valuation, the investment, and potential subsequent equity dilution.

Be familiar with risks, costs, structure, and terms that are unique to private equity funds. Know how to calculate management fees, carried interest, NAV, DPI, RVPI, and TVPI of a private equity fund. Using both the NPV and IRR venture capital methods, be able to calculate ownership fraction, number of new shares issued, and the price per share for the new investment.

BACKGROUND: PRIVATE EQUITY

Private equity is of increasing importance in the global economy. Private equity firms make investments ranging from those in venture capital firms (less mature firms) to those in buyout transactions (more mature firms).

The following diagram may help you understand the private equity investment process.

**Figure 1: The Typical Private Equity Investment Transaction**

We will use the term *portfolio company* to denote the companies that private equity firms invest in. Portfolio companies are sometimes referred to as investee companies.

We will use the term *private equity firm* (PE firm) to denote the intermediary in the illustrated transaction.

We will use the term *private equity investor* to denote the outside investor who makes an investment in a fund offered by the PE firm.

In this review, we examine the perspective of both private equity firms evaluating investments in portfolio companies and the perspective of an outside investor who is evaluating an investment in a private equity firm.
LOS 49.a: Explain the sources of value creation in private equity.

It is commonly believed that PE firms have the ability to add greater value to their portfolio companies than do publicly governed firms. The sources of this increased value are thought to come from the following:

1. The ability to re-engineer the firm and operate it more efficiently.

2. The ability to obtain debt financing on more advantageous terms.

3. Superior alignment of interests between management and private equity ownership.

Re-engineering the Firm

In order to re-engineer their portfolio companies, many private equity firms have an in-house staff of experienced industry CEOs, CFOs, and other former senior executives. These executives can share their expertise and contacts with portfolio firm management.

Obtaining Favorable Debt Financing

A second source of added value is from more favorable terms on debt financing. During 2006 and the first half of 2007, the availability of cheap credit with few covenants led many private equity firms to use debt for buyout transactions. In PE firms, debt is more heavily utilized and is quoted as a multiple of EBITDA (earnings before interest, taxes, depreciation, and amortization) as opposed to a multiple of equity, as for public firms.

The central proposition of the Modigliani-Miller theorems is that the use of debt versus equity is inconsequential for firm value. However, once the assumption of no taxes is removed from their model, the tax savings from the use of debt (i.e., the interest tax shield) increases firm value. The use of greater amounts of financial leverage may increase firm value in the case of private equity firms. Because these firms have a reputation for efficient management and timely payment of debt interest, this helps to allay concerns over their highly leveraged positions and helps maintain their access to the debt markets.

Professor’s Note: The Modigliani-Miller theorems are discussed in detail in the corporate finance material, Study Session 8. In that corporate finance material, they are referred to as propositions.

The use of debt is thought to make private equity portfolio companies more efficient. According to this view, the requirement to make interest payments forces the portfolio companies to use free cash flow more efficiently because interest payments must be made on the debt.

Much of the debt financing for private equity firms comes from the syndicated loan market, but the debt is often repackaged and sold as collateralized loan obligations (CLOs). Private equity firms may also issue high-yield bonds which are repackaged as collateralized debt obligations (CDOs). These transactions have resulted in a large transfer of risk. However, the markets slowed beginning in 2007, creating less availability of financing for large buyouts.
A third source of value added for PE firms is the alignment of interests between private equity owners and the managers of the portfolio companies they own, as discussed in the next LOS.

**LOS 49.b: Explain how private equity firms align their interests with those of the managers of portfolio companies.**

In many private equity transactions, ownership and control are concentrated in the same hands. In buyout transactions, management often has a substantial stake in the company’s equity. In many venture capital investments, the private equity firm offers advice and management expertise. The private equity firm can also gain increased control if the venture capital company does not meet specified targets.

In private equity firms, managers are able to focus more on long-term performance because, unlike public companies, private companies do not face the scrutiny of analysts, shareholders, and the broader market. This also allows the private equity firms to hire managers that are capable of substantial restructuring efforts.

**Control Mechanisms**

Private equity firms use a variety of mechanisms to align their interests with those of the managers of portfolio companies. The following contract terms are contained in the term sheet that specifies the terms of the private equity firm’s investment.

*Compensation:* Managers of the portfolio companies receive compensation that is closely linked to the firm’s performance, and the compensation contract contains clauses that promote the achievement of the firm’s goals.

*Tag-along, drag-along clauses:* Anytime an acquirer acquires control of the company, they must extend the acquisition offer to all shareholders, including firm management.

*Board representation:* The private equity firm is ensured control through board representation if the firm experiences a major event such as a takeover, restructuring, initial public offering (IPO), bankruptcy, or liquidation.

*Noncompete clauses:* Company founders must sign such clauses that prevent them from competing against the firm within a prespecified period of time.

*Priority in claims:* Private equity firms receive their distributions before other owners, often in the form of preferred dividends and sometimes specified as a multiple of their original investment. They also have priority on the firm’s assets if the portfolio company is liquidated.

*Required approvals:* Changes of strategic importance (e.g., acquisitions, divestitures, and changes in the business plan) must be approved by the private equity firm.
**Earn-outs**: These are used predominantly in venture capital investments. Earn-outs tie the acquisition price paid by the private equity firm to the portfolio company’s future performance over a specified time period.

By specifying the appropriate control mechanisms in the investment contract, private equity firms can make investments in companies of considerable risk.

**LOS 49.c**: Distinguish between the characteristics of buyout and venture capital investments.

**Valuation Characteristics of Venture Capital vs. Buyout Investments**

Venture capital and buyout firms are the two main forms of private equity investments but have different focuses. As previously noted, firms financed with venture capital are usually less mature than targets defined as buyout firms. Venture capital firms usually have a specific industry focus, such as biotechnology, and emphasize revenue growth. When private equity firms make buyout purchases, the emphasis is on EBIT or EBITDA growth, and typically a portfolio of firms with stable earnings growth is purchased.

The following chart summarizes the key differences between venture capital and buyout investments.
### Figure 2: Key Differences Between Venture Capital and Buyout Investments

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Venture Capital Investments</th>
<th>Buyout Investments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flows</td>
<td>Low predictability with potentially unrealistic projections</td>
<td>Stable and predictable cash flows</td>
</tr>
<tr>
<td>Product Market</td>
<td>New product market with uncertain future</td>
<td>Strong market position with a possible niche position</td>
</tr>
<tr>
<td>Products</td>
<td>Product is based on new technology with uncertain prospects</td>
<td>Established products</td>
</tr>
<tr>
<td>Asset Base</td>
<td>Weak</td>
<td>Substantial base that can serve as collateral</td>
</tr>
<tr>
<td>Management Team</td>
<td>New team although individual members typically have a strong entrepreneurial record</td>
<td>Strong and experienced</td>
</tr>
<tr>
<td>Financial Leverage</td>
<td>Low debt use with a majority of equity financing</td>
<td>High amounts of debt with a large percentage of senior debt and substantial amounts of junior and mezzanine debt</td>
</tr>
<tr>
<td>Risk Assessment</td>
<td>Risk is difficult to estimate due to new technologies, markets, and firm history</td>
<td>Risk can be estimated due to industry and firm maturity</td>
</tr>
<tr>
<td>Exit</td>
<td>Exit via IPO or firm sale is difficult to forecast</td>
<td>Exit is predictable</td>
</tr>
<tr>
<td>Operations</td>
<td>High cash burn rate required due to firm and product immaturity</td>
<td>Potential exists for reduction in inefficiencies</td>
</tr>
<tr>
<td>Working Capital Required</td>
<td>Increasing requirements due to growth</td>
<td>Low requirements</td>
</tr>
<tr>
<td>Due Diligence Performed by</td>
<td>Private equity firms investigate technological and commercial prospects; investigation of financials is limited due to short history</td>
<td>Private equity firms perform extensive due diligence</td>
</tr>
<tr>
<td>Private Equity Firms</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Goal Setting</td>
<td>Goals are milestones set in business plan and growth strategy</td>
<td>Goals reference cash flows, strategic plan, and business plan</td>
</tr>
<tr>
<td>Private Equity Investment</td>
<td>High returns come from a few highly successful investments with writeoffs from less successful investments</td>
<td>Low variability in the success of investments with failures being rare</td>
</tr>
<tr>
<td>Returns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Market Presence</td>
<td>Generally not active in capital markets</td>
<td>Active in capital markets</td>
</tr>
<tr>
<td>Sales Transactions</td>
<td>Most firms are sold as a result of the relationship between venture capital firm and entrepreneurs</td>
<td>Firms are typically sold in an auction-type process</td>
</tr>
<tr>
<td>Ability to Grow Through</td>
<td>Firms are less scalable as subsequent funding is typically smaller</td>
<td>Strong performers can increase subsequent funding amounts</td>
</tr>
<tr>
<td>Subsequent Funding</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Source of General Partner’s</td>
<td>Carried interest is most common, transaction and monitoring fees are less common</td>
<td>Carried interest, transaction fees, and monitoring fees</td>
</tr>
<tr>
<td>Variable Revenue</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Terms related to private equity, such as carried interest, and revenue of private equity general partners are discussed in greater detail in LOS 49.g.

Professor’s Note: Many of these characteristics can be more easily remembered if you keep in mind that, relative to buyout firms, venture capital portfolio firms are immature firms with risky prospects and cash flows. They require a great deal of funding but may have limited access to financing, especially debt. The returns on venture capital come from a small number of highly successful investments.

GENERAL VALUATION ISSUES FOR PRIVATE EQUITY

Public firms are bought and sold on regulated exchanges daily. Private firms, however, are bought by buyers with specific interests at specific points in time, with each potential buyer possibly having a different valuation for the firm. Furthermore, valuing a private firm is more difficult than valuing public firms because, as discussed previously, PE firms often transform and reengineer the portfolio company such that future cash flow estimates are difficult to obtain.

Private Equity Valuation Methodologies

There are six methodologies used to value private equity portfolio companies.

- **Discounted cash flow (DCF) analysis** is most appropriate for companies with a significant operating history because it requires an estimate of cash flows.
- A relative value or market approach applies a price multiple, such as the price-earnings ratio, against the firm’s earnings to get an estimate of the firm’s valuation. This approach requires predictable cash flows and a significant history.
- A third approach uses real option analysis and is applicable for immature firms with flexibility in their future strategies.

Professor’s Note: Real options are covered in more detail in the topic review on capital budgeting in Study Session 8.

- The fourth approach uses the replacement cost of the business. It is generally not applicable to mature firms whose historical value added would be hard to estimate.
- The last two approaches, the venture capital method and the leveraged buyout method, are discussed at the end of this review.

Other Considerations

Other considerations for valuing private equity portfolio companies are control premiums, country risk, and marketability and illiquidity discounts. In buyouts, the private equity investors typically have complete control. In venture capital investments, however, these investors usually have a minority position, and their control of the firms depends on the alignment of their interests with that of controlling shareholders. When valuing firms in emerging markets, country risk premiums may be added, thereby increasing the discount rate applied to the firm’s cash flows. Illiquidity and marketability discounts refer to the ability and right to sell the firm’s shares, respectively.
Professor’s Note: Country risk and premiums in emerging markets are covered in more detail in the topic review on return concepts in Study Session 10.

Price Multiples

To value private equity portfolio companies, many investors use market data from similar publicly traded firms, most commonly the price multiples from comparable public firms. However, it is often difficult to find public firms at the same stage of development, same line of business, same capital structure, and same risk. A decision must also be made as to whether trailing or future earnings are used. For these reasons, a relative value or market approach should be used carefully.

Discounted Cash Flow Analysis

Market data is also used with discounted cash flow (DCF) analysis, with beta and the cost of capital estimated from public firms while adjusting for differences in operating and financial leverage between the private and public comparables. In DCF analysis, an assumption must be made regarding the firm’s future value. Typically a terminal value (i.e., an exit value) is calculated using a price multiple of the firm’s EBITDA.

Professor’s Note: Adjusting beta for differences in operating and financial leverage between comparables is covered in more detail in the topic review on return concepts in Study Session 10.

Given the uncertainty associated with private firms, a variety of valuation techniques is typically applied to a range of different potential scenarios.

Buyout Valuation Issues

Types of Buyouts

In a buyout transaction, the buyer acquires a controlling equity position in a target company. Buyouts include takeovers, management buyouts (MBOs), and leveraged buyouts (LBOs). This review focuses on LBOs, in which a high amount of debt is used to finance a substantial portion of the acquisition. The financing of a LBO typically involves senior debt, junk bonds, equity, and mezzanine finance. Mezzanine finance is a hybrid between debt and equity and can be structured to suit each particular transaction.

Leveraged Buyout (LBO)

The view of an LBO transaction, referred to as the LBO model, is not a form of valuation but rather a method of factoring in the firm’s capital structure and other parameters to determine the return the private equity firm should expect from the transaction. The objective is not to value the firm but to determine the maximum price in negotiation that the private equity firm should pay for its stake.
LBO Model

The LBO model has three main inputs:
1. The target firm's forecasted cash flows.
2. The expected returns to the providers of the financing.
3. The total amount of financing.

The cash flow forecasts are provided by the target's management but scrutinized by the private equity firm. The exit date (when the target firm is sold) is evaluated at different dates to determine its influence on the projected returns. The value of the firm at that time is forecast using a relative value or market approach.

Professor's Note: LOS 49.d is inserted here in the discussion of LOS 49.c for the purposes of clarity.

LOS 49.d: Discuss the valuation issues in buyout and venture capital transactions.

Exit Value

The exit value can be viewed as:

\[
\text{investment cost} + \frac{\text{earnings growth}}{\text{price multiple}} + \frac{\text{increase in debt}}{\text{reduction in debt}} = \text{exit value}
\]

As previously mentioned, private equity firms are known for their reengineering and improved corporate governance of target firms, which should result in operational efficiencies and higher earnings growth. As a result, the target firm should see an increase in price multiples and increased ability to pay down its debt. Each of the three variables should be examined using scenario analysis to determine the plausibility of their forecasted values and the forecasted exit value. One purpose for calculating the exit value is to determine the investment's internal rate of return sensitivity in the exit year.
Example: Calculating payoff multiples and IRRs for equity investors

Suppose an LBO transaction is valued at $1,000 million and has the following characteristics (amounts are in millions of dollars):

- Exit occurs in five years at a projected multiple of 1.80 of the firm’s initial cost.
- It is financed with 60% debt and 40% equity.
- The $400 equity investment is composed of:
  - $310 in preference shares held by the private equity firm.
  - $80 in equity held by the private equity firm.
  - $10 in equity held by management equity participation (MEP).
- Preference shares are guaranteed a 14% compound annual return payable at exit.
- The equity of the private equity firm is promised 90% of the firm’s residual value at exit after creditors and preference shares are paid.
- Management equity receives the other 10% residual value.
- By exit, the firm will have paid off $350 of the initial $600 in debt using operating cash flow.

Calculate the payoff for the firm’s claimants and the internal rate of return (IRR) and payoff multiple for the equity claimants.

Answer:

First calculate the exit value as: $1,000 x 1.8 = $1,800.

Next calculate the claimants’ payoffs:

- **Debt:** The claim of debtholders is their initial investment minus the amount that has been paid down: $600 – $350 = $250.
- **Preference shares:** Earn a return of 14% so their claim is: $310 x (1.14)^5 = $596.88.
- **Private equity firm:** Receives 90% of the residual exit value: 0.90(1,800 - $250 - $596.88) = $857.81.
- **Management:** Receives 10% of the residual exit value: 0.10(1,800 - $250 - $596.88) = $95.31.

The total investment by the private equity firm is $310 + $80 = $390.

The total payoff is $596.88 + $857.81 = $1454.69.

The payoff multiple for the private equity firm is: 1454.69 / 390 = 3.7.

Using your TI BA II Plus, the IRR is calculated as:

PV = –$390; FV = $1454.69; N = 5; CPT I/Y ⇒ 30.1%.

For the management equity, the IRR is:

PV = –$10; FV = $95.31; N = 5; CPT I/Y ⇒ 57.0%.

The payoff multiple for the management equity program (MEP) is: 95.31 / 10 = 9.5.

In the example, the equity held by the private equity firm and management experiences a significant increase in value. The IRR for each is attractive at 30.1% and 57.0%, respectively.
The components of the return are:
- The return on the preference shares for the private equity firm.
- The increased multiple upon exit.
- The reduction in the debt claim.

In most LBOs, most of the debt is senior debt that will amortize over time. In the preceding example, the debtholders’ claim on assets was reduced from $600 to $250. The use of debt in this example is advantageous and magnifies the returns to the equityholders. However, the use of debt also increases risk to the equityholders. Use of debt becomes disadvantageous if a firm experiences difficulties and cannot make the payments on the debt. In this case, the equityholders could lose control of the firm if it is forced into bankruptcy.

VALUATION ISSUES IN VENTURE CAPITAL INVESTMENTS

Pre- and Post-Money Valuation

The two fundamental concepts in venture capital investments are pre-money (PRE) valuation and post-money (POST) valuation. A private equity firm makes an investment (INV) in the venture capital firm.

The post-money valuation of the firm is:

\[ \text{PRE} + \text{INV} = \text{POST} \]

The ownership proportion of the venture capital (VC) investor is:

\[ = \frac{\text{INV}}{\text{POST}} \]

Example: Calculating post-money valuation and proportional ownership

A firm is valued at $3,000,000 prior to a capital infusion of $1,000,000 by a VC investor.

Calculate the post-money valuation and the VC investor's proportional ownership.

Answer:

The post-money valuation is:

\[ $3,000,000 + $1,000,000 = $4,000,000 \]

The ownership proportion of the VC investor is:

\[ = \frac{$1,000,000}{$4,000,000} = 25\% \]
Appropriate Methods for Venture Capital Valuation

The pre-money valuation and investment will be negotiated between the firm and the VC investor. Additionally, the VC investor should keep in mind that his ownership could be diluted in the future due to future financing, conversion of convertible debt into equity, and the issuance of stock options to management.

As discussed previously, it is difficult to forecast the cash flows for a VC portfolio firm. Therefore, discounted cash flow analysis (the income approach) is not usually used as the primary valuation method for VC firms. It is also difficult to use a relative value or market approach. This is because a VC firm is often unique, and there may be no comparable firms to estimate a benchmark price multiple from. A replacement cost approach may also be difficult to apply. Alternative methodologies include real option analysis and the venture capital method, which will be addressed later in this review.

To estimate the pre-money valuation, the VC investor typically examines the company’s intellectual property and capital, the potential for the firm’s products, and its intangible assets. Sometimes a cap (e.g., $3,000,000) is placed on the pre-money valuation due to its uncertain value.

### Valuation Issues: Buyout vs. Venture Capital

The following table highlights the different issues when valuing buyouts versus private equity.

<table>
<thead>
<tr>
<th>Valuation Issue</th>
<th>Buyout</th>
<th>Venture Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applicability of DCF Method</td>
<td>Frequently used to estimate value of equity</td>
<td>Less frequently used as cash flows are uncertain</td>
</tr>
<tr>
<td>Applicability of Relative Value Approach</td>
<td>Used to check the value from DCF analysis</td>
<td>Difficult to use because there may be no truly comparable firms</td>
</tr>
<tr>
<td>Use of Debt</td>
<td>High</td>
<td>Low as equity is dominant form of financing</td>
</tr>
<tr>
<td>Key Drivers of Equity Return</td>
<td>Earnings growth, increase in multiple upon exit, and reduction in the debt</td>
<td>Pre-money valuation, investment, and subsequent dilution</td>
</tr>
</tbody>
</table>

Professor’s Note: Valuation methodologies for buyouts need to factor in the level and pattern of leverage over the investment term. Initially, debt levels are high but are expected to decrease to “normal” levels by the time of exit. We address this issue near the end of this topic review.
LOS 49.e: Explain alternative exit routes in private equity and their impact on value.

Types of Exit Routes

The exit value is a critical element in the return for the private equity firm and is considered carefully before the investment is undertaken. The means and timing of the exit strongly influence the exit value. There are four exit routes that private equity firms typically use: (1) an initial public offering (IPO), (2) secondary market sale, (3) management buyout (MBO), and (4) liquidation.

Initial Public Offering (IPO)

In an IPO, a firm's equity is offered for public sale. An IPO usually results in the highest exit value due to increased liquidity, greater access to capital, and the potential to hire better quality managers. However, an IPO is less flexible, more costly, and a more cumbersome process than the other alternatives.

IPOs are most appropriate for firms with strong growth prospects and a significant operating history and size. The timing of an IPO is key. After the bursting of the U.S. tech bubble in 2000, the IPO market withered and venture capital firms had to find other means of exit.

Secondary Market Sale

In a secondary market sale, the firm is sold to another investor or to another firm interested in the purchase for strategic reasons (e.g., a firm in the same industry wishes to expand its market share). Secondary market sales from one investor to another are quite frequent, especially in the case of buyouts. VC firms are sometimes exited via a buyout to another firm, but VC firms are usually too immature to support a large amount of debt. Secondary market sales result in the second highest firm valuations after IPOs.

Management Buyout (MBO)

In an MBO, the firm is sold to management, who utilize a large amount of leverage. Although management will have a strong interest in the subsequent success of the firm, the resulting high leverage may limit management's flexibility.

Liquidation

Liquidation, the outright sale of the firm's assets, is pursued when the firm is deemed no longer viable and usually results in a low value. There is potential for negative publicity as a result of displaced employees and from the obvious implications of the firm's failure to reach its objectives.
Exit Timing

The timing of the exit is also very important for firm value, and the venture capital firm should be flexible in this regard. For example, if a firm cannot be sold due to weak capital markets, the venture capital firm may want to consider buying another firm at depressed prices, merging the two firms, and waiting until capital market conditions improve to sell both as one firm.

When an exit is anticipated in the next year or two, the exit valuation multiple can be forecasted without too much error. Beyond this time horizon, however, exit multiples become much more uncertain and stress testing should be performed on a wide range of possible values.

Professor’s Note: Don’t lose sight of the purpose of valuation: (1) to assess the ability of the firm to generate cash flow and (2) to represent a benchmark for negotiations.

LOS 49.f: Explain private equity fund structures, terms, valuation, and due diligence in the context of an analysis of private equity fund returns.

Limited Partnership

The most common form of ownership structure for private equity funds is the limited partnership. In a limited partnership, the limited partners (LPs) provide funding and do not have an active role in the management of the investments. Their liability is limited to what they have invested (i.e., they cannot be held liable for any amount beyond their investment in the fund). The general partner (GP) in a limited partnership is liable for all the firm’s debts and, thus, has unlimited liability. The GP is the manager of the fund.

Another form of private equity fund structure is the company limited by shares. It offers better legal protection to the partners, depending on the jurisdiction. Most fund structures are closed end, meaning that investors can only redeem the investment at specified time periods.

Private equity firms must both raise funds and manage the investment of those funds. The private equity firm usually spends a year or two raising funds. Funds are then drawn down for investment, after which returns are realized. Most private equity funds last 10 to 12 years but can have their life extended another 2 to 3 years.

Private Equity Fund Terms

As mentioned previously, private equity investments are often only available to qualified investors, the definition of which depends on the jurisdiction. In the United States, the individual must have at least $1 million in assets.
The terms in a fund prospectus are a result of negotiation between the GP and the LPs. If the fund is oversubscribed (i.e., has more prospective investors than needed), the GP has greater negotiating power.

The terms of the fund should be focused towards aligning the interests of the GP and LPs and specifying the compensation of the GP. The most important terms can be categorized into economic and corporate governance terms.

**Economic Terms of a Private Equity Fund**

*Management fees:* These are fees paid to the GP on an annual basis as a percent of paid-in capital invested and usually range from 1.5% to 2.5%.

*Transaction fees:* These are paid to the GP for fund investment banking services, such as arranging a merger. These fees are usually split evenly with the LPs and, when paid, are deducted from management fees.

*Carried interest:* This is the GP’s share of the fund profits and is usually 20% after management fees.

*Ratchet:* This specifies the allocation of equity between stockholders and management of the portfolio firm and allows management to increase their allocation, depending on firm performance.

*Hurdle rate:* This is the IRR that the fund must meet before the GP can receive carried interest. It usually varies from 7% to 10% and incentivizes the GP.

*Target fund size:* The stated total maximum size of the PE fund, specified as an absolute figure. It signals the GP’s ability to manage and raise capital for a fund. It is a negative signal if actual funds ultimately raised are significantly lower than targeted.

*Vintage:* This is the year the fund was started and facilitates performance comparisons with other funds.

*Term of the fund:* As discussed previously, this is the life of the firm and is usually ten years.

*Professor’s Note:* There are several “capital” terms used throughout this reading. *Committed capital* is the amount of funds promised by investors to private equity funds. *Paid-in capital* is the amount of funds actually received from investors (also referred to as invested capital in this reading).
Example: Calculating carried interest with a hurdle rate

Suppose a fund has committed capital of $100 million, carried interest of 20%, and a hurdle rate of 9%. The firm called 80% of its commitments in the beginning of Year 1. Of this, $50 million was invested in Firm A and $30 million in Firm B.

At the end of Year 2, a $7 million profit is realized on the exit from Firm A. The investment in Firm B is unchanged. The carried interest is calculated on a deal-by-deal basis (i.e., the IRR for determining carried interest is calculated for each deal upon exit).

Determine the theoretical carried interest and the actual carried interest.

Answer:

The theoretical carried interest is: 20% × $7,000,000 = $1,400,000.

The IRR for Firm A is: PV = −$50; FV = $57; N = 2; CPT I/Y ⇒ 6.8%.

Because the 6.8% IRR is less than the hurdle rate of 9%, no carried interest is actually paid.

Corporate Governance Terms of a Private Equity Fund

The corporate governance terms in the prospectus provide the legal arrangements for the control of the fund and include the following:

Key man clause: If a key named executive leaves the fund or does not spend a sufficient amount of time at the fund, the GP may be prohibited from making additional investments until another key executive is selected.

Performance disclosure and confidentiality: This specifies the fund performance information that can be disclosed. Note that the performance information for underlying portfolio companies is typically not disclosed.

Clawback: If a fund is profitable early in its life, the GP receives compensation from the GP's contractually defined share of profits. Under a clawback provision, if the fund subsequently underperforms, the GP is required to pay back a portion of the early profits to the LPs. The clawback provision is usually settled at termination of the fund but can also be settled annually (also known as true-up).

Distribution waterfall: This provision specifies the method in which profits will flow to the LPs and when the GP receives carried interest. Two methods are commonly used. In a deal-by-deal method, carried interest can be distributed after each individual deal. The disadvantage of this method from the LPs’ perspective is that one deal could earn $10 million and another could lose $10 million, but the GP will receive carried interest on the first deal, even though the LPs have not earned an overall positive return.
In the total return method, carried interest is calculated on the entire portfolio. There are two variants of the total return method: (1) carried interest can be paid only after the portfolio value exceeds committed capital; or (2) carried interest can be paid when the value of the portfolio exceeds invested capital by some minimum amount (typically 20%). Notice that the former uses committed capital whereas the latter uses only the capital actually invested.

Tag-along, drag-along clauses: Anytime an acquirer acquires control of the company, they must extend the acquisition offer to all shareholders, including firm management.

Removal for cause: This clause allows a GP to be fired if a supermajority (usually 75% or more) of the LPs agree to do so.

Divorce for cause: This provision allows for the firing of a manager or the termination of a fund given sufficient cause (e.g., a felony conviction of a senior manager).

Investment restrictions: These specify leverage limits, a minimum amount of diversification, etc.

Co-investment: This provision allows the LPs to invest in other funds of the GP at low or no management fees. This provides the GP another source of funds. The provision also prevents the GP from using capital from different funds to invest in the same portfolio company. A conflict of interest would arise if the GP takes capital from one fund to invest in a troubled company that had received capital earlier from another fund.

Example: Applying distribution waterfalls methods

Suppose a fund has committed capital of $100 million and carried interest of 20%. An investment of $40 million is made. Later in the year, the fund exits the investment and earns a profit of $22 million.

Determine whether the GP receives any carried interest under the three waterfall distribution methods.

Answer:

In the deal-by-deal method, carried interest can be distributed after each individual deal, so carried interest of 20% x $22,000,000 = $4,400,000 is paid to the GP.

In the total return method #1, carried interest can be paid only after the portfolio value exceeds committed capital. Committed capital is $100 million and total proceeds from the exit are only $62 million, so no carried interest is paid.

In the total return method #2, carried interest can be paid when the value of the portfolio exceeds invested capital by some minimum amount (typically 20%).

Invested capital plus the 20% threshold is: $40,000,000 x 1.20 = $48 million.

The total proceeds from the exit are $62 million, so carried interest of $4,400,000 is paid to the GP.
Example: Applying clawback provision methods

Continuing with the previous example, assume in the second year, another investment of $25 million is exited and results in a loss of $4 million. Assume the deal-by-deal method and a clawback with annual true-up apply.

Determine whether the GP must return any former profits to the LPs.

Answer:

In the deal-by-deal method, the GP had received carried interest of $4,400,000.

With a subsequent loss of $4 million, the GP owes the LPs 20% of the loss:

\[
20\% \times \$4,000,000 = \$800,000
\]

**NET ASSET VALUE (NAV)**

Because there is no ready secondary market for private equity investments, they are difficult to value. In a prospectus, however, the valuation is related to the fund’s net asset value (NAV), which is the value of fund assets minus liabilities.

**Ways to Determine NAV**

The assets are valued by the GP in one of six ways:

1. At cost, adjusting for subsequent financing and devaluation.
2. At the minimum of cost or market value.
3. By revaluing a portfolio company anytime there is new financing.
4. At cost with no adjustment until exit.
5. By using a discount factor for restricted securities (e.g., those that can only be sold to qualified investors).
6. Less frequently, by applying illiquidity discounts to values based on those of comparable publicly traded firms.

**Issues in Calculating NAV**

There are several issues with calculating NAV for a private equity fund:

- First, if the NAV is only adjusted when there are subsequent rounds of financing, then the NAV will be more stale when financings are infrequent.
- Second, there is no definitive method for calculating NAV for a private equity fund because the market value of portfolio companies is usually not certain until exit.
• Third, undrawn LP capital commitments are not included in the NAV calculation but are essentially liabilities for the LP. The value of the commitments depends on the cash flows generated from them, but these are quite uncertain. When a GP has trouble raising funds, this implies that the value of these commitments is low.
• Fourth, the investor should be aware that funds with different strategies and maturities may use different valuation methodologies. In the early stages, a venture capital investment is typically valued at cost. In the later stages, a method based on comparables may be used. Mature funds may use market comparables for their investments that are near exit. Asset price bubbles would inflate the value of these firms.
• Finally, it is usually the GP who values the fund. LPs are increasingly using third parties to value private equity funds.

Due Diligence of Private Equity Fund Investments

Before investing, outside investors should conduct a thorough due diligence of a private equity fund due to the following characteristics:

• First, private equity funds have returns that tend to persist. Hence, a fund’s past performance is useful information. In other words, outperformers tend to keep outperforming and underperformers tend to keep underperforming or go out of business.
• Second, the return discrepancy between outperformers and underperformers is very large and can be as much as 20%.
• Third, private equity investments are usually illiquid, long-term investments. The duration of a private equity investment, however, is usually shorter than expected because when a portfolio company is exited, the funds are immediately returned to the fund investors.

LOS 49.g: Explain the risks and costs of investing in private equity.

Post-Investment Investor Expectations

Once an investment is made by a private equity firm, the outside investors in the private equity fund expect to be apprised of the firm’s performance. The following material now takes the perspective of this outside investor.

There are two important differences between investing in public equity and in a private equity fund. First, funds are committed in the private investments and later drawn down as capital is invested in portfolio companies. In a public firm, the committed capital is usually immediately deployed. Second, the returns on a private equity investment typically follow a J-Curve pattern through time. Initially, returns are negative but then turn positive as portfolio firms are sold at exit.

Private equity investments are usually regulated such that they are only available to “qualified” investors, usually defined as institutions and wealthy individuals. These regulations exist because of the high risks associated with private equity investing, which are disclosed in the private equity prospectus.
Risks of Investing in Private Equity

Classifying private equity risks broadly, the categories of private equity risk are general private equity risk (discussed in the following), risks specific to the investment strategy, industry risks, risks specific to the investment vehicle, and any regional or country risk.

General Risk Factors

The general private equity risk factors are as follows:

Liquidity risk: Because private equity investments are not publicly traded, it may be difficult to liquidate a position.

Unquoted investments risk: Because private equity investments do not have a publicly quoted price, they may be riskier than publicly traded securities.

Competitive environment risk: The competition for finding reasonably-priced private equity investments may be high.

Agency risk: The managers of private equity portfolio companies may not act in the best interests of the private equity firm and investors.

Capital risk: Increases in business and financial risks may result in a withdrawal of capital. Additionally, portfolio companies may find that subsequent rounds of financing are difficult to obtain.

Regulatory risk: The portfolio companies’ products and services may be adversely affected by government regulation.

Tax risk: The tax treatment of investment returns may change over time.

Valuation risk: The valuation of private equity investments reflects subjective, not independent, judgment.

Diversification risk: Private equity investments may be poorly diversified, so investors should diversify across investment development stage, vintage, and strategy of private equity funds.

Market risk: Private equity is subject to long-term changes in interest rates, exchange rates, and other market risks. Short-term changes are usually not significant risk factors.

Costs of Private Equity Investing

The costs of investing in private equity are significantly higher than that with publicly traded securities and include the following:

Transaction costs: These costs include those from due diligence, bank financing, legal fees from acquisitions, and sales transactions in portfolio companies.
Investment vehicle fund setup costs: The legal and other costs of setting up the fund are usually amortized over the life of the fund.

Administrative costs: These are charged on a yearly basis and include custodian, transfer agent, and accounting costs.

Audit costs: These are fixed and charged annually.

Management and performance costs: These are typically higher than that for other investments and are commonly 2% for the management fee and a 20% fee for performance.

Dilution costs: As discussed previously, additional rounds of financing and stock options granted to portfolio company management will result in dilution. This is also true for options issued to the private equity firm.

Placement fees: Placement agents who raise funds for private equity firms may charge up-front fees as much as 2% or annual trailer fees as a percent of funds raised through limited partners.

Professor's Note: A trailer fee is the compensation paid by the fund manager to the person selling the fund to investors.

LOS 49.h: Interpret and compare financial performance of private equity funds from the perspective of an investor.

INTERNAL RATE OF RETURN (IRR)

The return metric recommended for private equity by the Global Investment Performance Standards (GIPS) is the IRR. The IRR is a cash-weighted (a.k.a. money-weighted) return measure. Although the private equity fund portfolio companies are actually illiquid, IRR assumes intermediate cash flows are reinvested at the IRR. Therefore, the IRR calculation should be interpreted cautiously.

Gross IRR

The IRR can be calculated gross or net of fees. Gross IRR reflects the fund’s ability to generate a return from portfolio companies and is the relevant measure for the cash flows between the fund and portfolio companies.

Net IRR

Net IRR can differ substantially from Gross IRR because it is net of management fees, carried interest, and other compensation to the GP. Net IRR is the relevant measure for the cash flows between the fund and LPS and is therefore the relevant return metric for the LPS.
MULTIPLES

Multiples are also used to evaluate fund performance. Multiples are a popular tool of LPs due to their simplicity, ease of use, and ability to differentiate between realized and unrealized returns. Multiples, however, ignore the time value of money.

Quantitative Measures

The more popular multiples and those specified by GIPS include the following:

PIC (paid-in capital). This is the capital utilized by the GP. It can be specified in percentage terms as the paid-in capital to date divided by the committed capital. Alternatively, it can be specified in absolute terms as the cumulative capital utilized or called down.

DPI (distributed to paid-in capital). This measures the LP’s realized return and is the cumulative distributions paid to the LPs divided by the cumulative invested capital. It is net of management fees and carried interest. DPI is also referred to as the cash-on-cash return.

RVPI (residual value to paid-in capital). This measures the LP’s unrealized return and is the value of the LP’s holdings in the fund divided by the cumulative invested capital. It is net of management fees and carried interest.

TVPI (total value to paid-in capital). This measures the LP’s realized and unrealized return and is the sum of DPI and RVPI. It is net of management fees and carried interest.

Qualitative Measures

In addition to quantitative analysis of the fund, the investor should also analyze qualitative aspects of the fund, including the following:

- The realized investments, with an evaluation of successes and failures.
- The unrealized investments, with an evaluation of exit horizons and potential problems.
- Cash flow projections at the fund and portfolio company level.
- Fund valuation, NAV, and financial statements.

As an example, consider a fund that was started before the financial market collapse of 2007. If the RVPI is large relative to the DPI, this indicates that the firm has not successfully harvested many of its investments and that the fund may have an extended J-curve (it is taking longer than realized to earn a positive return on its investments). The investor should carefully examine the GP’s valuations of the remaining portfolio companies, potential write-offs, and whether the routes for future exit have dried up.
Benchmarks

The benchmarking of private equity investments can be challenging. Private equity funds vary substantially from one to another; so before performance evaluation is performed, the investor should have a good understanding of the fund’s structures, terms, valuation, and the results of due diligence. Because there are cyclical trends in IRR returns, the Net IRR should be benchmarked against a peer group of comparable private equity funds of the same vintage and strategy.

Professor’s Note: The vintage refers to the year the fund was set up.

Note also that the private equity IRR is cash flow weighted whereas most other asset class index returns are time weighted. One solution to this problem has been to convert publicly traded equity benchmark returns to cash weighted returns using the cash flow patterns of private equity funds. This method, however, has some significant limitations.

Example: Comparing the financial performance of private equity funds

Two private equity funds, Fund A and Fund B, are being considered by an investor.

Financial Performance of Private Equity Fund A and Fund B

<table>
<thead>
<tr>
<th></th>
<th>Fund A</th>
<th>Fund B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross IRR</td>
<td>22.1%</td>
<td>2.4%</td>
</tr>
<tr>
<td>Net IRR</td>
<td>17.6%</td>
<td>-0.3%</td>
</tr>
<tr>
<td>Performance quartile</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>DPI</td>
<td>1.43</td>
<td>0.29</td>
</tr>
<tr>
<td>RVPI</td>
<td>1.52</td>
<td>1.03</td>
</tr>
<tr>
<td>TVPI</td>
<td>2.95</td>
<td>1.32</td>
</tr>
<tr>
<td>Maturity of fund</td>
<td>6 years</td>
<td>4 years</td>
</tr>
</tbody>
</table>

Interpret and compare the financial performance of private equity funds A and B.
Answer:

Examining its DPI, Fund A has distributed $1.43 in return for every dollar invested. Additionally, the RVPI implies that it will return $1.52 as other investments are harvested. Its Gross IRR of 22.1% is attractive, and after fees, the Net IRR is 17.6%. The fund ranks in the first quartile in its peer group of the same strategy and vintage.

At four years, Fund B is a less mature fund than Fund A. Fund B’s DPI is 0.29, indicating that the realized returns for the fund are not substantial. Unrealized returns (RVPI) indicate that its investments not yet harvested should provide an additional return. The low Gross and Net IRRs indicate that the firm may still be affected by the J-curve, where a fund experiences initial losses before experiencing later profits. Currently, the firm is lagging its peers, as it ranks in the third quartile.

Note that in this illustrative example, we compared two funds of different maturities. As noted, a fund should be benchmarked against peers of the same vintage.

LOS 49.i: Calculate management fees, carried interest, net asset value, distributed to paid in (DPI), residual value to paid in (RVPI), and total value to paid in (TVPI) of a private equity fund.

In this section, we calculate the quantitative measures previously discussed using an example.

Example: Calculating performance measures

The GP for private equity Fund C charges a management fee of 2% and carried interest of 20%, using the first total return method. The total committed capital for the fund was $150 million. The statistics for years 2004–2009 are shown in the following table (in millions).

<table>
<thead>
<tr>
<th>Year</th>
<th>Capital Called Down</th>
<th>Paid-in Capital</th>
<th>Management Fees</th>
<th>Operating Results</th>
<th>NAV before Distributions</th>
<th>Carried Interest</th>
<th>Distributions</th>
<th>NAV after Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>50</td>
<td>50</td>
<td>1.0</td>
<td>-10</td>
<td>39.0</td>
<td></td>
<td></td>
<td>39.0</td>
</tr>
<tr>
<td>2005</td>
<td>20</td>
<td>70</td>
<td>1.4</td>
<td>-25</td>
<td>32.6</td>
<td></td>
<td></td>
<td>32.6</td>
</tr>
<tr>
<td>2006</td>
<td>30</td>
<td>100</td>
<td>2.0</td>
<td>25</td>
<td>85.6</td>
<td></td>
<td></td>
<td>85.6</td>
</tr>
<tr>
<td>2007</td>
<td>20</td>
<td>120</td>
<td>2.4</td>
<td>50</td>
<td>153.2</td>
<td>0.6</td>
<td>20</td>
<td>132.6</td>
</tr>
<tr>
<td>2008</td>
<td>10</td>
<td>130</td>
<td>2.6</td>
<td>60</td>
<td>200.0</td>
<td>9.4</td>
<td>40</td>
<td>150.6</td>
</tr>
<tr>
<td>2009</td>
<td>10</td>
<td>140</td>
<td>2.8</td>
<td>110</td>
<td>267.8</td>
<td>13.6</td>
<td>80</td>
<td>174.2</td>
</tr>
</tbody>
</table>
Professor's Note: In the table, assume the capital called down, operating results, and distributions were given. The other statistics can be calculated.

Calculate the management fees, carried interest, NAV before distributions, NAV after distributions, distributed to paid in (DPI), residual value to paid in (RVPI), and total value to paid in (TVPI) of private equity Fund C.

Answer:

*Paid-in capital:* This is just the cumulative sum of the capital called down. For example, in 2005, it is the sum of the capital called down in 2004 and 2005: $50 + $20 = $70.

*Management fees:* In each year, these are calculated as the percentage fee (here 2%) multiplied by the paid-in capital. For example, in 2005, it is $2\% \times 70 = 1.4$.

*Carried interest:* Carried interest is not paid until the GP generates realized and unrealized returns (as reflected in the NAV before distributions) greater than the committed capital of $150.

In 2007, the NAV before distributions exceeded the committed capital for the first time. In this first year, the carried interest is 20% multiplied by the NAV before distributions minus the committed capital: $20\% \times (153.2 - 150) = 0.6$.

In subsequent years, it is calculated using the increase in the NAV before distributions. For example, in 2008, it is: $20\% \times (200 - 153.2) = 9.4$.

*NAV before distributions:* These are calculated as:

\[
\text{NAV after distributions in prior year} = \text{NAV after distributions in prior year} + \text{capital called down} - \text{management fees} + \text{operating results}
\]

For example in 2008, NAV before distributions is: $132.6 + 10 - 2.6 + 60 = 200$.

*NAV after distributions:* These are calculated as:

\[
\text{NAV before distributions} = \text{NAV before distributions} - \text{carried interest} - \text{distributions}
\]

For example in 2008, NAV after distributions is: $200 - 9.40 - 40 = 150.60$. 

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For DPI, RVPI, and TVPI, we will calculate these as of the most recent year (2009):

**DPI**: The DPI multiple is calculated as the cumulative distributions divided by the paid-in capital: 
\[
\frac{($20 + $40 + $80)}{$140} = 1.0
\]
This indicates that, in terms of distributed returns, the fund has returned every dollar invested.

**RVPI**: The RVPI multiple is calculated as the NAV after distributions (i.e., the net non-distributed value of the fund) divided by the paid-in capital: 
\[
\frac{$174.2}{$140} = 1.24
\]
This indicates that, although the distributed returns are not impressive for this fund, the fund has unrealized profits that should accrue to the LPs as investments are harvested.

**TVPI**: The TVPI multiple is the sum of the DPI and RVPI: 
\[
1.0 + 1.24 = 2.24
\]
This indicates that on a realized and unrealized basis, the GP has more than doubled the investment of the LPs.

**LOS 49.j**: Calculate pre-money valuation, post-money valuation, ownership fraction, and price per share applying the venture capital method 1) with single and multiple financing rounds and 2) in terms of IRR.

Here, we describe the valuation of an investment in an existing firm using the venture capital (VC) method.

At the time of a new investment in the firm, the discounted present value of the estimated exit value, \( PV(\text{exit value}) \), is called the **post-money value** (after the investment is made). The value before the investment is made can be calculated as the post-money value minus the investment amount and is called the **pre-money value**.

\[
\text{POST} = PV(\text{exit value})
\]

\[
\text{PRE} = \text{POST} - \text{INV}
\]

In order to determine the number of new shares issued to the venture capital firm (\( \text{shares}_{\text{VC}} \)) for an investment in an existing firm, we need to determine the fraction of the firm value (after the investment is made) that the investment represents. Based on the expected future value of the firm (exit value) and the expected or required rate of return on the investment, we can do this in either of two ways with the same result.
The fraction of VC ownership \( f \) for the VC investment can be computed as:

The first method (NPV method):

\[
f = \frac{\text{INV}}{\text{POST}}
\]

where:

- \( \text{INV} \) = amount of new investment for the venture capital investment.
- \( \text{POST} \) = post-money value after the investment.

\[
\text{POST} = \frac{\text{exit value}}{(1 + r)^n}
\]

The second method (IRR method):

\[
f = \frac{\text{FV(INV)}}{\text{exit value}}
\]

where:

- \( \text{FV(INV)} \) = future value of the investment in round 1 at the expected exit date
- \( \text{exit value} \) = value of the firm upon exit.

As long as the same compound rate is used to calculate the present value of the exit value and to calculate the future value of the VC investment, the fractional ownership required \( (f) \) is the same under either method.

Once we have calculated \( f \), we can calculate the number of shares issued to the VC(\( \text{shares}_{\text{VC}} \)) based on the number of existing shares owned by the firm founders prior to investment (\( \text{shares}_{\text{Founders}} \)).

\[
\text{shares}_{\text{VC}} = \text{shares}_{\text{Founders}} \left( \frac{f}{1 - f} \right)
\]

The price per share at the time of the investment (price) is then simply the amount of the investment divided by the number of new shares issued.

\[
\text{price} = \frac{\text{INV}}{\text{shares}_{\text{VC}}}
\]
Example: Calculations using the NPV venture capital method and a single financing round

Ponder Technologies is a biotech firm, and its entrepreneur founders believe they can sell the firm for $40 million in five years. They need $5 million in capital now, and the entrepreneurs currently hold 1 million shares.

The venture capital firm, VC Investors, decides that given the high risk of this firm, a discount rate of 40% is appropriate.

Calculate the pre-money valuation, post-money valuation, ownership fraction, and price per share applying the NPV venture capital method with a single financing round.

Answer:

**Step 1:** The post-money (POST) valuation is the present value of the expected exit value (this assumes the investment was made in the firm):

\[
\text{POST} = \frac{40,000,000}{(1 + 0.40)^5} = 7,437,377
\]

**Step 2:** The pre-money (PRE) valuation is what the company would hypothetically be worth without the investment:

\[
\text{PRE} = 7,437,377 - 5,000,000 = 2,437,377
\]

**Step 3:** To put $5 million in a firm worth $7.4 million, the private equity firm must own 67.23% of the firm:

\[
f = \frac{5,000,000}{7,437,377} = 67.23%
\]

Note that under the IRR method, \(f\) is the same:

\[
f = \frac{5\ \text{million}(1.40^5)}{40\ \text{million}} = 67.23%
\]

**Step 4:** If the entrepreneurs want 1 million shares, the private equity firm must get 2.05 million shares to get 67.23% ownership:

\[
S_{VC} = 1,000,000 \left[ \frac{0.6723}{(1 - 0.6723)} \right] = 2,051,572
\]

**Step 5:** Given a $5 million investment and 2.05 million shares, the stock price per share (\(P\)) must be:

\[
P = \frac{5,000,000}{2,051,572} = $2.44\text{ per share}
\]
Professor's Note: For the purpose of differentiating terms between multiple rounds of venture capital investment, we are using subscripts 1 and 2 in this section to denote first and second round, respectively. For multiple rounds of VC financing, we work backwards (from last round to first).

If there is a second round of VC financing ($INV_2$), we can calculate the new fractional ownership from the new investment ($f_2$) and the number of new shares required ($shares_{VC2}$) using the NPV method, as:

$$f_2 = \frac{INV_2}{POST_2}$$

Where $POST_2$ is the discounted present value of the firm as of the time of the second financing round, its post-money value after the second round investment.

$$POST_2 = \frac{exit \ value}{(1 + r_2)^n_2}$$

and

$$PRE_2 = POST_2 - INV_2$$

$POST_1$ is the discounted present value of the firm as of the time of the first financing round, its post-money value after the first round investment.

$$POST_1 = \frac{PRE_2}{(1 + r_1)^n_1}$$

As before, we can calculate the fractional ownership from the first round investment ($f_1$) using the NPV method, as:

$$f_1 = \frac{INV_1}{POST_1}$$

The new shares required to be issued to the VC in return for the first round financing amount ($INV_1$) and the price per share can then be calculated as:

$$shares_{VC1} = shares_{Founders} \left( \frac{f_1}{1 - f_1} \right)$$

$$price_1 = \frac{INV_1}{shares_{VC1}}$$

The new shares required to be issued to the VC in return for the second round financing amount ($INV_2$) and the price per share can also be calculated as:

$$shares_{VC2} = \left( shares_{VC1} + shares_{Founders} \right) \left( \frac{f_2}{1 - f_2} \right)$$

$$price_2 = \frac{INV_2}{shares_{VC2}}$$
If the second round of financing is considered less risky than the first round (since the firm has survived longer), a different, lower discount rate may be used in calculating the PV of the exit value at the time of the second round of financing. In the following example, we use a discount rate of 30% in calculating the firm value to reflect this fact.

Example: Calculating shares issued and share price for a second round financing

Suppose that instead of a single round of financing of $5 million, the firm will need $3 million in the first round and a second round of financing (three years later) of $2 million to finance firm expansion to the size expected at exit.

Use a discount rate of 40% for the first three years and 30% for the last two years. The firm is still expected to be worth $40 million after five years, and founders will hold 1 million shares.

The value of the firm at the time of the second round of financing (two years remaining to exit) is:

\[
\text{POST}_2 = \frac{\text{exit value}}{(1 + r_2)^{n_2}} = \frac{40,000,000}{(1.30)^2} = 23,668,639
\]

The fractional VC ownership required for the second round investment of $2 million is:

\[
\frac{\text{INV}_2}{\text{POST}_2} = \frac{2,000,000}{23,668,639} = 0.0845 \text{ or } 8.45\%
\]

The value of the firm before the second round financing would then be:

\[
\text{PRE}_2 = \text{POST}_2 - \text{INV}_2 = 23,668,639 - 2,000,000 = 21,668,639
\]

Value of the firm at the first round of financing is:

\[
\text{POST}_1 = \frac{\text{PRE}_2}{(1 + r_1)^{n_1}} = \frac{21,668,639}{(1.40)^3} = 7,896,734
\]

The fractional VC ownership required for the first round investment of $3 million is:

\[
\frac{\text{INV}_1}{\text{POST}_1} = \frac{3,000,000}{7,896,734} = 0.38 \text{ or } 38\%
\]

Number of shares issued at the time of first round of financing is:

\[
\text{shares}_\text{VC1} = \text{shares}_\text{Founders} \left( \frac{f_1}{1 - f_1} \right) = 1,000,000 \left( \frac{0.38}{1 - 0.38} \right) = 612,903
\]

The price per share at the time of first round of financing is:

\[
\text{price}_1 = \frac{\text{INV}_1}{\text{shares}_\text{VC1}} = \frac{3,000,000}{612,903} = $4.89
\]
Number of shares issued to the VC firm at the time of the second round of financing is:

\[ \text{shares}_{VC2} = \left( \text{shares}_{VC1} + \text{shares}_{Founders} \right) \left( \frac{f_2}{1 - f_2} \right) \]

\[ = (612,903 + 1,000,000) \left( \frac{0.0845}{1 - 0.0845} \right) = 148,870 \]

The price per share at the time of second round of financing is:

\[ \text{price}_2 = \frac{\text{INV}_2}{\text{shares}_{VC2}} = \frac{2,000,000}{148,870} = \$13.43 \]

**LOS 49.k: Demonstrate alternative methods to account for risk in venture capital.**

Our previous discussions have been highly dependent on the assumptions, and sensitivity analysis should be used to determine how changes in the input variables will affect firm valuation. The discount rate used and the estimate of terminal value will strongly influence the current valuation.

Projections by entrepreneurs are typically overly optimistic and based on an assumption that the firm will not fail. Instead of arguing over the validity of the projections with the entrepreneurs, most investors simply apply a high discount rate that reflects both the probability of failure and lack of diversification available in these investments.

**Adjusting the Discount Rate**

One approach to arriving at a more realistic valuation is to adjust the discount rate to reflect the risk that the company may fail in any given year. In the following formula, \( r^* \) is adjusted for the probability of failure, \( q \):

\[ r^* = \frac{1 + r}{1 - q} - 1 \]

where:

- \( r \) = discount rate unadjusted for probability of failure

**Example: Adjusting the discount rate for the probability of failure**

Assume a private equity investor has a discount rate of 30%. The investor believes, however, that the entrepreneur’s projection of the company’s success is overly optimistic and that the chance of the company failing in a given year is 25%.

Calculate a discount rate that factors in the company’s probability of failure.

**Answer:**

\[ r^* = \frac{1 + 0.30}{1 - 0.25} - 1 = 73.33\% \]
Alternatively, the investor could have deflated each future cash flow for the cumulative probability that the company will fail. The adjusted discount rate approach is more straightforward.

### Adjusting the Terminal Value Using Scenario Analysis

A second approach to generating a realistic valuation is to adjust the terminal value for the probability of failure or poor results. Typically to obtain the terminal value, the future earnings are estimated and multiplied by an industry multiple. The problem is that almost by definition, VC firms are innovative with few true comparables. Price multiples also fluctuate a great deal so that the current multiple may not be indicative of what can be obtained in the future. We should therefore use scenario analysis to calculate an expected terminal value, reflecting the probability of different terminal values under different assumptions.

In theory, we should just determine the present value of future cash flows to get the current value. But estimating future cash flows is subject to error, and this method may not be any better than a price multiple approach.

**Example: Using scenario analysis to arrive at an expected terminal value**

In the previous valuation example, we were given a terminal value of $40 million. Assume that the scenario analysis is performed and examines three possible scenarios:

1. The expected earnings are $4 million and the expected price-earnings multiple is 10, resulting in the $40 million (as before).
2. The firm is not as successful, and earnings are only $2 million. Growth is slower, so the expected price-earnings multiple is 5. The expected terminal value is $10 million.
3. The firm fails, and its terminal value is $0.

If each scenario is equally likely, each possible value is weighted by one-third, and the expected terminal value is:

\[
\frac{1}{3}($40) + \frac{1}{3}($10) + \frac{1}{3}($0) = $16.7 \text{ million}
\]

The terminal value of $16.7 million is then used instead of the $40 million in the valuation analysis above. This is an alternative to adjusting the discount rate for the probability of failure.

In summary, VC valuation is highly dependent on the assumptions used and how risk is accounted for. Additionally, scenario and sensitivity analysis should be used to determine how changes in the input variables will affect the valuation of the firm.

Note that the purpose of the valuation procedures discussed here is not to ascertain the exact value of the firm. Rather, the purpose is to place some bounds on the value of the firm before negotiations begin between the venture capital firm and the private equity firm. The final price paid for the venture capital firm will also be affected by the bargaining power of the respective parties.
**KEY CONCEPTS**

**LOS 49.a**
The sources of value creation in private equity are: (1) the ability to reengineer the firm, (2) the ability to obtain debt financing on more favorable terms, and (3) superior alignment of interests between management and private equity ownership.

**LOS 49.b**
Private equity firms use the following mechanisms to align their interests with those of the managers of portfolio companies:
- Manager’s compensation tied to the firm’s performance.
- Tag-along, drag-along clauses ensure that anytime an acquirer acquires control of the company, they must extend the acquisition offer to all shareholders, including firm management.
- Board representation by private equity firm.
- Noncompete clauses required for company founders.
- Priority in claims. PE firms have priority if the portfolio company is liquidated.
- Required approval by PE firm for changes of strategic importance.
- Earn-outs. Acquisition price paid is tied to portfolio company’s future performance.

**LOS 49.c**
Relative to buyouts, venture capital portfolio companies are characterized by: unpredictable cash flows and product demand; weak asset base and newer management teams; less debt; unclear risk and exit; high demand for cash and working capital; less opportunity to perform due diligence; higher returns from a few highly successful firms; limited capital market presence; firm sales that take place due to relationships; smaller subsequent funding; and general partner revenue primarily in the form of carried interest.

**LOS 49.d**
With regard to valuation issues, for venture capital firms relative to buyout firms: discounted cash flow analysis and the relative value approach are less frequently used; and equity, not debt, is relied on as the primary form of financing.

The key drivers of equity return for buyouts are earnings growth, the increase in multiple upon exit, and the reduction in the debt. The key drivers of equity returns for venture capital firms are the pre-money valuation, the investment, and potential subsequent equity dilution.

The components of performance from a leveraged buyout are the earnings growth, the increase in multiple upon exit, and the reduction in the debt for the buyout firm.
LOS 49.c
The means and timing of the exit strongly influence the exit value.

The four typical exit routes:
• Initial public offerings usually result in the highest exit value due to increased liquidity, greater access to capital, and the potential to hire better quality managers.
• Secondary market sales to other investors or firms result in the second highest firm valuations after IPOs.
• In an MBO, the firm is sold to management, who utilize a large amount of leverage.
• A liquidation is pursued when the firm is deemed no longer viable and usually results in a low exit value.

LOS 49.f
The most common form of ownership structure for private equity funds is the limited partnership where limited partners (LPs) provide funding and have limited liability. The general partner (GP) manages the investment fund.

The economic terms in a private equity prospectus address the following issues: management fees; transaction fees; carried interest (the GP’s share of the fund profits); ratchet (the allocation of equity between stockholders and management of the portfolio firm); hurdle rate (the IRR that the GP must meet before receiving carried interest); target fund size; vintage year; and term of the fund.

The corporate governance terms in the prospectus address the following issues: key man clause (the provisions for the absence of a key named executive); performance disclosure and confidentiality (specifies the fund performance information that can be disclosed); clawback (the provision for when the GP must return profits); distribution waterfall (the method in which profits will flow to the LPs before the GP receives carried interest); tag-along, drag-along clauses (give management the right to sell their equity stake if the private equity firm sells its stake); removal for cause (specify when a GP can be fired); no-fault divorce (provisions for the firing of a manager or the termination of a fund); investment restrictions; and co-investment (allows the LPs to invest in other funds of the GP at low or no management fees).

Valuations are difficult for private equity funds because there is no ready secondary market for their investments. Additional issues with NAV calculations include the following: (1) the NAV will be stale if it is only adjusted when there are subsequent rounds of financing; (2) there is no definitive method for calculating NAV; (3) undrawn LP capital commitments are not included in the NAV calculation but are essentially liabilities for the LP; (4) different strategies and maturities may use different valuation methodologies; and (5) it is the GP who usually values the fund.

Investors should conduct due diligence before investing in a private equity fund due to the persistence in returns in private equity fund returns, the return discrepancies between outperformers and underperformers, and their illiquidity.

LOS 49.g
The general private equity risk factors are liquidity risk, unquoted investments risk, competitive environment risk, agency risk, capital risk, regulatory risk, tax risk, valuation risk, diversification risk, and market risk.
The costs of investing in private equity are significantly higher than those associated with publicly traded securities and include transactions costs, investment vehicle fund setup costs, administrative costs, audit costs, management and performance fee costs, dilution costs, and placement fees.

**LOS 49.h**
The Gross IRR reflects the fund’s ability to generate a return from portfolio companies. The Net IRR is the relevant return metric for the LPs and is net of management fees, carried interest, and other compensation to the GP. The Net IRR should be benchmarked against a peer group of comparable private equity funds of the same vintage and strategy.

**LOS 49.i**
The following statistics are important for evaluating the performance of a PE fund:
- Management fees are calculated as the percentage fee multiplied by the total paid-in capital.
- The carried interest is calculated as the percentage carried interest multiplied by the increase in the NAV before distributions.
  - The NAV before distributions is calculated as:

\[
\text{NAV before distributions} = \text{NAV after distributions in prior year} + \text{capital called down} - \text{management fees} + \text{operating results}
\]

  - The NAV after distributions is calculated as:

\[
\text{NAV after distributions} = \text{NAV before distributions} - \text{carried interest} - \text{distributions}
\]

- The DPI multiple is the cumulative distributions divided by the paid-in capital.
- The RVPI multiple is the NAV after distributions divided by the paid-in capital.
- The TVPI multiple is the sum of the DPI and RVPI.

**LOS 49.j**
Under the NPV method, the proportion of the firm \( (f) \) received for an investment in the firm is calculated as the investment amount \( (\text{INV}) \) divided by the post-money (post-investment) value of the firm. The post-money value of the firm is calculated by discounting the estimated exit value for the firm to its present value \( (\text{PV(exit value)}) \), as of the time the investment is made.

\[
f = \frac{\text{INV}}{\text{POST}}
\]

Alternatively, under the IRR method, we can calculate the fraction, \( f \), as the future value of the VC investment at the time of exit (using the discount rate as a compound rate of return), divided by the value of the firm at exit:

\[
f = \frac{\text{FV(INV)}}{\text{exit value}}
\]

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Once we have calculated this post-money ownership share, we can calculate the number of shares issued to the venture capital investor for the investment ($\text{shares}_{\text{VC}}$) and the price per share as:

$$\text{shares}_{\text{VC}} = \text{shares}_{\text{Founders}} \left( \frac{f}{1-f} \right)$$

$$\text{price} = \frac{\text{INV}}{\text{shares}_{\text{VC}}}$$

If there is a second round of financing, we first calculate the proportion of the firm ($f_2$) purchased for the second round of financing as:

$$f_2 = \frac{\text{INV}_2}{\text{POST}_2}$$

where:

$$\text{POST}_2 = \frac{\text{exit value}}{(1 + \tau_2)^{n2}}$$

and

$$\text{PRE}_2 = \text{POST}_2 - \text{INV}_2$$

We then compute the fractional ownership from the first round of financing as:

$$f_1 = \frac{\text{INV}_1}{\text{POST}_1}$$

where:

$$\text{POST}_1 = \frac{\text{PRE}_2}{(1 + \eta)^{n1}}$$

We can finally compute the number of shares issued and price per share in each round as:

$$\text{shares}_{\text{VC}_1} = \text{shares}_{\text{Founders}} \left( \frac{f_1}{1-f_1} \right)$$

$$\text{price}_{1} = \frac{\text{INV}_1}{\text{shares}_{\text{VC}_1}}$$

$$\text{shares}_{\text{VC}_2} = \left( \text{shares}_{\text{VC}_1} + \text{shares}_{\text{Founders}} \left( \frac{f_2}{1-f_2} \right) \right)$$

$$\text{price}_{2} = \frac{\text{INV}_2}{\text{shares}_{\text{VC}_2}}$$
LOS 49.k
The valuation of a venture capital investment is highly dependent on the assumptions used. The risk of the investment can be assessed using two methods.

• In the first approach, the discount rate is adjusted to reflect the risk that the company may fail in any given year:

\[ r^* = \frac{1 + r}{1 - q} - 1 \]

where:
- \( r^* \) = discount rate adjusted for probability of failure
- \( r \) = discount rate unadjusted for probability of failure
- \( q \) = probability of failure in a year

• In the second approach, scenario analysis is used to calculate an expected terminal value, reflecting different values under different assumptions.
CONCEPT CHECKERS

1. Which of the following is least likely a source of value creation in private equity firms?
   A. The use of debt with few covenants.
   B. The overutilization of cheap equity financing in private equity firms.
   C. The ability to reengineer firms through the use of an experienced staff of former senior managers.

2. Which of the following is least likely to be contained in a private equity term sheet?
   A. Tag-along, drag-along clauses.
   B. Earn-outs that ensure portfolio company manager compensation.
   C. A clause that ensures private equity firm representation on the portfolio company board.

3. Which of the following is more likely to be associated with a venture capital investment as compared to a buyout investment?
   A. Valuation using a discounted cash flow model.
   B. High cash burn rate.
   C. Due diligence covering all aspects of the business.

4. Which of the following is most likely to be a key driver for the equity return in a buyout firm?
   A. The pre-money valuation.
   B. The reduction in debt's claim on assets.
   C. The potential subsequent equity dilution.

5. Which of the following exit routes typically results in the highest exit valuation?
   A. An initial public offering.
   B. A management buyout.
   C. A secondary market sale.

6. Which of the following best describes the competitive environment risk of investing in private equity?
   A. The competition for finding reasonably priced private equity investments may be high.
   B. The competition for funds from private equity investors has increased as financial markets have fallen in activity.
   C. The competitive environment in the product markets for portfolio companies has increased due to the economic slowdown.

7. Which of the following best describes the placement fee cost of investing in private equity?
   A. The general partner may charge the fund fees for finding perspective portfolio companies.
   B. Investment banking fees are paid when exiting a private equity portfolio company via an IPO.
   C. Placement agents who raise funds for private equity firms may charge up-front or annual trailer fees.
8. What is the most typical organizational structure of a private equity investment?
   A. An S-corporation.
   B. A limited partnership.
   C. A sole proprietorship.

9. A private equity general partner has invested in portfolio Company A that has been funded by private equity Fund A. Portfolio Company A is experiencing financial difficulty, so the general partner uses funds from a newly formed private equity fund, Fund B, to assist the company. Which of the following terms in the private equity prospectus has the general partner most likely violated?
   A. The co-investment clause.
   B. The no-fault divorce clause.
   C. The tag-along, drag-along clause.

10. Using the information in the table below, which of the following firms likely has the best corporate governance system?
    A. Firm A.
    B. Firm B.
    C. Firm C.

<table>
<thead>
<tr>
<th></th>
<th>Firm A</th>
<th>Firm B</th>
<th>Firm C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Man Clause</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Management Fees</td>
<td>1.5%</td>
<td>2.0%</td>
<td>2.3%</td>
</tr>
<tr>
<td>Transaction Fees</td>
<td>The split between LPs and GP is 50/50</td>
<td>The split between LPs and GP is 50/50</td>
<td>GP share is 100%</td>
</tr>
<tr>
<td>Carried Interest</td>
<td>25%</td>
<td>20%</td>
<td>22%</td>
</tr>
<tr>
<td>Hurdle Rate</td>
<td>10%</td>
<td>8%</td>
<td>9%</td>
</tr>
<tr>
<td>Clawback Provision</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Distribution Waterfall</td>
<td>Total return</td>
<td>Total return</td>
<td>Deal-by-deal</td>
</tr>
<tr>
<td>Removal for Cause Clause</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

11. Which of the following best describes the method that most private equity funds use to incorporate undrawn capital commitments into NAV calculations?
    A. The GP uses public comparables to determine their value.
    B. There is no straightforward method for calculating the value of the commitments.
    C. The GP estimates the net present value of the capital commitments using the historical record of previous allocations to the portfolio companies.

12. Which of the following measures the limited partner's unrealized return in a private equity fund?
    A. The DPI.
    B. The RVPI.
    C. The TVPI.
Study Session 13
Cross-Reference to CFA Institute Assigned Reading #49 – Private Equity Valuation

Professor’s Note: From this point on, this set of Concept Checkers contains several multi-part questions where questions “nest” on each other—meaning that you need the answer to one question to complete the next. Note that it is unlikely you will encounter this situation on the exam. We recommend that after you complete a question, you check your answer to ensure that you begin the next question with the correct information.

Use the following information to answer Questions 13 through 21.

The GP for the private equity fund charges a management fee of 2% and carried interest of 20%, using the first total return method. The total committed capital for the fund was $200 million. The figures in the table are in millions.

<table>
<thead>
<tr>
<th>Capital Called Down</th>
<th>Paid-in Capital</th>
<th>Management Fees</th>
<th>Operating Results</th>
<th>NAV Before Distributions</th>
<th>Carried Interest</th>
<th>Distributions</th>
<th>NAV After Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>60</td>
<td>60</td>
<td>1.2</td>
<td>-15</td>
<td>43.8</td>
<td>?</td>
<td>43.8</td>
</tr>
<tr>
<td>2005</td>
<td>20</td>
<td>80</td>
<td>1.6</td>
<td>-20</td>
<td>42.2</td>
<td>?</td>
<td>42.2</td>
</tr>
<tr>
<td>2006</td>
<td>10</td>
<td>90</td>
<td>1.8</td>
<td>30</td>
<td>80.4</td>
<td>?</td>
<td>80.4</td>
</tr>
<tr>
<td>2007</td>
<td>20</td>
<td>110</td>
<td>2.2</td>
<td>50</td>
<td>148.2</td>
<td>?</td>
<td>118.2</td>
</tr>
<tr>
<td>2008</td>
<td>25</td>
<td>135</td>
<td>2.7</td>
<td>70</td>
<td>210.5</td>
<td>?</td>
<td>158.4</td>
</tr>
<tr>
<td>2009</td>
<td>10</td>
<td>?</td>
<td>120</td>
<td>?</td>
<td>?</td>
<td>90</td>
<td>?</td>
</tr>
</tbody>
</table>

13. What is the paid-in capital for 2009?
   A. $125.
   B. $142.
   C. $145.

14. What are the management fees for 2009?
   A. $2.7.
   B. $2.9.
   C. $15.4.

15. In what year is carried interest first paid?
   B. 2008.
   C. 2009.

16. What is the NAV before distributions for 2009?
   A. $275.50.
   B. $285.50.
   C. $288.40.

17. What is the carried interest for 2009?
   A. $2.9.
   B. $15.0.
   C. $17.9.
18. What is the NAV after distributions for 2009?
   A. $180.50.
   B. $195.50.
   C. $270.50.

19. What is the DPI after 2009?
   A. 0.62.
   B. 0.83.
   C. 1.17.

20. What is the RVPI after 2009?
   A. 1.24.
   B. 1.35.
   C. 1.97.

21. What is the TVPI after 2009?
   A. 1.76.
   B. 2.41.

Use the following information to answer Questions 22 through 26.

ScaleIt is a startup specializing in mobile applications. The firm's founders believe they can sell the firm for $50 million in four years. They need $7 million in capital now, and the founders wish to hold 1 million shares. The venture capital investor firm decides that, given the high risk of this firm, a discount rate of 45% is appropriate. Use the NPV venture capital method, assuming a single financing round.

22. What is the post-money valuation?
   A. $4,310,922.
   B. $11,310,922.
   C. $50,000,000.

23. What is the pre-money valuation?
   A. $4,310,922.
   B. $7,310,922.
   C. $43,000,000.

24. What is the ownership fraction for the venture capital firm?
   A. 14.00%.
   B. 38.11%.
   C. 61.89%.

25. What is the number of shares for the venture capital firm?
   A. 615,846.
   B. 1,623,983.
   C. 2,603,078.

26. What is the stock price per share?
   A. $2.69.
   B. $4.31.
   C. $11.37.
Use the following information to answer Questions 27 through 32.

A firm's founders believe that their firm can be sold for $60 million in four years. The firm needs $6 million in capital now and $3 million in three years. The entrepreneurs want to hold 1 million shares. The venture capital firm uses a discount rate of 50% over all four years.

27. What is the post-money valuation at the time of second-round financing?
   A. $17,777,778.
   B. $40,000,000.
   C. $57,000,000.

28. What is the post-money valuation at the time of first-round financing?
   A. $4,962,963.
   B. $9,851,259.
   C. $10,962,963.

29. What is the required fractional ownership for the second-round investors?
   A. 5.00%.
   B. 7.50%.
   C. 16.88%.

30. What is the fractional ownership for the first-round investors, after dilution by the second-round investors?
   A. 50.63%.
   B. 54.73%.
   C. 92.50%.

31. What is the stock price per share after the first round of financing?
   A. $4.96.
   B. $5.85.
   C. $6.00.

32. What is the stock price per share after the second round of financing?
   A. $5.77.
   B. $16.75.
   C. $37.00.

Use the following information to answer Questions 33 through 36.

The venture capital firm's founders believe they can sell the firm for $70 million in five years. They need $9 million in capital now, and the entrepreneurs wish to hold 1 million shares. The venture capital investor requires a return of 35%. Use the IRR venture capital method, assuming a single financing round.

33. What is the investor's ownership fraction?
   A. 12.86%.
   B. 42.35%.
   C. 57.65%.
34. What is the stock price per share?
   A. $2.39.
   B. $6.61.
   C. $12.25.

35. What is the post-money valuation?
   A. $6.61 million.
   B. $15.61 million.
   C. $70.00 million.

36. What is the pre-money valuation?
   A. $6.61 million.
   B. $9.00 million.
   C. $61.00 million.

37. A private equity investor has a discount rate of 30%. The investor believes, however, that the entrepreneur's projection of the company's success is overly optimistic and that the chance of the company failing in a given year is 20%. What is the discount rate that factors in the company's probability of failure?
   A. 50.0%.
   B. 62.5%.
   C. 71.4%.
### Answers – Concept Checkers

1. **B** It is actually the overutilization of cheap *debt* financing in private equity firms that leads to value creation. Private equity firms carry more debt than public firms but have a reputation for paying it back.

2. **B** Earn-outs do not ensure portfolio company manager compensation. Earn-outs tie the acquisition price paid by private equity firms to the portfolio company’s future performance. These are used predominantly in venture capital investments.

3. **B** Venture capital investments typically have significant cash burn rates. Discounted cash flow analysis is typically used for companies with substantial operating history and is, therefore, more likely to be associated with a buyout investment rather than a venture capital investment. Full due diligence is conducted for a buyout investment. Due diligence for typical venture capital investment is limited to technological feasibility and commercial potential due to limited operating results history.

4. **B** The pre-money valuation, investment, and potential subsequent equity dilution are issues for venture capital equity return. The key drivers of equity return for buyouts are earnings growth, the increase in multiple upon exit, and the reduction in the debt.

5. **A** Initial public offerings usually result in the highest exit value due to increased liquidity, greater access to capital, and the potential to hire better-quality managers.

6. **A** Competitive environment risk examines risk from the perspective of an investor who is considering an investment in private equity. It refers to the fact that the competition for finding reasonably priced private equity investments may be high.

7. **C** Placement fees are those charged by placement agents who raise funds for private equity firms. They may charge up-front fees as much as 2% or annual trailer fees as a percent of funds raised from limited partners.

8. **B** The most typical organizational structure of a private equity investment is a limited partnership. In a limited partnership, the limited partners provide funding and have limited liability. The general partner manages the investment fund.

9. **A** The clause in the private equity prospectus that the general partner has likely violated is the co-investment clause. The co-investment clause prevents the GP from using capital from different funds to invest in the same portfolio company. A conflict of interest arises here because portfolio Company A may be a poor use of the funds from Fund B investors.

10. **A** Firm A likely has the best corporate governance system. A large amount of the GP’s compensation comes in the form of incentive-based compensation as the carried interest and hurdle rate necessary to obtain carried interest is the highest, but the compensation unrelated to performance (the management and transactions fees are the lowest). The clawback provision also incentivizes the GP because they have to return previously received profits.

   Furthermore, the key man clause and the removal for cause clause give the LPs the right to dismiss an underperforming GP. The total return distribution waterfall method is used instead of the deal-by-deal method, in which the GP can receive carried interest even in cases when the LPs have not earned a net positive return.
11. B There is no straightforward method for calculating the value of the commitments, which are essentially liabilities for the LP. The value of the commitments depends on the cash flows generated from them, but these are quite uncertain.

12. B The RVPI (residual value to paid-in capital) measures the limited partner’s unrealized return in a private equity fund. It is the value of the LP’s holdings in the fund divided by the cumulative invested capital. It is net of management fees and carried interest. The DPI (distributed to paid-in capital) measures the LP’s realized return, and the TVPI (total value to paid-in capital) measures both the LP’s realized and unrealized return.

13. C This is the cumulative sum of the capital called down, and in 2009 is: $135 + $10 = $145.

14. B These are calculated as the percentage fee of 2% times the paid-in capital: 2% x $145 = $2.9.

15. B Carried interest is not paid until the NAV before distributions exceeds the committed capital of $200 million, which is the year 2008.

16. B NAV before distributions is calculated as:

\[
\text{NAV after distributions} = \frac{\text{NAV after distributions in prior year}}{\text{capital called down}} - \text{management fees} + \text{operating results}
\]

For 2009, NAV before distributions is: $158.4 + $10 - $2.9 + $120 = $285.50.

17. B It is calculated as the percentage carried interest times the increase in the NAV before distributions. In 2009, it is: 20% x ($285.50 - $210.50) = $15.00.

18. A NAV after distributions is calculated as:

\[
\text{NAV after distributions} = \frac{\text{NAV before distributions}}{\text{carried interest} - \text{distributions}}
\]

In 2009, NAV after distributions is: $285.50 - $15.00 - $90 = $180.50.

19. C The DPI multiple is calculated as the cumulative distributions divided by the paid-in capital: ($30 + $50 + $90) / $145 = 1.17. The GP has distributed more than the paid-in capital.

20. A The RVPI multiple is calculated as the NAV after distributions divided by the paid-in capital: ($180.50) / $145 = 1.24. The net unrealized returns are more than the paid-in capital.

21. B The TVPI multiple is the sum of the DPI and RVPI: 1.17 + 1.24 = 2.41.

22. B The post-money valuation is the present value of the expected exit value:

\[
\text{POST} = \frac{50,000,000}{(1 + 0.45)^4} = 11,310,922
\]

23. A The pre-money valuation is what the company is worth before the investment:

\[
\text{PRE} = 11,310,922 - 7,000,000 = 4,310,922
\]
24. C To put up $7 million in a firm worth $11.3 million, the venture capital firm must own 61.89% of the firm:

\[ f = \frac{7,000,000}{11,310,922} = 61.89\% \]

25. B If the entrepreneurs want 1 million shares, the venture capital firm must get 1.6 million shares to get 61.89% ownership:

\[ \text{Shares}_{\text{VC}} = 1,000,000 \times \frac{0.6189}{(1 - 0.6189)} = 1,623,983 \]

26. B Given a $7 million investment and 1.6 million shares, the stock price per share must be:

\[ p = \frac{7,000,000}{1,623,983} = $4.31 \text{ per share} \]

27. B Discount the terminal value of the company at exit back to the time of second round financing to obtain the post-money \( (\text{POST}_2) \) valuation:

\[ \text{POST}_2 = \frac{60,000,000}{(1 + 0.5000)} = $40,000,000 \]

28. C First, calculate the second-round pre-money \( (\text{PRE}_2) \) valuation by netting the second-round investment \( \text{INV}_2 \) from the post-money \( \text{POST}_2 \) valuation:

\[ \text{PRE}_2 = 40,000,000 - 3,000,000 = $37,000,000 \]

Next, discount the second-round pre-money valuation back to the time of the first-round financing to obtain the post-money \( \text{POST}_1 \) valuation:

\[ \text{POST}_1 = \frac{37,000,000}{(1 + 0.50)^3} = $10,962,963 \]

29. B The required fractional ownership for the second-round investors is:

\[ f_2 = \frac{3,000,000}{40,000,000} = 7.50\% \]

30. A The required fractional ownership for the first-round investors is:

\[ f_1 = \frac{6,000,000}{10,962,963} = 54.73\% \]
31. A First determine the number of shares the first-round venture capital investors (Shares\(_{VC1}\)) need to obtain their fractional ownership:

\[
\text{Shares}_{VC1} = 1,000,000 \left[ \frac{0.5473}{(1 - 0.5473)} \right] = 1,208,968
\]

To obtain a 54.73% share of the company, the first-round investors must receive 1,208,968 shares.

Next, determine the stock price per share after the first round of financing (\(P_1\)):

\[
P_1 = \frac{6,000,000}{1,208,968} = 4.96
\]

32. B First determine the number of shares the second-round venture capital investors (Shares\(_{VC2}\)) need to obtain their fractional ownership:

\[
\text{Shares}_{VC2} = (1,000,000 + 1,208,968) \left[ \frac{0.0750}{(1 - 0.0750)} \right] = 179,106
\]

To obtain a 7.50% share of the company, the second-round investors must receive 179,106 shares.

Next, determine the stock price per share after the second round of financing (\(P_2\)):

\[
P_2 = \frac{3,000,000}{179,106} = 16.75
\]

33. C First, calculate the investor’s expected future wealth (\(W\)):

\[
W = 9,000,000 \times (1 + 0.35)^5 = 40,356,301
\]

Given this expected wealth, we determine the required fractional ownership (\(f\)) by calculating how much of the terminal value should be the investor’s:

\[
f = \frac{40,356,301}{70,000,000} = 57.65\%
\]

34. B First, determine the number of shares the venture capital firm (Shares\(_{VC}\)) requires for its fractional ownership:

\[
\text{Shares}_{VC} = 1,000,000 \left[ \frac{0.5765}{(1 - 0.5765)} \right] = 1,361,275
\]

Next, determine the stock price per share (\(P\)):

\[
P = \frac{9,000,000}{1,361,275} = 6.61
\]
35. B Divide the investment by the fractional ownership to obtain the post-money (POST) valuation:
\[
POST = \frac{9,000,000}{0.5765} = 15.61 \text{ million}
\]

36. A Determine the pre-money (PRE) valuation by netting the investment (INV) from the post-money (POST) valuation:
\[
PRE = 15.61 \text{ million} - 9 \text{ million} = 6.61 \text{ million}
\]

37. B The discount rate that factors in the company's probability of failure is calculated as:
\[
r^* = \frac{1 + r}{1 - q} - 1
\]
\[
r^* = \frac{1 + 0.30}{1 - 0.20} - 1 = 62.5\%
\]
The following is a review of the Alternative Asset Valuation principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**INVESTING IN COMMODITIES**

Study Session 13

**EXAM FOCUS**

This new, short topic review examines some relatively recent changes in the composition of commodity futures market participants and the impact of these changes on commodity futures prices. It also considers the question as to whether or not commodity futures should be considered an asset class and examines some of the fundamental issues the analyst needs to be aware of when commodity futures are included in an investment portfolio.

*Professor's Note: This topic review assumes you have a basic understanding of futures contracts. As a result, before you start reading it, we suggest that you read the topic review on futures markets and contracts in Study Session 16 in Book 5. Pay particular attention to LOS 61.e on backwardation and contango.*

**LOS 50.a:** Explain why commodity futures such as gold have limited “contango,” whereas others such as oil often have natural “backwardation,” and indicate why these conditions might be less prevalent in the future.

**Contango and Backwardation**

When futures prices are higher than spot prices, the basis is negative and the market is said to be in contango. When futures prices are lower than spot prices, the market is said to be in backwardation.

**Why Contango Has Limits**

In a risk-neutral world, for an underlying asset that is costless to store and provides no income, the relationship between spot and futures prices is:

\[
\text{futures price} = \text{spot price} \times (1 + r)^{(T - t)}
\]

where:

- \(r\) = the risk-free rate of interest
- \((T - t)\) = the time from today (t) to the contract maturity (T) in years

*Professor's Note: You will become very familiar with this formula in Study Session 16. Note that the formula can be modified to include the net effect of costs (including storage and insurance costs) and benefits. Costs increase the futures price because they are a cost of holding the asset.*
Maximum Contango

Suppose that a given commodity costs $100 at spot, and an investor is holding $100 in the risk-free asset, earning 5%. The theoretical 1-year futures price is:

\[
\text{futures price} = 100 \times (1.05)^{1.0} = 105
\]

For most commodities that can be readily purchased at spot and stored, such as precious metals, this represents a theoretical maximum price—the amount of contango is limited to $5. If the futures price exceeds $105, investors will sell futures and buy spot until the price is forced back to the equilibrium. For commodities that can readily be borrowed and sold short, this also represents a theoretical minimum price. If the futures price falls below $105, investors will buy futures and sell spot short until the price is forced back to the equilibrium. However, for many physical commodities, practical restrictions can effectively preclude an investor from establishing a spot position that will enable them to achieve arbitrage profits by buying or selling futures contracts. This implies that for many physical commodities, the futures price can deviate from its theoretical value.

Decreased Prevalence of Backwardation

Historically, the demand for hedging has exceeded the demand for speculation for most commodity forward and futures contracts. Because of this difference in the number of hedgers and speculators, hedgers often needed to offer a price inducement in order to get speculators to take the other side of the transaction. In recent years, we have observed a shift in the composition in the marketplace, with more parties interested in speculation entering the market.

In addition to the relative number of hedgers and speculators, whether markets are in contango or backwardation also depends upon whether the perceived price risk is greater for commodity consumers or commodity producers. When prices are perceived as relatively high, commodity consumers are exposed to larger risks and their demand to buy futures drives prices higher (contango). When prices are perceived as relatively low, commodity producers are exposed to larger risks and seek to sell futures. This excess supply drives futures prices lower (backwardation). Historically, the need for commodity producers to hedge has tended to exceed that for commodity consumers, implying a net hedging need to sell futures. Consequently, for most commodities there is a natural tendency for markets to be in backwardation.

In summary, the actual value of the futures contract is a function of the theoretical price, the net demand for hedging (short or long futures), and the supply of funds for speculation (long futures). The combination of relatively high commodity prices and increased interest in long commodity futures positions (speculation) in investment portfolios is expected to decrease the level of backwardation and increase the level of contango compared to what has been observed historically.
LOS 50.b: Discuss how “roll yield” in a commodity futures position can be positive (negative).

Yield Types

The yield on a commodity futures is comprised of the following types of yields: price, collateral, and roll (also referred to as convenience).

When long positions in commodities are established in investment portfolios using futures, as opposed to buying the spot commodity directly, these futures positions are usually collateralized. A collateralized futures position is one in which the investor holds futures contracts and the risk-free asset. The value of the risk-free asset is the amount required to purchase the commodity by the exercise of the futures contract upon maturity.

Since the spot commodity is seldom purchased in practice, the usual situation is one in which the maturing futures contract is replaced with a new futures contract, which results in roll yield. Maturing risk-free asset proceeds also must be rolled into a new investment in the risk-free asset. The return realized by the investor when this process is undertaken affects the collateral yield the investor earns from holding the risk-free asset.

Roll Yield

The roll yield is independent of the spot price and is the return from closing out a futures contract at one price and entering into another contract at another price. This “rolling” of contracts allows the investor to keep a long position.

If futures prices are in backwardation, the maturing futures are consistently being rolled into new contracts that are relatively cheaper, and the result is that the investor is achieving a positive roll yield. If futures prices are in contango, the maturing contracts are being rolled into more expensive contracts, and the investor realizes negative roll yield.

Historically, because the natural state was for backwardation in commodity futures markets, roll yields have been positive. The increased presence of institutional investors in commodity futures markets taking long speculative positions can be expected to decrease roll yield, or even make it negative, on average.

LOS 50.c: Discuss the argument that commodity futures are not an asset class.

The main arguments against commodity futures as an asset class are as follows:

- Commodity prices have tended to decline in the long run.
- Roll yield is not guaranteed (and may be eliminated).
- Rolling costs reduce returns.
- Rebalancing, rather than any change in the asset price, drives returns.
- Commodity futures do not produce cash flows.
Here we take a brief look at each one of these points.

**Declining Prices**

Commodity prices have declined over the very long term, primarily due to increased efficiencies in production through time. However, one can argue that the driver of price changes can shift to the demand side, as we are now seeing with petroleum, and that when demand increases for a commodity with limited supply, prices will increase. Moreover, as discussed in LOS 50.e, the value of commodity futures is denominated in government currencies. Hence, if the real value of commodities is relatively constant, they will tend to increase in value if the value of the currency of denomination is eroded by inflation.

**Roll Yield**

The historical roll yield has been positive because the market was normally in backwardation. While the market may be moving towards a greater propensity for contango, there is no reason to suspect that it will not reverse course again in the future. The commodity futures markets have many more participants with varying motives than in the past, and the interactions among these participants creates opportunities for the market to move either way.

**Rolling Costs**

The rolling cost is the transaction costs incurred when selling one futures contract and buying another. While these costs exist, careful timing can help to minimize the impact on returns.

**Rebalancing Returns**

The source of the return premium in commodity index futures is primarily an academic debate. As we will see in LOS 50.d, under certain circumstances, rebalancing strategies can contribute positively to return.

**Lack of Cash Flows**

An asset is defined as something valuable. While commodities themselves do not generate cash flow, most commodities clearly have value. This is true in both the sense of value as a production input, such as grains or petroleum, or as a store of value, such as precious metals. So, while a bushel of grain does not generate cash flow in and of itself, it can be turned into a cash flow when it is sold to a user of grain who needs it as a production input to manufacture the end product.

The question as to whether commodity futures are an asset class is a bit more fraught. The answer to this depends upon whether they are, on average, valuable. When commodity futures markets are in backwardation, speculators in commodities expect to achieve positive roll yields as futures contracts mature. In this case, the answer would
appear to be yes, commodity futures are an asset. When markets are in contango, expected roll yields are negative, and the answer is not clear. The expected roll yield is negative, so the speculator with a long position is betting on an increase in the price of the underlying commodity. In this case, active management of the commodity futures position may be required to achieve positive returns.

LOS 50.d: Demonstrate how the geometric return of an actively managed commodity basket can be positive, whereas the underlying average commodity has a geometric return near zero.

During some observation periods, actively managed baskets of commodities have had positive returns, while the average return of the commodities in these baskets have been zero. How is this possible? The answer is that the prices of the individual commodities are not perfectly correlated, and the need to adhere to a strategic allocation across commodities forces the manager to sell commodities that have increased in value, while purchasing those that have decreased in value. If the returns are, on average, zero, the net result is a gain from the rebalancing.

Example: Calculating the geometric average for commodity returns

Consider a 2-commodity portfolio with a beginning value of $100. Over the upcoming two periods, the return on Commodity A is 100% and -50%, and that to Commodity B is 0% and 0%. Thus, the geometric average return to each commodity is exactly zero. The allocation to each is 50%, and the portfolio is rebalanced after each period. The portfolio values are as follows:

<table>
<thead>
<tr>
<th>Value</th>
<th>t = 0</th>
<th>t = 1</th>
<th>Rebalance</th>
<th>t = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity A</td>
<td>$50</td>
<td>$100</td>
<td>$75</td>
<td>$37.5</td>
</tr>
<tr>
<td>Commodity B</td>
<td>$50</td>
<td>$50</td>
<td>$75</td>
<td>$75.0</td>
</tr>
<tr>
<td>Portfolio</td>
<td>$100</td>
<td>$150</td>
<td>$150</td>
<td>$112.5</td>
</tr>
</tbody>
</table>

Calculate the geometric return of the portfolio and compare it to the geometric return of the components.

Answer:

The portfolio’s geometric average return is:

\[ R = \left( \frac{112.5}{100} \right)^{0.5} - 1 = 6.07\% \]

The geometric return of portfolio is 6.07%, even though the geometric return of each asset in the portfolio is zero.
LOS 50.e: Discuss why investing in commodities offers diversification opportunities during periods of economic fluctuation in the short run and inflation in the long run.

Returns to most asset classes, both positive and negative, tend to be concentrated across time. The early stages in an economic expansion, when demand for commodities may still remain weak, are often characterized by strong performance for equities. Conversely, during the late states of an expansion, demand for commodities is typically strong, while returns to equities often begin to falter. Thus, in the short run, commodity futures can provide diversification benefits for equity (and bond) portfolios across the economic cycle.

As we noted earlier, most commodities have value as either a production input or as a store of value. Their value is denominated in government currencies. Hence, if the real value of commodities is constant, they will tend to increase in value if the value of the currency of denomination is eroded by inflation. Thus, they provide a natural hedge against inflation. Moreover, many organizations have liabilities with nominal values that are increased by an erosion of currency value, such as inflation-indexed pension payments. The presence of commodity futures in the portfolio supporting these liabilities can provide a long-run hedge against inflation.
LOS 50.a
If there are no trading restrictions or storage costs, the relationship between the futures price and the spot price is governed by arbitrage. In practice, trading restrictions on the spot commodity often result in futures prices above or below their theoretical price.

Whether the futures price is above the spot price (contango) or below (backwardation) depends upon the relative demand for speculation and hedging between commodity producers, consumers, and investors.

If demand for investment in long futures increases, this is likely to reduce the prevalence of backwardation and increase the prevalence of contango.

LOS 50.b
Roll yield is the gain or loss when a commodity futures position matures and must be re-established.

Roll yield is positive during periods of backwardation, but negative under contango.

LOS 50.c
The main arguments against commodity futures as an asset class are as follows:
• Commodity prices have tended to decline in the long run.
• Roll yield is not guaranteed (and may be eliminated).
• Rolling costs will reduce returns.
• Rebalancing, rather than any change in the asset price, drives return premiums.
• Commodity futures do not produce cash flows.

LOS 50.d
An actively managed commodity futures portfolio can have positive returns during periods when the underlying commodities have zero returns, if the rebalancing framework leads the portfolio manager to sell high and buy low.

LOS 50.e
Commodity prices tend to be inversely correlated with stock returns over the economic cycle and positively correlated with inflation over the long term.
CONCEPT CHECKERS

1. Analyze the following two statements:

Statement 1: Commodity futures prices are observed to deviate from their theoretical prices because of trading frictions relating to the futures contracts.

Statement 2: When futures prices exceed spot prices, this is known as contango.

Are these statements correct or incorrect?
A. Both statements are correct.
B. Only one statement is correct, because deviations from theoretical prices are caused by trading frictions relating to the underlying commodity.
C. Only one statement is correct, because when futures prices exceed spot prices, this is known as backwardation.

2. When maturing futures positions are replaced with new futures contract positions, the resulting return outcome is called roll yield. This yield:
A. cannot be positive if futures markets are in backwardation.
B. is likely to be negative if the presence of long speculators increases, resulting in contango.
C. cannot be negative because the investor will realize a return equal to the risk-free rate on the portfolio collateral.

3. Evaluate the following two statements:

Statement 1: Commodity futures are not an asset class because they do not generate cash flows.

Statement 2: When markets are in contango, it will not be possible to generate positive returns from holding a long position in commodity futures.

Are these statements correct or incorrect?
A. Both statements are correct.
B. One is correct, because it will not be possible to generate positive returns with long futures if markets are in contango.
C. Neither is correct.

4. If the average return on a set of commodities is zero, but the returns are not perfectly positively correlated, the average returns on:
A. a passively managed portfolio holding those commodities can be greater than zero because of diversification benefits.
B. an actively managed portfolio holding those commodities can be greater than zero if the portfolio is periodically rebalanced to the strategic allocations.
C. an actively managed portfolio holding those commodities must be equal to zero if the portfolio is periodically rebalanced to the strategic allocations.
5. Consider the following two statements:

Statement 1: Commodity futures provide diversification across economic cycles, because commodity prices tend to move in the opposite direction as equity prices during some phases of the economic cycle.

Statement 2: In the long run, commodity futures provide diversification benefits for firms with liabilities that are related to currency values, because commodity prices tend to increase when inflation increases.

Are these statements correct or incorrect?
A. Both are correct.
B. One is correct, because commodity values decrease during inflationary periods.
C. Neither is correct.
1. B Only one of the statements is correct—when futures prices exceed spot, this is known as contango. When futures prices deviate from theoretical value, this is due to trading frictions relating to the underlying spot commodity. For example, it may be difficult to go short.

2. B Roll yields are likely to be negative when markets are in contango. This is because, on average, the price paid for the new futures contract will exceed the amount realized upon expiration.

3. C Neither is technically correct. Commodity futures would almost certainly be considered an asset class when markets are in backwardation because they would have positive expected returns based upon the cash flows derived from rolling the contracts over at maturity. It is possible to generate positive returns to long commodities futures positions when markets are in contango if the underlying commodity appreciates in value.

4. B The return on the actively managed portfolio can be greater than zero if the returns on the underlying commodities are not perfectly positively correlated and the rebalancing strategy results in the manager selling high and buying low after significant price movements. But, whether or not this is actually the case depends upon the rebalancing framework.

5. A Both of these statements are correct with regard to the diversification benefits of commodities and commodity futures.
The following is a review of the Alternative Asset Valuation principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

EVALUATING THE PERFORMANCE OF YOUR HEDGE FUNDS

Study Session 13

EXAM FOCUS

Hedge funds are a relatively new topic at Level 2. Only the basics of hedge fund performance are covered in this and the next topic review as a prelude to the more extensive treatment you’ll encounter next year at Level 3. For the exam, understand how the characteristics of hedge funds make their performance evaluation difficult. The use of market indexes, hedge fund indexes, and positive risk-free rates can serve as guideposts of performance, but should not be used as the final determinant of hedge fund performance.

WARM-UP: CLASSIFICATIONS OF HEDGE FUNDS

Recall from Level 1 that hedge funds can usually be classified by investment strategy; however, there is a great deal of overlap among categories. Some hedge fund classifications are:

- **Long/short funds** make up the largest category of hedge funds in terms of asset size. These funds take long and short common stock positions, use leverage, and are invested in markets worldwide. By definition, they are not market neutral but seek to profit from greater returns on the long positions than on the short positions.

- **Market-neutral funds** are a type of long/short fund that strive to hedge against general market moves. Managers may try to achieve this through any of several strategies, some involving derivatives. The fund may still have long and short positions, but the positions will offset each other so that the effect is a net zero exposure to the market.

- **Global macro funds** make bets on the direction of a market, currency, interest rate, or some other factor. Global macro funds are typically highly leveraged and rely heavily upon derivatives.

- **Event-driven funds** strive to capitalize on some unique opportunity in the market. This may involve investing in a distressed company or in a potential merger and acquisition situation.

WARM-UP: BACKGROUND ON TRADITIONAL PERFORMANCE MEASURES

Many different measures are typically used to analyze hedge fund performance. To provide some context for the discussion that follows, we provide here a brief review of two important measures: Jensen’s alpha and the Sharpe ratio. See Study Session 18 for more detail on each.
Jensen’s Alpha (ex post alpha)

Alpha is the difference between the account return and the return required to compensate for systematic risk. Alpha uses the ex post security market line (SML) as a benchmark to appraise performance. Alpha is a direct measure of performance (i.e., it yields the performance measure without being compared to other portfolios).

Recall the SML:

\[
E(R_A) = R_F + \beta_A \left[ E(R_M) - R_F \right]
\]

where:
- \( E(R_A) \) = the expected return on the account (portfolio)
- \( R_F \) = the risk-free rate of return
- \( E(R_M) \) = the expected return on the market
- \( \beta_A \) = the account’s beta (systematic risk)

Using data on actual returns, a simple linear regression is used to calculate ex-post alpha:

\[
\alpha_A = R_A - E(R_A)
\]

where:
- \( \alpha_A \) = the ex post alpha
- \( R_A \) = the return on the account
- \( E(R_A) = R_F + \beta_A \left[ E(R_M) - R_F \right] \)

Note:
A portfolio that generates a positive alpha would plot above the SML.
A portfolio that generates a zero alpha would plot on the SML.
A portfolio that generates a negative alpha would plot below the SML.

The Sharpe ratio

The Sharpe ratio calculates excess returns above the risk-free rate, relative to total risk measured by standard deviation. Sharpe is used to compare the portfolio performance to other portfolios.

\[
S_A = \frac{\bar{R}_A - \bar{R}_F}{\sigma_A}
\]

where:
- \( \bar{R}_A \) = average account return
- \( \bar{R}_F = \) average risk-free return
- \( \sigma_A \) = standard deviation of account returns
LOS 51.a: Discuss how the characteristics of hedge funds affect traditional methods of performance measurements.

The particular characteristics of a hedge fund pose challenges in measuring its performance. It is difficult to compare a hedge fund to a benchmark due to potential changes in leverage, changes in hedging techniques, changes in style, and portfolio turnover.

Changes in Leverage

Hedge funds often use leverage, with fixed income funds using more leverage than equity funds. Some funds lever 20 times the amount of their capital base. Furthermore, the amount of leverage used often varies within a month, with leverage amounts usually changing more dramatically for fixed income funds.

Although leverage changes the performance of a fund, leverage is usually not accounted for in performance evaluation. For example, if an equity fund has levered twice its capital base, it is logical that the return for the market index benchmark should also be adjusted to reflect the amount of leverage being used. However, this adjustment is not usually made.

Changes in Hedging Techniques

A market neutral fund typically strives to hedge long positions with short positions so that it has a zero beta position. The method that a fund uses to establish short positions, either by buying equity put options or short selling the stock, can have a large impact on performance. Using derivatives provides less protection in a down market than short selling, but caps losses at the option premium in an up market.

Professor's Note: Some market neutral funds are dollar-neutral, which means the dollar amount of the long position is equal to the dollar amount of the short position. However, this may not result in a zero-beta position if the betas of the long and short positions are different.

If the market (as measured by a market index) increases, the put option provides a more advantageous position because it expires worthless and allows the investor to participate in the higher market return. The investor is out only the put premium. In contrast, the short stock position decreases in value as much as the market increases, decreasing the fund's return.

If the market decreases, the put position increases in value according to the option delta. This change in value may not fully protect the fund's position, especially in the case of out-of-the-money puts. In contrast, the short stock position does not require a premium to be paid and can better protect the fund's position.

Professor's Note: See Study Session 17 for more on delta-neutral hedging.
Style Drift

As is the case with most hedge funds, a hedge fund manager can change her style over time to exploit perceived mispricing in various sectors. When she does so, the benchmarks originally assigned for evaluating the fund are no longer appropriate.

Portfolio Turnover

As markets become volatile, as they did in the bear market of 2000–2002, hedge fund managers often trade frequently to exploit the volatility. As a result, managers may stray from their stated strategy, which could result in inappropriate risk exposures for the investor. The return generated from such trading may reflect the manager’s skill, but it may also reflect luck.

LOS 51.b: Compare and contrast the use of market indices, hedge fund indices, and positive risk-free rates to evaluate hedge fund performance.

Broad Market Indexes

Individual investors tend to view hedge fund performance in absolute terms. In other words, did the fund provide a positive return or not, regardless of how the market did? In contrast, institutional investors view performance in relative terms: did the hedge fund outperform the market?

The problem in the latter case is that it is difficult to determine what benchmark index should be used to proxy for “the market.” Hedge funds themselves do not help in this matter because their portfolios experience high turnover and are not transparent. As a result, investors often use a broad-based market index such as the Russell 3000 to measure performance, even though that may be inappropriate.

Both the Sharpe ratio and Jensen’s alpha are used to measure hedge fund performance. There are problems using these measures to assess hedge fund performance. First, they require the use of a market portfolio benchmark. The S&P 500, which has been used for long-only funds, is not appropriate for hedge funds. Second, a hedge fund’s beta will be quite volatile over time as the fund manager adapts to changing market conditions or changes the fund’s risk exposures. Third, alpha can vary as the periodicity of the return interval changes (e.g., a monthly alpha will differ from a 5-year alpha). Alphas tend to deteriorate over time, with older fund alphas being lower than newer fund alphas.

The strategies for fixed income hedge funds are differentiated by CSFB/Tremont as being fixed income arbitrage, distressed securities, or convertible arbitrage. When they are compared against the Lehman Bond index, the Merrill Lynch High-Yield index, the S&P 500, and the Russell 3000, all three fixed income strategies have the highest correlation with the high yield index. This suggests that the best market index to use for a fixed income hedge fund is the high yield index.
Equity hedge funds can be differentiated as long/short equity, equity market neutral, global macro, or emerging markets. The Russell 3000 explains the returns of long/short equity and emerging markets funds the best, but it does about the same as the S&P 500 at explaining the returns of equity market neutral and global macro funds.

In sum, the Merrill Lynch High Yield index may be the best market index for fixed income hedge funds, and the Russell 3000 may be the best market index for equity hedge funds. However, a combination of indexes may be the best market index through the use of multifactor models. Multifactor models do the best in explaining mutual fund returns, which suggests that they might also be useful in explaining hedge fund returns.

Although market indexes are only rough measures of fund performance, returns much higher or lower than the benchmark index returns can be a cause for further investigation. Such excess performance could be the result of luck, skill, or extreme risk positions.

Hedge Fund Indexes

CSFB/Tremont provides hedge fund indexes for the previously described hedge fund types. These will be better benchmarks than the broad market indexes described previously, especially because the correlation among hedge fund returns is quite high.

However, there can be large differences in strategies between a hedge fund and a hedge fund index. For example, an interest rate swap arbitrage hedge fund and a forward yield curve arbitrage could be both classified as a fixed income arbitrage hedge fund, yet they could have very different strategies and different performance through time.

In addition, hedge fund indexes are susceptible to data problems and questionable statistics.

Data problems include:

- Managers voluntarily report performance to hedge fund index providers, and it is more likely managers will not report the results from poorly performing funds.
- Indexes may omit the returns of some large funds.
- Self-reported hedge fund data is not confirmed by index providers.

Problems with questionable statistics include:

- Constituent funds change over time, so the index is not consistent.
- Funds are subject to survivorship and backfill bias.
- Some constituent hedge funds are closed to new investors.
- Serial correlation in hedge fund data results in artificially low standard deviations.
- The historical record of hedge fund data is limited.
Professor’s Note: An example of survivorship bias is as follows. A hedge fund provider decides to begin a new index on January 1, 2008. To provide a historical record of returns from 2000 to 2007, the provider obtains the historical returns for all funds in existence in January 2008. The problem is that only the returns for survivors will be reported and the index return will be biased upwards. The performance of many poorly performing funds over the period 2000 to 2007 that are no longer in existence will not be reflected in the index return.

Backfill bias results when a new hedge fund is added to an index and the fund’s historical performance is added to the index’s historical performance. Again, the problem is that only funds that survived will have their performance added to the index, resulting in an upward bias in index returns.

Because of these problems, and because a hedge fund index may not match the strategy of a particular hedge fund, the indexes are best used as methods for detecting unusual performance. They can be misleading if they are used as the only basis for evaluating hedge fund performance.

The Risk-Free Rate

The risk-free rate is often advocated as a benchmark for hedge funds, sometimes with a spread of 3% to 6% added to it. The argument for using a positive risk-free rate is that investors desire a positive return, that arbitrage strategies are risk free, and that the interest earned on short positions is often tied to the risk-free rate. If a spread is added to the risk-free rate, it should reflect management costs and an expected excess return from active management.

Of the equity hedge funds, market neutral strategies should have a return that is closest to risk free. However, they are not completely risk free. Furthermore, although they are said to have a zero beta return, this is not always true, and there is no agreement on what the “market” is in the zero beta strategy. Lastly, if the fund is not well diversified, it will contain unsystematic risk.

On the fixed income side, arbitrage funds seek to exploit the spread between different risk securities. For example, a fund might go long a corporate bond and short a Treasury bond, thereby earning the difference in yields. However, these funds can suffer large losses if the yield on the risky bond rises while the yield on the Treasury falls. The use of leverage, sometimes 20 times the capital base, can further magnify risk. Thus, the risk-free rate is not an appropriate benchmark for fixed income hedge funds.

The Evaluation Process

To summarize, market indexes, hedge fund indexes, and positive risk-free rates should serve only as rough guideposts to performance. If a manager performs outside of these benchmarks, the investor should investigate whether the manager has shifted her strategy, taken more risk, and/or is just lucky.
An investor should have an understanding of a fund’s style, strategy, risk, and leverage. Competent performance evaluation should use more than a single index to assess a manager. However, there is no assurance that a combination of indexes will result in a correct benchmark to assess performance, and may ignore the investor’s particular investment objective.

For equity funds, the weighted average beta can be used to determine expected return. This is preferable to just using the net dollar amount invested long, because different positions can have very different risk exposures. For fixed income funds, the expected return can be estimated by multiplying the change in the portfolio credit spread times the amount of leverage. For either type of fund, a benchmark based on several factors may also be used.

In any case, the evaluation process should be viewed as an opportunity to better understand a manager’s strategy, risk exposures, and suitability for the investment objective.
Key Concepts

LOS 51.a  
*Changes in leverage:* Hedge funds often use leverage, while fixed income funds use more leverage than equity funds. The amount of leverage used often varies within a month with leverage amounts usually changing more dramatically for fixed income funds.

*Changes in hedging techniques:* A market neutral fund typically strives to hedge long positions with short positions so that it has a zero beta position. The method that a fund uses to establish short positions can have a large impact on performance. Using derivatives provides less protection in a down market than short selling, but caps losses at the option premium in an up market.

*Style drift:* A hedge fund manager can change her style over time to exploit perceived mispricing in various sectors. When she does so, the benchmarks originally assigned for evaluating the fund are no longer appropriate.

*Portfolio turnover:* As markets become volatile, hedge fund managers often trade frequently to exploit the volatility. As a result, managers may stray from their stated strategy, which could result in inappropriate risk exposures for the investor. The return generated from such trading may reflect the manager’s skill, but it could also reflect luck.

LOS 51.b  
Market indexes, hedge fund indexes, and positive risk-free rates can be used as guidelines for hedge fund performance, but should not be used for the final assessment of manager performance. If a manager performs outside of these benchmarks, the investor should investigate whether the manager has shifted her strategy, taken more risk, and/or is just lucky.

Hedge fund indexes are susceptible to the following data problems:
- Managers voluntarily report performance to hedge fund index providers, and it is more likely managers will not report the results from poorly performing funds.
- Indexes may omit the returns of some large funds.
- Self-reported hedge fund data is not confirmed by index providers.

Hedge fund indexes are susceptible to the following sources of questionable statistics:
- Constituent funds change over time, so the index is not consistent.
- Funds are subject to survivorship and backfill bias.
- Some constituent hedge funds are closed to new investors.
- Serial correlation in hedge fund data results in artificially low standard deviations.
- The historical record of hedge fund data is limited.
CONCEPT CHECKERS

1. A market neutral hedge fund will establish a short equity position. The best protection in a declining stock market would be provided by a(n):
   A. short sale of stock.
   B. in-the-money put.
   C. out-of-the-money put.

2. In general, which of the following market indexes would be the more appropriate index to use for a(n):
   - Fixed income hedge fund?
   - Equity hedge fund?
   A. Lehman Bond index  S&P 500
   B. Lehman Bond index  Russell 3000
   C. Merrill Lynch High Yield index  Russell 3000

Use the following information for Questions 3 through 5.

Portfolio manager George Mason-Stevens, CFA uses a series of custom tables to monitor the funds he recommends. He is currently reviewing three hedge funds: Pearl Street SuperFund series 1, 2, and 3. Each of these funds is classified by CSFB/Tremont as a long/short equity fund.

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<thead>
<tr>
<th>Quarter</th>
<th>Russell 3000</th>
<th>Merrill Lynch High Yield index</th>
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</thead>
<tbody>
<tr>
<td>3</td>
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<td>NM</td>
</tr>
<tr>
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<tr>
<td>---------</td>
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<td>---------------------</td>
</tr>
<tr>
<td>3</td>
<td>2006</td>
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<tr>
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**Correlation** 0.693

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<th>Eq LS Custom 2 Benchmark Return</th>
<th>Pearl St SF3 Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
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<td>-2%</td>
</tr>
<tr>
<td>4</td>
<td>2006</td>
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</tr>
<tr>
<td>1</td>
<td>2007</td>
<td>6%</td>
</tr>
<tr>
<td>2</td>
<td>2007</td>
<td>-5%</td>
</tr>
<tr>
<td>3</td>
<td>2007</td>
<td>7%</td>
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<td>6%</td>
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<td>2009</td>
<td>9%</td>
</tr>
<tr>
<td>2</td>
<td>2009</td>
<td>-11%</td>
</tr>
</tbody>
</table>

**Correlation** 0.522

3. All else equal, Pearl Street SuperFund series 1 has *most likely* experienced:
   A. changes in leverage.
   B. changes in hedging techniques.
   C. style drift.

4. All else equal, Pearl Street SuperFund series 2 has *most likely* experienced:
   A. changes in leverage.
   B. changes in hedging techniques.
   C. style drift.
5. All else equal, Pearl Street SuperFund series 3 has most likely experienced:
   A. changes in leverage.
   B. changes in hedging techniques.
   C. style drift.

6. Hedge fund indexes are least susceptible to which of the following data problems?
   A. Poorly performing funds tend to report performance to hedge fund index providers after the close of the reporting period.
   B. Indexes may omit the returns of some large funds.
   C. Self-reported hedge fund data is not confirmed by index providers.

7. Hedge fund indexes are least susceptible to which of the following questionable statistics?
   A. Funds are subject to survivorship and backfill bias.
   B. Some constituent hedge funds are closed to new investors.
   C. Serial correlation in hedge fund data results in artificially high standard deviations.
ANSWERS – CONCEPT CHECKERS

1. A In a declining stock market, a short sale of stock provides the best protection and an out-of-the-money put provides the worst protection. When the market decreases, the put increases in value according to the option delta. This change in value may not fully protect the fund's position, especially in the case of deep out-of-the-money puts. In contrast, the short stock position has no explicit cost (price) and can better protect a fund's position.

2. C In general, the best index to use for a fixed income hedge fund is the Merrill Lynch High Yield index, because it has the highest correlation with fixed income hedge funds. For equity hedge funds, the best index to use is the Russell 3000 index, because it typically has a significantly higher correlation with equity hedge funds than other indexes.

3. C Pearl Street SF 1 originally had significant correlation to the Russell 3000, but this correlation has fallen. Meanwhile, Pearl Street SF 1's correlation to the High-Yield Index has increased. This suggests the fund may have changed style.

4. A Pearl Street SF 2 closely tracks its benchmark, but suddenly changes from 1x the benchmark to almost 4x the benchmark. This suggests leverage is a significant factor in the fund's results.

5. B Pearl Street SF 3 originally had large changes in returns which are highly coincident with the benchmark. Lately, the returns have been much more moderate, but consistent despite wide variations in the benchmark. This suggests active hedging techniques have been implemented.

6. A Hedge fund indices have problems with data, including:
   • Managers voluntarily report performance to hedge fund index providers, and it is more likely managers will never report the results from poorly performing funds.
   • Indexes may omit the returns of some large funds.
   • Self-reported hedge fund data is not confirmed by index providers.

7. C Hedge fund indices have problems with questionable statistics, including:
   • Constituent funds change over time, so the index is not consistent.
   • Funds are subject to survivorship and backfill bias.
   • Some constituent hedge funds are closed to new investors.
   • Serial correlation in hedge fund data results in artificially low standard deviations.
   • The historical record of hedge fund data is limited.
The following is a review of the Alternative Asset Valuation principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**BUYERS BEWARE: EVALUATING AND MANAGING THE MANY FACETS OF THE RISKS OF HEDGE FUNDS**

**Study Session 13**

**EXAM FOCUS**

This topic review is a continuation of the relatively new topic of hedge funds at Level 2 that was introduced in the previous review. Here, we address the issue of hedge fund risk. For the exam, you should know that hedge funds are exposed to three broad areas of risk: investment risk, fraud risk, and operational risk. In terms of investment risk, although many hedge funds claim to be market neutral or low risk, both equity and fixed income funds have exposure to the equity market, style drift, and leverage. Fixed income funds are also exposed to changes in the credit spread. Because hedge fund risk is not easily quantified by the standard deviation of returns, investors should evaluate a downside measure of risk, such as maximum drawdown and/or value at risk. These two methods measure the risk of far left-tail events, as opposed to standard deviation, which measures total risk.

**LOS 52.a: Discuss common types of investment risks for hedge funds.**

The three broad categories of hedge fund risk discussed in this topic review are investment risk, fraud risk, and operational risk. Additional detail on each category follows.

**Investment Risk**

Investment risks are those due to the nature of the markets, manager styles, manager strategies, securities held, and derivatives used. Because hedge fund managers have so much flexibility in implementing their investment strategies, the potential investment risk is great. Note that a manager's style does not necessarily define the risks, as two managers pursuing the same style can have very different risks.

**Limited Information on Hedge Fund Risk**

In order to understand the risks of hedge funds, it is useful to have a history of performance to review. Unfortunately, most data for hedge funds only goes back as far as the late 1980s or early 1990s. Coincidentally, the historical period in the interim consists of a what is considered a fairly stable period in the financial markets, so it is difficult to forecast the future risks a hedge fund might face.
Credit Spreads and Fixed Income Funds

For fixed income hedge funds, the largest risk factor is a significant widening of credit spreads and when the widening persists for long periods. During the 1990s, which represents the largest span of data available for hedge funds, credit spreads were fairly stable, so the risk of fixed income hedge funds may be greater in the future.

Equity Market Risk and Fixed Income Funds

Widening credit spreads, declines in the stock market, and losses at fixed income hedge funds are correlated with one another. This relationship demonstrates that fixed income funds are also subject to equity risk.

It may seem that a declining stock market would be a positive for fixed income funds, given that investors often shift their funds from stocks to bonds when economic growth slows down. However, most fixed income arbitrage funds are short Treasuries and long bonds of higher credit risk. When economic growth slows down, credit spreads widen and the loss from the short Treasury position creates a net loss for a fixed income hedge fund, especially if the bond issuers on the long side are also experiencing financial difficulty.

Equity Market Risk and Equity Funds

Although many equity hedge funds claim to have zero beta positions, equity hedge funds do have equity market risk exposure. Merger arbitrage funds, for example, have a fairly high correlation with the equity markets. Although the volatility of returns for equity market neutral and risk arbitrage funds are quite low, the returns on these funds are also significantly correlated with equity market returns. So while these equity hedge funds are often successful in reducing the return volatility, they still have equity market exposure. Long/short equity funds have higher standard deviations and a higher equity market correlation than either equity market neutral or risk arbitrage funds.

Emerging markets arbitrage funds actually have low betas (e.g., less than 0.2) in up markets and positive betas (e.g., greater than 1.5) in down markets. This indicates that they don’t fully participate in bull markets and generate losses in bear markets. Thus, not only are they not market neutral, but their performance pattern is disadvantageous from the investor’s perspective.

For example, suppose a hedge fund has an up-market beta of 0.1 and a down-market beta of 1.5. When the market increases by 10%, the fund will generate a return of about 1%; however, then the market falls by 10%, the fund will realize losses of about 15%.

To summarize, both fixed income and equity hedge funds have more exposure to interest rate risk and equity market risk than commonly thought. In addition, both types of funds often experience significant losses at the same time, as they did in 1998 after the Russian debt defaults.

Style Drift and Leverage

It is often tempting for managers to change their investment style when a particular market segment is doing well. This often occurs during market downturns when
managers seek the safety of less risky investments. However, over the long-term, the risk-return trade-off is not favorable from a manager straying from his area of expertise.

Another risk arises from the use of leverage. Fixed income hedge funds typically use the most leverage, often as much as 20 to 30 times their capital base. These funds often seek to exploit yield spreads of less than 100 basis points, so to earn a return greater than 10%, they must leverage their asset base by at least 10 times.

**Fraud Risk**

The risk of investing in hedge funds is usually characterized by the variability of investment returns. However, investors also face the risk of fraud and operational risk.

The typical pattern is for the fund manager to misrepresent his background, qualifications, track record, or assets under management. The capital raised from investors is then used for the manager's personal welfare or other purpose. For these reasons, investors should conduct a background check on potential managers. The scrutiny of the manager's record should include a check of employment history, references, and court records.

The history of SEC prosecutions is both tragic and comical, given the lengths that managers go to in order to defraud investors. One fraud was carried out by a 22 year old living with his parents who claimed to be managing $250 million. In a more serious case, the Beacon Hill Asset Management firm obscured losses in its master fund in reported gains in other funds. The SEC charged the firm with misrepresenting its return by the way in which it calculated its net asset value. Beacon Hill allegedly did so in order to conceal large losses caused by the use of leverage.

To paraphrase an old adage, if the returns appear too good to be true, they probably are. This is particularly so when the managers are unknown to investors. Outsized returns should be investigated to determine if they are the result of luck, skill, excessive risk, or unusual trades.

**Operational Risk**

Operational risk arises from deficiencies in procedures, infrastructure, technology, resources, supervision, or trade data. Poor supervision, for example, can result in unauthorized trading, a breach of trading procedures, or questionable security valuations. The latter is a problem when a security is traded off an exchange because there may be few bid and asks quotes for the security. This is particularly a problem for illiquid securities and in times of crisis, when there may be no dealers willing to make a market in the security.

The more complex securities, (e.g., derivatives) may be especially hard to value because to do so requires sophisticated mathematical models. One problem with these models is that they can break down in volatile markets. This happens because the valuation produced by the model is very sensitive to its underlying assumptions (which may not be valid under certain market conditions). Exacerbating this problem, some over-the-counter contracts are tailored to an investor's specific needs. The valuation of the
position will then depend on the willingness of a specific dealer to take the other side of the customized contract. Furthermore, sometimes these contracts will specify the return as a multiple of an underlying asset so that the position is levered, even though the leverage is not apparent to outside investors. When the problems with valuation are combined with the lax regulatory environment and weak internal controls present in many hedge funds, potential problems arise.

Counterparty risk is a problem when securities are traded off an exchange. One glaring example was the case of Enron, which was a dominant energy trader. When the firm went bankrupt, the counterparties to their trades suffered. Transactions risk, the risk that a trade will not settle on the settlement date, is another potential problem. Settlement risk can be particularly problematic for hedge funds with high leverage that use derivatives extensively when trading in emerging markets.

LOS 52.b: Evaluate maximum drawdown and value-at-risk for measuring risks of hedge funds.

The discussion in the previous LOS suggests that the risks of hedge funds are numerous and not easily quantified by their standard deviation of returns. Hedge fund risk should be carefully monitored using a risk management system that identifies worst case scenarios, detects departures from stated investment strategies, assesses performance, and identifies the sources of the manager’s performance. Risk management is an ongoing process: risk should not just be assessed before the investment is made.

When measuring risk with the standard deviation, we make the assumption that returns are normally distributed. However, hedge fund returns are often not normally distributed because their return distributions have fat tails on the left hand side. In other words, the probability of large losses is greater than that expected from a normal distribution. For this reason, it is imperative that investors evaluate a downside measure of risk, such as maximum drawdown and/or value at risk.

Maximum Drawdown

Drawdown is the percentage decrease in investment value from its peak to its valley. Maximum drawdown is the largest drawdown that has ever occurred for a fund within a specified time period. The maximum drawdown is quoted so that investors can have an idea of the largest loss they can expect from a hedge fund investment. It measures left tail risk. For example, a maximum drawdown of 15% would imply less risk than a maximum drawdown of 25%.

The problem with this measure is that it does not provide the probability of a large loss. Using the broad stock market as an example, there was a 25% decline in the market in October 1987. However, this occurrence is quite rare and not typical for the stock market. Furthermore, stocks performed strongly during the 1980s as a whole and the overall increase in investment value was quite high during that decade.
Value at Risk

Value at Risk (VAR) provides both the amount of an expected largest loss as well as its probability. For example, the VAR could be stated as “there is a 95% chance that a fund will not lose more than 11.4% of its value over the next quarter.” In this example, the investor is provided with both the left tail risk value and the chance that it will occur.

However, the measure is not frequently used for hedge funds because it has several shortcomings.

- First, given that VAR is usually estimated using historical data, it is not indicative of future risk when a fund changes its investment strategy over time.
- Second, the interpretation of VAR is meaningful only when the return distribution is normal. Hedge fund returns are rarely normally distributed.
- Third, VAR is often computed assuming that component risks are additive, when in fact they are often multiplicative. For example, the risk and return of an investment in Japanese stocks by a U.S. investor is a multiplicative function of both the stock return in yen terms and the change in the yen/dollar exchange rate.

The value of maximum drawdown and VAR as risk measures for hedge funds is that if they have been historically high, the fund will typically continue to have high risk. Once an investment is made, the measures should be monitored, and if they signal higher risk, then investors should be alert for changes in operational and investment risk. For example, an increase in VAR accompanied by a resignation of a hedge fund officer could be a warning sign.

Alternative Risk Measures

Other risk measures assess the entire left side of the distribution, not just the extremes, as do maximum drawdown and VAR. The loss standard deviation is analogous to the standard deviation, except that it focuses on the left side of the return distribution. The downside deviation is similar to the loss standard deviation except that a minimum acceptable return is specified. The Sortino ratio is like the Sharpe ratio, except that it uses the downside deviation and the minimum acceptable return in place of the risk-free rate.
**Key Concepts**

**LOS 52.a**
Hedge funds are exposed to investment risk, fraud risk, and operational risk.

The investment risks of investing in hedge funds are high due to a limited availability of information, changes in credit spreads, equity market risk, style drift, and leverage.

**LOS 52.b**
Because hedge fund returns are often not normally distributed, investors should evaluate a downside measure of risk, such as maximum drawdown and/or value at risk. Maximum drawdown provides an estimate of the magnitude of the largest percentage loss.

Value at Risk (VAR) provides both the amount of an expected largest loss as well as its probability. VAR is not frequently used for hedge funds because of the following shortcomings:

- VAR is usually estimated using historical data, which is not indicative of future risk when a fund changes its investment strategy over time.
- The interpretation of VAR is meaningful only when the return distribution is normal. Hedge fund returns are rarely normally distributed.
- VAR is often computed assuming that component risks are additive, when in fact they can be multiplicative.

The Sortino ratio (similar to the Sharpe ratio) uses the downside deviation and the minimum acceptable return in place of the risk-free rate.
CONCEPT CHECKERS

1. Which of the following best describes the overall volatility of equity market neutral hedge funds and their correlation with equity markets?
   - Overall volatility Correlation with equity markets
   A. Low Significant
   B. Low Insignificant
   C. High Insignificant

2. In general, which of the following best characterizes the assumptions of the VAR calculation regarding the distribution of returns and the computation of component risks?
   - Distribution of returns Computation of component risks
   A. Normal Additive
   B. Normal Multiplicative
   C. Non-normal Multiplicative

3. Does the maximum drawdown method provide an estimate of the:
   - Magnitude of the Probability of the
     largest % loss? largest % loss occurring?
   A. Yes Yes
   B. Yes No
   C. No No

CHALLENGE PROBLEMS

Use the following information to answer Questions 4 through 9.

Endowment fund manager Shelly Davis wants to increase her portfolio’s exposure to alternative investments and has hired Davey Jarvis, econometrics consultant, to recommend a series of risk metrics to use in evaluating prospective hedge funds. She also asks for a list of key operational risks, which hedge funds typically face, and potential consequences of operational risk.

4. Which of the following is Jarvis most likely to recommend as a measure of the entire left side of the probability-density function?
   A. Sharpe ratio.
   B. Value at Risk.
   C. Sortino ratio.

5. Which of the following is Jarvis is most likely to recommend as a measure of risk at a given probability?
   A. Value at Risk.
   B. Maximum Drawdown.
   C. Projected Drawdown.
6. Which of the following risk metrics incorporates the concept of a minimum acceptable return?
   A. Maximum drawdown.
   B. Clawback ratio.
   C. Sortino ratio.

7. The maximum drawdown is best thought of as:
   A. the sum of all downside deviations since fund inception.
   B. the lowest value of the fund since inception.
   C. the largest drawdown that has ever occurred at a fund within a specified time period.

8. Which of the following is most likely to make Jarvis' list of key operational risks?
   A. Model misspecification.
   B. Deficient infrastructure.
   C. Employee slippage.

9. Which of the following are least likely to appear on Jarvis' list of potential consequences of operational risk?
   A. Breach of trading procedures.
   B. Unauthorized trading.
   C. Investor drawdown.
Answers - Concept Checkers

1. A The volatility of returns for equity market neutral and risk arbitrage funds is quite low. However, the fund returns are significantly correlated with equity market returns. So, although these equity hedge funds are often successful in reducing the volatility of returns, they still have equity market exposure.

2. A VAR is usually calculated under the assumption that the left side returns are normally distributed. Note that hedge funds returns are rarely normally distributed. VAR is computed assuming that component risks are additive, when in fact they are often multiplicative.

3. B Maximum drawdown provides an estimate of only the magnitude of the largest percentage loss. VAR provides estimates of both the magnitude and probability of the largest loss.

Answers - Challenge Problems

4. C The Sortino ratio is like the Sharpe ratio, except that it uses the downside deviation and the minimum acceptable return in place of the risk-free rate. The downside deviation is similar to the loss standard deviation except that a minimum acceptable return is specified. VAR can be helpful in some asset classes but needs to be heavily modified to be meaningful for hedge fund evaluation.

5. A Value at Risk (VAR) provides both the amount of an expected largest loss as well as its probability. VAR is not frequently used for hedge funds but is the best choice. Maximum drawdown is a historical change, not a forecast probability. “Projected drawdown” has no meaning in this context.

6. C The Sortino ratio is like the Sharpe ratio, except that it uses the downside deviation and the minimum acceptable return in place of the risk-free rate.

7. C The maximum drawdown is defined as the largest percentage drawdown that has ever occurred at a fund within a specified time period.

8. B Operational risk arises from deficiencies in procedures, infrastructure, technology, resources, supervision, or trade data. Employee slippage (i.e., shoplifting) is unlikely at a hedge fund.

9. C Examples of consequences of operational deficiencies (i.e., poor supervision) include unauthorized trading and breaches of trading procedures. “Investor drawdown” has no meaning in this context.
Self-Test: Alternative Investments

Use the following information to answer Questions 1 through 6.

Eva Williams is an investment manager for Straughn Capital Management (SCM). Williams believes that it would be beneficial to add some real estate investments to SCM’s existing portfolio. She has asked a local real estate broker, Steven Riley, to present some investment ideas to her. Riley is not certain which type of property might be most suitable for SCM, so he has prepared information regarding three different types of investment property. The first property is an undeveloped plot of land in an area that is not very heavily populated, but is on the fringe of a rapidly growing city. The second property is a hotel in the downtown district of the same city. The third property is a small shopping center in a well-developed, but declining section of the city.

While describing each of the properties, Riley makes the following statements:

Statement 1: The raw land is really a great opportunity. An investor is practically guaranteed steady price appreciation with any raw land deal given the way the city is growing. Another benefit is that if an investor buys the land and decides to sell it later, the investor will find that undeveloped land is very liquid. This is because the possible uses for undeveloped land are virtually unlimited.

Statement 2: Hotel properties do not require active management, so an investor in this type of property would not have to be very involved with the day-to-day operations. The current income from operations would be considerable, but hotels tend not to provide returns in the form of price appreciation. The current managers don’t seem to be very efficient, but that is a secondary concern with this type of property. One considerable risk with hotels is potential competition from major chains that appear to have an interest in moving into the downtown area.

Statement 3: The community where the shopping center is located is not growing, so that may cause future rental income growth to slow. Also, median incomes are falling in that area, but that should not affect the value of the property because the property draws customers from a larger area. The center has a great mix of tenants and very low vacancy rates.

Riley also provides certain operating data for the hotel and the shopping center properties. The market value of the hotel is $2,500,000. Net operating income for the upcoming year is expected to be $275,000, and is expected to grow at a constant annual rate of 6% for the foreseeable future.

The shopping center would require an initial investment of $1,525,000 and is expected to generate after-tax cash flow of $330,000 for the next five years. In year six, a significant renovation would be required. This would result in after-tax cash flow of −$700,000 during that year. Following the renovation, after-tax cash flows would be $450,000 for years seven through twelve. Riley believes that a reasonable risk-adjusted after-tax return on this type of property would be 16%.
1. With respect to Statement 1, Riley’s assertions regarding the price appreciation and the liquidity of raw land investments are:

<table>
<thead>
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<th>Liquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Correct</td>
<td>Incorrect</td>
</tr>
<tr>
<td>B. Incorrect</td>
<td>Correct</td>
</tr>
<tr>
<td>C. Incorrect</td>
<td>Incorrect</td>
</tr>
</tbody>
</table>

2. In Statement 2, Riley makes several generalizations about hotel investments. Which of Riley’s statements is most likely to be true given the actual characteristics of hotels as investments?

A. Competition is a primary risk for hotel investments.
B. Hotels provide a return from income, but not price appreciation.
C. Hotels do not require active management, so the investor would not have to be involved in the day-to-day operations.

3. Based on the information provided in Statement 3, which factors are most likely to have a negative impact on the market value of the shopping center property?

A. Median incomes and vacancy rates.
B. Community growth and vacancy rates.
C. Community growth and median incomes.

4. What is the market capitalization rate for the hotel?

A. 5%.
B. 6%.
C. 11%.

5. What is the required ROE for the hotel?

A. 6%.
B. 11%.
C. 17%.

6. What are the Net Present Value (NPV) and Internal Rate of Return (IRR) for the shopping center?

A. NPV = −$51,226, IRR = 15.21%.
B. NPV = −$51,226, IRR is unreliable.
C. NPV = $523,394, IRR = 23.19%.
1. C One of the principal characteristics of raw land investments is an appreciation in value. However, one of the primary risks is the unstable and unpredictable pattern of that appreciation. Riley's assertion that the price appreciation will be stable makes this part of the statement incorrect. Another principal characteristic of raw land is relatively low liquidity. Therefore, Riley's claim that this type of property is very liquid is also incorrect.

2. A Competition and competent management are two of the primary risks for hotel properties. Therefore, his assertion regarding competition is correct, but his assessment of the importance of competent managers is incorrect. Hotels typically require active management that would be likely to involve the investor on some level, maybe even in the day-to-day operations. Hotels typically provide a return from income AND price appreciation.

3. C Limited community growth and declining median incomes would tend to put downward pressure on the value of the property. A strong tenant mix and low vacancy rates would tend to increase the value of the property.

4. C Within the direct income capitalization framework, which is equivalent to the constant growth model, \( MV_0 = \frac{NOI_1}{R_0} \), where \( MV_0 \) is the current market value, \( NOI_1 \) is the net operating income for the coming year, and \( R_0 \) represents the market capitalization rate. Therefore, \( R_0 = \frac{NOI_1}{MV_0} = \frac{275,000}{2,500,000} = 0.11 \) or 11%.

5. C The market capitalization rate equals the difference between the required rate of return on equity minus the constant growth rate of net operating income (\( R_0 = r - g \)). So, 0.11 = r - 0.06. Solving for \( r \) results in a required return on equity of 0.11 + 0.06 = 0.17, or 17%.
6. B The NPV of the shopping center is -$51,226. In this case the IRR is unreliable for investment purposes because there is more than one sign change in the cash flow stream. The IRR should not be used when there is more than one sign change in the cash flow stream. The shopping center investment should be rejected because the NPV is negative. The NPV can be calculated as follows using the TI BA II Plus:

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<thead>
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<th>Procedure</th>
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<td>[CF]</td>
<td>CF0 (old contents)</td>
</tr>
<tr>
<td>Clear worksheet</td>
<td>[2nd] [CLR WORK]</td>
<td>CF0 = 0.00</td>
</tr>
<tr>
<td>Enter initial cash flow</td>
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<td>CF0 = -1,525,000</td>
</tr>
<tr>
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<td>C01 = 330,000</td>
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<tr>
<td></td>
<td>↓ 5 [ENTER]</td>
<td>F01 = 5.00</td>
</tr>
<tr>
<td>Enter cash flow for year 6</td>
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<td>C02 = -700,000</td>
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<tr>
<td></td>
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</tr>
<tr>
<td>Enter cash flows for years 7–12</td>
<td>↓ 450000 [ENTER]</td>
<td>C03 = 450,000</td>
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<td>Access NPV portion of cash flow worksheet</td>
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<td>Enter interest rate per period</td>
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<tr>
<td>Compute net present value</td>
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The following is a review of the Fixed Income: Valuation Concepts principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**GENERAL PRINCIPLES OF CREDIT ANALYSIS**

**EXAM FOCUS**

This topic review covers the analysis of the credit risk of corporate, high-yield, asset-backed, municipal, and sovereign bonds. It is the only credit analysis material in the Level 2 curriculum, so it is an important topic that you should spend some study time on to make sure you've mastered the details. In particular, you must be able to identify and discuss the factors that are most important in the analysis of each type of issue.

**CREDIT RISK**

**LOS 53.a: Distinguish among default risk, credit spread risk, and downgrade risk.**

Credit risk encompasses three distinct types of risk:

1. **Default risk** is the risk that the borrower will not repay the obligation.
2. **Credit spread risk** is the risk that the credit spread will increase and cause the value of the issue to decrease and/or cause the bond to underperform its benchmark.
3. **Downgrade risk** is the risk that the issue will be downgraded by the credit rating agencies, which will also cause the bond price to fall, and/or cause the bond to underperform its benchmark.

Credit ratings issued by the nationally recognized rating agencies (Standard & Poor's, Moody's, and Fitch) assess only default risk. There are three sources of information released by the rating agencies that are useful for investors and analysts assessing the default risk of a bond:

1. **Credit rating.** For long-term debt, the credit rating reflects the probability of default (the default rate) and the loss to the investor if default occurs (the default loss rate). For short-term debt, the credit rating reflects only probability of default. The higher the credit rating, the lower the default rate and the default loss rate.

2. **Rating watch.** The rating agencies typically announce that they are reviewing a particular issue in advance of a potential upgrade or downgrade in the short term. For example, an upgrade watch means the agency may issue an upgrade, usually within three months.
3. **Rating outlook.** The agencies also issue longer-term projections (which look out six months to two years) of whether an issue is likely to be upgraded (a positive outlook), downgraded (a negative outlook), or keep its current rating (a stable outlook).

Moody's suggests that the analyst or investor use these three sources of information in the following manner when assessing the default risk of a particular issue:

- Reduce the current rating by two rating notches (e.g., from A1 to A3) for a downgrade watch.
- Reduce the current rating by one notch (e.g., from A1 to A2) for a negative outlook.
- Maintain the current rating for a stable outlook.
- Increase the current rating by one notch (e.g., from A1 to Aa3) for a positive outlook.
- Increase the current rating by two notches (e.g., from A1 to Aa2) for an upgrade watch.

**LOS 53.b: Explain and analyze the key components of credit analysis.**

A common way to categorize the key components of credit analysis is by the “four Cs of credit analysis”: character, covenants, collateral, and capacity.

**Character**

Character refers to management’s integrity and its commitment to repay the loan. Other factors, such as management’s business qualifications and operating record, are also important. Management’s ability to react appropriately to unexpected events is one of the most important factors considered by the rating agencies in assigning a credit rating. Credit analysts also place emphasis on management’s strategic direction, financial philosophy, conservatism, track record, succession planning, and control systems.

An important focus of the analysis of the quality of management involves the firm’s corporate governance structure. As part of our discussion of credit analysis of corporate bonds, we will simply note a few key concepts you should keep in mind for the exam. Please see the topic review “Corporate Governance” in Study Session 9 for specific details about the objectives and attributes of a corporate governance system, methods of analyzing its strengths and weaknesses, and the valuation implications of corporate governance.

Here is a short list of four key corporate governance best practices to reduce the influence of the CEO on the board of directors:

1. A larger board reduces the influence of the CEO and allows for separate auditing, compensation, and nominating committees.
2. The majority of the board members should be independent directors, and the three committees mentioned in the first bullet point should be composed entirely of independent directors.
3. The nominating committee, not the CEO, should be responsible for identifying potential new board members.
4. The CEO should not be the chairman of the board.
In a study of the relationship between bond ratings and corporate governance structures, Bhojraj and Sengupta\(^1\) report the following findings:

- Companies with a greater degree of institutional ownership and stronger outside control of the board of directors had lower bond yields and higher credit ratings.
- Corporate governance is especially important for lower-rated corporate bonds because traditional credit analysis (ratio and cash flow analysis) may not be useful in assessing the issuer’s ability to satisfy its future financial obligations.

A number of firms (such as Standard & Poor’s) provide corporate governance ratings. The higher the rating, the more effective the company’s corporate governance mechanisms, and presumably the lower the credit risk, all else equal.

**Covenants**

Covenants are the terms and conditions the borrowing and lending parties have agreed to as part of the bond issue. They include restrictions on management’s ability to make operating and financial decisions in the normal course of business. There are two types of covenants: affirmative covenants and negative covenants.

1. **Affirmative covenants** require the debtor to take certain actions, such as paying interest, principal, and taxes, complying with loan agreements, and maintaining properties.

2. **Negative covenants** prohibit the borrower from taking certain actions, usually by requiring the borrower to maintain certain ratios at specified levels. They place limitations on the borrower’s ability to incur additional debt, maintain sufficient coverage of interest expense and other fixed charges, place restrictions on borrowings by subsidiaries, maintain certain levels of cash flow and working capital, and place limitations on dividend payments and stock repurchases.

Covenants impose limitations and restrictions on a borrower’s activities that may lead to a decline in the issuer’s ability to repay. At the same time, covenants also create a legally binding contractual framework for the repayment of the debt obligation, which reduces uncertainty to the debtholders. Therefore, careful credit analysis should also include an assessment of whether the covenants protect the interests of the bondholders while not unduly restricting the operating and strategic decisions of the borrower. Analysis of covenants is particularly important for high-yield issuers.

**Collateral**

Collateral includes the assets offered as security for the debt as well as other assets controlled by the issuer. A company’s borrowings may be secured with a pledge of assets, or they may be unsecured. The value of the unpledged assets is also an important consideration when evaluating collateral. However, the priority of a secured claim is sometimes a questionable issue in corporate reorganizations and liquidations under the bankruptcy laws in the United States. For these reasons, collateral analysis is probably the least useful in assessing corporate credit risk.

Capacity to Pay

Professor's Note: We will discuss three specific topics related to capacity to pay: sources of liquidity, ratio analysis, and cash flow analysis. The first topic is mentioned in this LOS (53.b). The other two have their own LOS (53.c and 53.e, respectively).

Capacity refers to the corporate borrower's ability to generate cash flow or liquidate short-term assets to repay its debt obligations. The firm's liquidity position is a key determining factor in its capacity to repay its debt obligations. Ultimately, the firm must have cash available at the time debt payments are due from one of five sources. Many apparently successful firms have gone bankrupt because of a lack of liquidity, despite impressive sales growth and profit margins.

Moody's uses the following factors (in addition to management quality) in assessing the issuer's capacity to pay (note the links to other places in the Level 2 curriculum):

- Industry trends (see Porter's five forces in Study Session 11).
- Regulatory environment (see the topic review of government regulation in Study Session 4).
- Operating and competitive position (see Porter's generic strategies in Study Session 11).
- Financial position and sources of liquidity, company structure, parent company support agreements, and special event risk (see the discussion in this topic review).

In determining the ability of a firm to pay its debt, analysts examine the corporation's financial position as well as its liquidity sources. Creditors can rely on a number of the debtor's sources of liquidity to repay debt:

- The firm's working capital (the difference between its current assets and current liabilities) is an important source of liquidity, particularly in certain industries. For example, traditional manufacturing firms buy raw materials on credit, convert them to finished inventory, sell the product on credit, collect from their customers, and pay their suppliers. The ability to quickly turn inventory and accounts receivable into cash and pay their bills is crucial to financial success. The analyst can use short-term solvency ratios to assess the company's working capital position, which we will discuss in LOS 53.c. High solvency ratios indicate a greater capacity to repay.
- Steady, dependable cash flow is an extremely important source of a firm's liquidity. We discuss cash flow analysis later in LOS 53.e.
- There are a variety of forms of back-up facilities, and the relative strength of these must be assessed. For instance, a lending agreement whereby the lender is contractually obligated to provide back-up financing is very strong. In contrast, an informal agreement, such as a line of credit, is relatively weak because the lender has the right to refuse to provide funds.
- Firms can generate liquidity by securitizing assets as a substitute for short-term bank debt. See the topic reviews of mortgage-backed securities and asset-backed securities in Study Session 15 for a detailed discussion of asset securitization.
- Additional third-party guarantees (by a parent company, for example) require a credit analysis of that entity.
CREDIT ANALYSIS WITH RATIOS

LOS 53.c: Calculate and interpret the key financial ratios used by credit analysts.

LOS 53.d: Evaluate the credit quality of an issuer of a corporate bond, given such data as key financial ratios for the issuer and the industry.

Professor’s Note: As you may recall, we spent a lot of time and effort on analyzing financial statements using ratio analysis back in Study Sessions 5, 6, and 7. The material here is a discussion of ratio analysis specifically focused on credit analysis from the viewpoint of a bondholder or creditor. Because the rating agencies use standardized ratios, when you run into a credit analysis item set on the exam, use the ratios and techniques in this topic review to answer the questions.

Profitability ratios assess the issuer’s ability to generate earnings sufficient to pay interest and repay principal. The DuPont framework is very useful in this regard. See Study Session 11 for more detail on the DuPont model.

Return on equity (ROE) can be estimated with the DuPont formula, which presents the relationship between margin, sales, and leverage as determinants of ROE.

\[
\text{ROE} = \frac{\text{net income}}{\text{stockholders' equity}} = \left(\frac{\text{net income}}{\text{sales}}\right) \times \left(\frac{\text{sales}}{\text{total assets}}\right) \times \left(\frac{\text{total assets}}{\text{stockholders' equity}}\right)
\]

Short-term solvency ratios are a measure of the firm’s ability to repay its short-term debt obligations by liquidating short-term assets. Working capital is important in determining a firm’s ability to pay its obligations as they come due. The analyst can use the current ratio or the acid-test ratio to determine whether the firm is able to meet its short-term obligations. Use industry averages as a benchmark. Companies with solvency ratios greater than the industry average have a greater ability to repay short-term debt.

\[
\text{current ratio} = \frac{\text{current assets}}{\text{current liabilities}}
\]

\[
\text{acid-test ratio} = \frac{\text{current assets} - \text{inventories}}{\text{current liabilities}}
\]

Example: Analyzing short-term solvency ratios

A firm has current assets of $60, $80, and $90 in years 1 through 3, respectively. The firm’s current liabilities were $50, $80, and $120 over the same time period. Compute and analyze the trend in the firm’s current ratio.

Answer:

The current ratio is 1.20 in year 1, 1.00 in year 2, and 0.75 in year 3. The trend is negative, indicating an erosion of the firm’s liquidity.
Capitalization (financial leverage) ratios are evaluated with reference to the industry in order to determine the firm's ability to take on the additional risk associated with increased borrowing. Two of the many measurements of the firm's ability to service its debt are:

\[
\text{long-term debt-to-capitalization ratio} = \frac{\text{long-term debt}}{\text{long-term debt + minority interest + shareholders' common and preferred equity}}
\]

\[
\text{total debt-to-capitalization ratio} = \frac{\text{current liabilities + long-term debt}}{\text{current liabilities + long-term debt + minority interest + shareholders' common and preferred equity}}
\]

Companies with capitalization ratios greater than the industry average have less capacity to take on more long-term debt.

Operating leases represent a long-term obligation that should be considered long-term debt. Thus, rating agencies often adjust long-term debt to reflect the present value of future payments under operating leases.

Example: Analyzing capitalization ratios

MidWest Electric is an electric utility. The firm has current liabilities of $60,000,000, long-term debt of $330,000,000, minority interest in subsidiaries of $110,000,000, and shareholders' equity of $620,000,000. The present value of expected operating lease payments (as disclosed in the footnotes) is $40,000,000. If the electric utilities industry has a long-term debt-to-capitalization ratio of 0.8 and a total debt-to-capitalization ratio of 0.85, assess MidWest's leverage position relative to the industry.

Answer:

MidWest's long-term debt-to-capitalization ratio is 0.34 = \(\frac{330 + 40}{330 + 110 + 620 + 40}\), and its total debt-to-capitalization ratio is 0.37 = \(\frac{60 + 330 + 40}{330 + 60 + 110 + 620 + 40}\). Given that MidWest has capitalization ratios much lower than the industry average, MidWest's ability to service additional debt is probably high.
Coverage ratios measure the firm’s ability to repay its debt and lease obligations out of operating cash flow. The earnings before interest and taxes (EBIT) coverage ratio is:

\[
\frac{\text{EBIT}}{\text{annual interest expense}}
\]

The earnings before interest, taxes, depreciation, and amortization (EBITDA) coverage ratio is:

\[
\frac{\text{EBITDA}}{\text{annual interest expense}}
\]

For both ratios, interest expense includes capitalized interest.

Coverage ratios may fluctuate widely over time. Thus, the variability of these ratios is an important consideration to a bondholder; companies with stable, comfortable coverage ratios are usually more creditworthy.

### Example: Analyzing coverage ratios

Use the information below to calculate coverage ratios for Southern States Power over the last three years.

#### Southern States Selected Income Statement Items

<table>
<thead>
<tr>
<th>Income Statement Item</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT</td>
<td>$500</td>
<td>$600</td>
<td>$200</td>
</tr>
<tr>
<td>Depreciation and amortization</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Interest expense</td>
<td>150</td>
<td>175</td>
<td>200</td>
</tr>
</tbody>
</table>

#### Answer:

\[
\text{EBIT coverage (year 1)} = \frac{\$500}{\$150} = 3.3
\]

\[
\text{EBITDA coverage (year 1)} = \frac{\$500 + \$100}{\$150} = 4.0
\]

If you do the same calculations for years 2 and 3, you'll find:

#### Southern States Coverage Ratios

<table>
<thead>
<tr>
<th>Income Statement Item</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT coverage</td>
<td>3.3</td>
<td>3.4</td>
<td>1.0</td>
</tr>
<tr>
<td>EBITDA coverage</td>
<td>4.0</td>
<td>4.0</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Southern’s ability to generate funds from operations to meet its debt obligations deteriorated significantly to dangerously low levels in year 3 as both coverage ratios fell. The analyst should be concerned with coverage ratios close to 1.

**Application of Credit Ratio Analysis**

We can use ratio analysis to identify firms that are in a weak financial position and are candidates for downgrading by the rating agencies. The rating agencies specify ranges of short-term solvency, capitalization, and coverage ratios for each debt rating. A firm whose liquidity ratios have fallen, whose capitalization ratios have increased, and/or whose coverage ratios have decreased, is a candidate for a downgrade.

<table>
<thead>
<tr>
<th>Example: Identify credit downgrade candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>An analyst has compiled the following information on the bonds of Winter Sports, Inc., which is currently rated “A.”</td>
</tr>
</tbody>
</table>

**Ratio Analysis of Winter Sports, Inc.**

<table>
<thead>
<tr>
<th></th>
<th>Current Ratio</th>
<th>Debt to Capitalization</th>
<th>EBIT/Interest Expense</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guideline for A-rated issues</td>
<td>1.00 to 1.20</td>
<td>0.35 to 0.45</td>
<td>3.00 to 4.00</td>
</tr>
<tr>
<td>Winter Sports Inc.</td>
<td>0.98</td>
<td>0.49</td>
<td>2.60</td>
</tr>
</tbody>
</table>

Discuss whether Winter Sports is a candidate for a ratings downgrade.

**Answer:**

Winter Sports’ credit quality has declined relative to the ratings guidelines for A-rated issues, suggesting a downgrade is likely. The firm’s liquidity has deteriorated, leverage has increased, and coverage has fallen.

**Cash Flow Analysis**

**LOS 53.e:** Analyze why and how cash flow from operations is used to assess the ability of an issuer to service its debt obligations and to assess the financial flexibility of a company.

The primary resource for repayment of debt is cash flow. Hence, an analysis of operating cash flows is critical to understanding the firm’s capacity to pay. However, the rating agencies often use cash flow measures that are slightly different than the statement of
cash flows or the FCFF/FCFE framework from Study Session 12. S&P, for example, uses the following cash flow measures:\(^2\)

- Net income
- + depreciation
- +/- other noncash items
- Funds from operations
- + decrease (increase) in noncash current assets
- + increase (decrease) in nondebt current liabilities
- Operating cash flow
- - capital expenditures
- Free operating cash flow
- - cash dividends
- Discretionary cash flow
- - acquisitions
- + asset disposals
- + other sources (uses)
- Prefinancing cash flow

Professor’s Note: These cash flow definitions are similar to, and in some cases the same as, other cash flow definitions found in the equity material in SS12:

- “Free operating cash flow” is similar to “free cash flow to the firm” (FCFF) from Topic Review 43, except it’s not adjusted by adding back after-tax interest expense, which is equal to interest × (1 – tax rate).
- “Funds from operations” is the same as “earnings-plus-noncash charges” (CF) from Topic Review 44.
- “Operating cash flow” is “cash flow from operations” from the statement of cash flows.

S&P uses this cash flow framework to analyze several cash flow ratios.

\[
\frac{\text{funds from operations}}{\text{total debt}}
\]

\[
\frac{\text{funds from operations}}{\text{capital spending requirements}}
\]

\[
\frac{\text{free operating cash flow} + \text{interest}}{\text{interest}}
\]

\[
\frac{\text{free operating cash flow} + \text{interest}}{\text{interest} + \text{annual principal repayment}} = \text{“debt service coverage”}
\]

\[
\frac{\text{total debt}}{\text{discretionary cash flow}} = \text{“debt payback period”}
\]

2. Standard and Poor’s, Corporate Ratings Criteria, undated.
The first four ratios can be interpreted like traditional coverage ratios: the higher the ratio, the stronger the issuer’s capacity to pay. The fifth ratio can be interpreted as a leverage ratio: the lower the ratio, the stronger the firm’s capacity to repay.

Stronger issuers with higher credit ratings are best assessed with ratios involving funds from operations. Weaker credits with lower credit ratings are best assessed with free operating cash flow ratios.

**HIGH-YIELD CORPORATE BONDS**

The analysis of high-yield issuers requires a particular focus on two areas:

- Debt structure.
- Corporate structure.

**Debt Structure Analysis of High-Yield Issuers**

**LOS 53.f:** Explain and interpret the typical elements of the corporate structure and debt structure of a high-yield issuer and the effect of these elements on the risk position of the lender.

The typical debt structure of a high-yield issuer often includes the following types of issues: bank debt, reset notes, and senior and subordinated debt (which may be zero-coupon). The following is based on Cornish (1990).

High-yield borrowers typically rely on short-term, floating-rate, senior bank debt to a greater extent than investment-grade borrowers. The presence of bank loans in the debt structure of a high-yield issuer has three effects on the credit analysis of the high-yield issue:

1. Because bank debt is *floating rate*, a cash flow analysis under different interest rate scenarios is necessary.

2. Because bank debt is *short-term*, the analyst must determine how and where funds will be obtained to pay off the bank debt that is about to mature:
   - If the source is operating cash flow, cash flow projections are crucial.
   - If repayment is expected to come from refinancing, an analysis of capital market conditions is necessary.
   - The analyst must assess the effect on future cash flows of the firm’s plans to sell assets to pay off bank debt. Asset sales generate cash today to pay down short-term debt but may reduce future cash flow available to pay down long-term debt.

3. Because bank debt is *senior*, it has a higher claim against the assets of the firm than debt that is carried on the books as “senior” debt. Thus, the holder of senior high-yield debt securities is really holding debt that is subordinate to bank debt.

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Reset notes trade at a specific premium to their par value because the coupon rate on a reset note is adjusted periodically. Reset notes also make the credit analyst’s job more complicated for the following reasons:

- The effects of changing credit spreads must be incorporated in the interest rate scenario analysis.
- The firm may sell assets in order to avoid higher interest costs on the reset notes if rates or spreads rise. This also leads to an analysis of the effect of asset sales on future cash flows.

An important point of concern for the subordinated bondholders of high-yield issuers is the relative amount of zero-coupon bonds in the debt structure. Assuming no other changes, as the unpaid interest to zero-coupon bonds accrues over time, the relative proportion of the zero-coupon bonds in the firm's debt structure increases relative to the subordinated issues. If these bonds are senior to the subordinated issues, the credit risk of the subordinated issues will increase. This may also have an adverse impact on the subordinated debtholders in case of bankruptcy.

**Corporate Structure Analysis of High-Yield Issuers**

High-yield issuers are often structured as a holding company. Debt is borrowed at the parent level, and funds to pay the obligation in the future are obtained from operating subsidiaries. This makes it imperative that the operating subsidiaries’ financial ratios be examined to determine whether the subsidiaries’ financial position will help the parent meet its obligations and whether the subsidiaries’ own debt covenants will restrict their cash contributions to the parent. In particular, look for the following:

- Are there restrictions on dividend payments to the parent in the debt covenants of the subsidiary’s borrowing agreements?
- Are intercompany loans permitted by the subsidiary’s debt covenants?
- Are there restrictions on asset sales?

**Asset-Backed Securities Credit Analysis**

**LOS 53.g:** Discuss the factors considered by rating agencies in rating asset-backed securities.

Collateral credit quality. The evaluation of corporate credit risk requires an analysis of collateral quality plus myriad operating and business factors that affect cash flows to the firm. For an asset-backed security (ABS) and non-agency mortgage-backed securities (MBS), however, rating agencies consider the quality of the collateral as the single most important issue. Rating agencies evaluate whether the collateral is of sufficient quality to be able to provide cash flows to pay principal and interest over the life of the issues in the asset-backed structure.

Another important aspect of collateral analysis for ABS is the concentration of loans in the collateral pool because many small loans reduce the total credit risk of a loan pool through diversification. Thus, a few large loans in the pool will provide less diversification and result in concentration risk.
Professor's Note: The rating agency will determine the credit enhancements to the issue necessary for it to receive a particular rating. Internal and external credit enhancements are discussed in LOS 56.d in Study Session 15.

### Seller/servicer quality

An additional complication of credit analysis of ABS is that the quality of the ABS seller/servicer must be evaluated. The third-party ABS servicer is responsible for certain administrative functions, such as collecting payments, notifying the issuer of delinquencies, and recovering and liquidating collateral. The servicer is also responsible for distributing the cash flows generated from the collateral pool to the various bondholders in the ABS transaction. In this role, the servicer may be responsible for advancing payments to the bondholders when temporary cash flow shortages occur. Obviously, the servicer can play an extremely important role in supporting ABS transactions.

Therefore, before assigning a rating, the rating agency looks at the servicer's performance history, experience, underwriting standards adopted for loan originations, servicing capabilities (including databases, systems, and personnel), financial strength, and growth relative to its competitive and business environment.

When the role of the servicer is exclusively administrative in nature, the ABS transaction is referred to as a true securitization. When the servicer's responsibility goes beyond the administrative tasks associated with the generation of cash flow, the ABS transaction is referred to as a hybrid transaction. In the case of a hybrid transaction, credit analysis entails two components: (1) the evaluation of the ABS, which emphasizes the quality of the collateral pool, and (2) an evaluation of the servicer using a corporate credit analysis approach. As the servicer's role in the generation of cash flow in an ABS transaction increases, the significance of the credit quality of the servicer takes on greater weight.

### Cash flow stress and payment structure

The rating agencies analyze ABS cash flow projections under different scenarios related to losses, delinquencies, and economic conditions, to assess how these cash flows are distributed to the various tranches (bonds) in the asset-backed security structure. This is an important point because the cash flow from the collateral pool may only be sufficient to meet the cash flow requirements of some, but not all, of the various ABS tranches.

Professor's Note: Please see Study Session 15 for a discussion of the basic features and payment structures of a typical ABS transaction.

### Legal structure

A firm that securitizes assets can obtain a credit rating on the securities it issues that is higher than the issuer's corporate credit rating. To accomplish this, the firm must structure the securitized assets (e.g., mortgages) in such a way that, in the event of bankruptcy, the courts will not apply the cash flow from the collateral toward satisfaction of general corporate liabilities. In order to meet this requirement, the securitizing firm uses an entity known as a special purpose vehicle (SPV) to hold the collateralized assets. In the event of a corporate bankruptcy, the assets of the SPV are not included in corporate assets.
MUNICIPAL BOND CREDIT ANALYSIS

LOS 53.h: Explain how the credit worthiness of municipal bonds is assessed, and contrast the analysis of tax-backed debt with the analysis of revenue obligations.

Tax-backed debt is issued by municipalities and secured by some form of tax revenue. There are four factors involved in the risk assessment of tax-backed debt:

1. **Issuer's debt structure.** The debt burden is measured as debt per capita (resident) in the tax jurisdiction, as well as debt as a percentage of the total real estate value of properties subject to tax and personal incomes of the residents.

2. **Budgetary policy.** This is important because a balanced budget over a 3- to 5-year period is indicative of financial and political discipline.

3. **Local tax and intergovernmental revenue availability.** Evaluation of tax collection rates and historical information concerning local revenue sources along with secondary sources of financial support from the state government help in determining the repayment capacity.

4. **Issuer’s socioeconomic environment.** Evaluation of the trends in the local employment level and economic environment enables the analyst to assess the stability of the revenue base and its future potential for servicing the debt.

Municipalities issue revenue bonds to finance specific projects and enterprises. The issuer pledges the revenue generated by the operating project for the repayment of the loan. There are a number of considerations in the credit analysis of a revenue bond to ensure the project cash flows will be sufficient to repay the obligation:

- **Limits of the basic security.** The trust indenture explains how the project revenues may be limited by federal, state, and local governments. Any limitations reduce the bond’s credit quality.

- **Flow of funds structure.** The revenue bonds are promised “net” revenues after operating expenses. This is accomplished by first placing the revenues from the project in a revenue fund, from which disbursements for expenses are made to other funds, typically in the following order of priority: for operations and ordinary maintenance, debt service, major repairs and equipment replacement, and extraordinary maintenance. Any funds remaining go into a reserve fund. As an analyst (and on the exam), look for deviations from this structure in which the revenues can first be applied to the general obligations of the issuing municipality. This reduces the credit quality of the revenue bonds.

- **Rate, or user charge, covenants.** A popular covenant associated with revenue bond issues is the rate, or user charge, covenant. This covenant specifies how prices will be set on the product or service provided by the issuer. It is intended to ensure that the enterprise has sufficient cash flow to meet expenses and service its debt. The presence of a rate covenant improves the bond’s credit quality.

- **Priority-of-revenue claims.** Can other entities legally redirect the project cash flows for other purposes prior to their application to operating cost and debt service as outlined in the flow of funds structure?
• **Additional-bonds test.** This is a covenant that sets out the conditions under which the municipality can issue additional debt with the same claim against the project revenues. The tighter the restrictions on additional bond issues, the higher the bond’s credit quality.

**SOVEREIGN BOND CREDIT ANALYSIS**

**LOS 53.i: Discuss the key considerations used by Standard & Poor’s in assigning sovereign ratings, and describe why two ratings are assigned to each national government.**

The debt ratings of national governments are referred to as sovereign ratings.

The two general categories used by Standard & Poor’s in deriving sovereign ratings are economic risk and political risk. Economic risk is the ability of a national government to meet its debt obligations. Rating agencies look at a country’s:

- **Economic and income structure** (living standards, income distributions, market versus non-market economy, resource endowments, and economic and industrial diversification).
- **Prospects for economic growth** (savings, investment, and economic growth trends).
- **Degree of fiscal flexibility** (fiscal and budgetary policy, tax competitiveness, and government spending requirements).
- **Public debt burden** (amounts, currency composition, and structure of public debt; debt service burden; and pension and other contingent liabilities).
- **Monetary policy and price stability** (inflation trends, exchange rate policy, money and credit growth, and central bank autonomy).
- **Balance of payments flexibility** (fiscal and monetary policy effect on balance of payments, current account structure, and capital flow composition).
- **External debt and liquidity** (amounts, currency composition, and structure of external public debt; debt service burden and track records; amounts and composition of reserves and other public assets).

Professor’s Note: See Study Session 4 for a more complete discussion of economic growth, the balance of payments, and the effect of fiscal and monetary policy on exchange rates and the balance of payments.

Political risk is the willingness of a national government to meet its debt obligations. Factors include:

- Form of government and degree of citizen participation in the political process.
- Political stability and orderliness of leadership or political party succession.
- Degree of national agreement on economic policy goals.
- Integration of its economy in global trade and financial systems.
- Internal and external security risks.

The assessment of the willingness of a national government to pay is important because there is usually no legal recourse to the borrower if the issuer refuses to pay.
Local Currency and Foreign Currency Debt Ratings

Each national government is assigned two ratings: a local currency debt rating and a foreign currency debt rating. The ratings are assigned separately because defaults on foreign-currency-denominated debt have historically exceeded those on local currency debt. Foreign currency debt typically has a higher default rate and a lower credit rating because the sovereign government must purchase foreign currency in the open market to make interest and principal payments, which exposes it to the risk of a significant local currency depreciation. In contrast, local currency debt can be repaid by raising taxes or controlling domestic spending.

Local currency debt ratings are determined by an analysis of several key factors that influence debt service ability: political stability and extent of participation by the populace in the political process; income base and growth along with the economic infrastructure; tax discipline and budgetary record; monetary policy and rate of inflation; and government debt burden and debt service experience.

For foreign currency debt ratings, the focus is on domestic government’s economic and fiscal policies vis-à-vis foreign governments’ economic and fiscal policies. Analysis includes the country’s balance of payments and the composition of its external balance sheet relative to its external debt (foreign currency) obligations.

CREDIT ANALYSIS: KEY CONSIDERATIONS

LOS 53.j: Contrast the credit analysis required for corporate bonds to that required for 1) asset-backed securities, 2) municipal securities, and 3) sovereign debt.

Corporate Bond Credit Analysis

The two most important factors in an analysis of a corporate issuer are the capacity to pay (particularly a cash flow analysis) and the corporate governance structure. This requires an analysis of the issuer’s business and operating risks (e.g. profitability, solvency, and leverage).

Credit Analysis of ABS vs. Corporate Bonds

There are two important differences between corporate bond and ABS credit analysis:

1. There are no business or operating risks to assess with an ABS. Instead, the emphasis is on the quality of the collateral that backs the ABS, particularly the collateral’s capacity to generate cash flow to meet the repayment obligations of each tranche under various scenarios of default and delinquency experience.
2. The ABS servicer also plays an important role in an ABS, collecting cash flows from the underlying collateral pool and redistributing them to the various tranches. Therefore, the efficiency and quality of the servicer are important factors considered by the rating agencies.

Credit Analysis of Municipal Securities vs. Corporate Bonds

Credit analysis of municipal tax-backed bonds is similar to that of corporate bonds. It requires an analysis of the municipality’s willingness to pay (i.e., the character of the municipality’s public officials) as well as its capacity to repay. Cash flow to repay tax-backed debt comes from fees and taxes, which requires an analysis of industry, employment, and real estate valuation trends.

Credit analysis of revenue bonds requires exactly the same analysis of operating cash flow as corporate bonds. Because revenue bonds are backed by the cash flow from projects like toll roads, the analysis focuses on (1) the specific project’s capacity to generate revenues while controlling costs and (2) how trends in the regional economy affect the stream of cash flow over time.

The only important difference in the analysis of municipal bonds is that the rate covenants and priority-of-revenue claims clause are unique to the trust indenture of municipal revenue bonds, so they require an additional level of analysis not necessary with corporate bonds.

Credit Analysis of Sovereign Debt vs. Corporate Bonds

Analysis of the credit risk of a sovereign entity is similar in a number of ways to credit risk analysis of a corporate borrower. What we called capacity to pay, in terms of corporate credits, is referred to as the economic risk of sovereign credits. Likewise, the issue of the character of corporate management is analogous to the political risk of sovereign debt.
LOS 53.a
Credit risk encompasses three types of risk:
• Default risk—the risk that the borrower will not pay. Credit ratings only assess default risk.
• Credit spread risk—the risk that the credit spreads will increase and that the bond will underperform its benchmark.
• Downgrade risk—the risk that the issue will be downgraded, reducing return and perhaps causing the bond to underperform.

LOS 53.b
There are four Cs of credit analysis:
• Character includes management’s integrity and its commitment to pay.
• Capacity refers to the availability of cash flow to pay debt.
• Collateral includes the assets offered as security.
• Covenants refers to the terms and conditions contained in the lending agreement.

In assessing the quality of management, analysts consider strategic direction, financial philosophy, conservatism, track record, succession planning, and control systems. Corporate governance is also important: firms with stronger corporate governance structures have lower bond yields and higher credit ratings, all else equal.

Analysis of a firm’s sources of liquidity is an important step in the credit analysis process. Potential sources of liquidity include:
• Working capital.
• Cash flow.
• Back-up facilities.
• Securitization.
• Third-party guarantees.

Covenants provide limitations and restrictions on the borrower’s activities.
• Affirmative covenants require the debtor to take certain actions that will enhance the credit quality of the issuer (e.g., pay interest and principal).
• Negative covenants prohibit the borrower from taking certain actions that will reduce the issuer’s credit quality (e.g., incur additional debt).

LOS 53.c
Important credit analysis ratios relate to solvency, capitalization, and coverage.

Short-term solvency ratios:
• current ratio = \frac{current\ assets}{current\ liabilities}
• acid-test ratio = \frac{current\ assets - inventories}{current\ liabilities}
Capitalization ratios:
• long-term debt-to-capitalization ratio =
  \[
  \frac{\text{long-term debt}}{\text{long-term debt + minority interest} + \text{shareholders' common and preferred equity}}
  \]
• total debt-to-capitalization ratio =
  \[
  \frac{\text{current liabilities + long-term debt}}{\text{current liabilities + long-term debt + minority interest + shareholders' common and preferred equity}}
  \]

Coverage tests:
• EBIT coverage ratio = \( \frac{\text{earnings before interest and taxes}}{\text{annual interest expense}} \)
• EBITDA coverage ratio = \( \frac{\text{earnings before interest, taxes, depreciation, amortization}}{\text{annual interest expense}} \)

**LOS 53.d**
To identify corporate issues that may be downgraded, traditional ratio analysis can be used to look for declines in liquidity and coverage ratios and increases in leverage ratios, relative to the average ratios for firms in the same ratings class.

**LOS 53.e**
Cash flow formula:
  \[
  \text{net income} + \text{depreciation} + \text{/}/- \text{other noncash items}
  \]
  \[
  \text{Funds from operations} + \text{decrease (increase) in noncash current assets} + \text{increase (decrease) in nondebt current liabilities}
  \]
  \[
  \text{Operating cash flow} - \text{capital expenditures}
  \]
  \[
  \text{Free operating cash flow} - \text{cash dividends}
  \]
  \[
  \text{Discretionary cash flow} - \text{acquisitions} + \text{asset disposals} + \text{other sources (uses)}
  \]
  \[
  \text{Prefinancing cash flow}
  \]
Cash flow ratios:

- \(\frac{\text{funds from operations}}{\text{total debt}}\)
- \(\frac{\text{funds from operations}}{\text{capital spending requirements}}\)
- \(\frac{\text{free operating cash flow} + \text{interest}}{\text{interest}}\)
- \(\frac{\text{free operating cash flow} + \text{interest}}{\text{interest} + \text{annual principal repayment}} = \text{"debt service coverage"}\)
- \(\frac{\text{total debt}}{\text{discretionary cash flow}} = \text{"debt payback period"}\)

**LOS 53.f**
The typical debt structure of a high-yield issuer includes bank debt, reset notes, senior and subordinated debt (which may be zero-coupon).

High-yield borrowers typically rely on short-term, floating-rate, senior bank debt to a greater extent than investment grade borrowers.

Reset notes trade at a specific premium to their par value because the coupon rate on a reset note is adjusted periodically.

High-yield issuers are often structured as a holding company. Debt is borrowed at the parent level, while funds to pay the obligation in the future are obtained from operating subsidiaries. This makes it imperative that the operating subsidiaries’ financial ratios be examined to determine whether the subsidiaries’ financial position will help the parent meet its obligations and whether the subsidiaries’ own debt covenants will restrict their cash contributions to the parent.

**LOS 53.g**
These factors are considered by rating agencies when rating ABS and non-agency MBS:
- Quality of the collateral—extremely important for ABS.
- Quality of the servicer—importance increases as effect on cash flow increases.
- Cash flow stress and payment structure—not all tranches are paid the same way.
- Legal structure—affects how cash flow is distributed in the event of bankruptcy.

**LOS 53.h**
For tax-backed debt, credit worthiness is determined by:
- Issuer’s debt structure.
- Budgetary policy.
- Local tax and intergovernmental revenue availability.
- Issuers’ socioeconomic environment.
Revenue bonds are issued by municipalities for financing specific projects and enterprises. Key considerations in the credit analysis are:
- Limits of the basic security.
- Flow of funds structure.
- Rate, or user charge, covenants.
- Priority-of-revenue claims.
- Additional-bonds tests.

**LOS 53.i**
Factors considered by rating agencies when evaluating sovereign debt include:
- Economic risk is the ability of a national government to meet its debt obligations.
- Political risk is the willingness of a national government to meet its debt obligations.

Two ratings are assigned to each national government because defaults on foreign currency denominated debt have historically exceeded those on local currency debt.
- Local currency debt ratings—consider political stability, income base, and economic growth.
- Foreign currency debt ratings—focus on balance of payments and the external balance sheet composition relative to external debt (foreign currency) obligations.

**LOS 53.j**
The two most important factors in an analysis of a corporate issuer are the capacity to pay (particularly a cash flow analysis) and the corporate governance structure. This requires an analysis of the issuer's operational risk.
- ABS credit analysis requires an assessment of collateral credit quality, but analysis of business and operational risks is not important.
- Municipal bond credit analysis is very similar to corporate bond analysis, focusing on capacity to repay for both tax-backed and revenue bonds and willingness to repay (corporate character) for tax-backed bonds. The only important difference is that revenue bonds have rate covenants and a priority of revenue claims clause.
- Sovereign credit analysis requires an assessment of the country's ability to repay (economic risk) and willingness to repay (political risk).
CONCEPT CHECKERS

1. The three types of risk encompassed by credit risk are:
   A. sovereign risk, country risk, and political risk.
   B. political risk, economic risk, and social risk.
   C. credit spread risk, downgrade risk, and default risk.

2. Which of the following is least likely to be a major factor rating agencies consider in assessing the quality of management?
   A. Employee turnover.
   B. Strategic direction of management.
   C. Succession planning.

3. A covenant limiting the issuer to an annual dividend payment of no more than 20% of net income before extraordinary items is an example of a(n):
   A. affirmative covenant.
   B. negative covenant.
   C. payment covenant.

4. Henry Adams is a credit analyst who has been assigned to Precision Paper, Inc. The company manufactures paper products for the retail market. For the current year, the firm's funds from operations are $100 million (operating cash flow is net income + depreciation + deferred taxes). No changes in working capital are anticipated. The company pays dividends of $25 million. Based on discussion with management, Adams believes that the company's capital expenditure for the year will be $50 million.

Free operating cash flow for Precision Paper, Inc. is closest to:
   A. $100 million.
   B. $50 million.
   C. $25 million.

5. Rating agencies assign both a local currency debt rating and a foreign currency debt rating to the bonds of a sovereign government because:
   A. the default rate of sovereign debt denominated in the local currency has historically been higher.
   B. the default rate of sovereign debt denominated in foreign currency has historically been higher.
   C. sovereign debt denominated in the local currency is guaranteed against default.

6. The quality of the collateral is the single most important factor for:
   A. high-yield bonds.
   B. asset-backed securities.
   C. municipal bonds.
7. An analyst has compiled the following information on the bonds of Ledd Computers, which is currently rated “A.”

<table>
<thead>
<tr>
<th>Guideline for Issues Rated A</th>
<th>Current Debt-to-EBIT/Interest Expense</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.00 to 1.20</td>
</tr>
<tr>
<td></td>
<td>0.35 to 0.45</td>
</tr>
<tr>
<td></td>
<td>3.00 to 4.00</td>
</tr>
<tr>
<td>Ledd Computers</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>4.80</td>
</tr>
</tbody>
</table>

Which of the following statements best describes Ledd’s position concerning a ratings change?
A. Stable to down one notch in rating.
B. Downward one notch in rating.
C. Stable to up one notch in rating.

8. Tax-backed municipal bond analysis involves the issuer’s:
A. budgetary policy, expected revenues, expected expenses, and surplus funds.
B. debt structure, budgetary policy, local tax and intergovernmental revenue availability, and socioeconomic environment.
C. expected revenue, expected expenses, debt structure, and surplus funds.

9. Which of the following is the debt payback ratio? Total debt divided by:
A. operating cash flow.
B. discretionary cash flow.
C. prefinancing cash flow.

10. Which of the following is the formula for the acid-test ratio?
A. \[ \frac{\text{current assets} - \text{inventories}}{\text{current liabilities}} \]
B. \[ \frac{\text{current assets} - \text{prepaid items}}{\text{current liabilities} + \text{accruals}} \]
C. \[ \frac{\text{current assets}}{\text{current liabilities} - \text{inventories} - \text{accruals} - \text{prepaid items}} \]
11. Henry Adams is discussing the credit analysis of various types of credits with Grace Roberts, an intern in Adams’ firm. Adams makes the following statements:

**Statement 1:** The credit analysis of municipal securities is very different than that for corporate bonds because the municipality is a tax-exempt issuer with the ability to raise taxes as necessary to repay principal and interest.

**Statement 2:** Although the quality of the servicer is an important factor in the analysis of asset-backed securities but not for corporate bonds, in general the analysis of both types is very similar because the key factor for both is an assessment of the business and operating risks of the issuer.

Roberts disagrees with both statements. Is Roberts correct in her assessment?
- A. Yes, she is correct to disagree with both.
- B. She is correct to disagree with one, but not the other.
- C. She is not correct to disagree with either.

Use the following information for Questions 12 and 13.

Kale Kind, FRM, is the controller for Northern Cities Power Source, Inc. Kind has gathered selected financial information for the year ended December 31, 2009, and has projected amounts for the year ended December 31, 2010 (noted 2010E in the following table). The company’s bond covenant requires an EBITDA coverage ratio of 3.5 times and an EBIT coverage ratio of 2.0 times. Kind is concerned about how the increased energy prices and the high energy demand from the hot summer in the northern United States will affect the company’s covenant compliance.

### Northern Cities Power

Selected Financial Information (in millions of dollars)

<table>
<thead>
<tr>
<th>Year Ended December 31,</th>
<th>2009</th>
<th>2010E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation and amortization</td>
<td>250</td>
<td>300</td>
</tr>
<tr>
<td>Interest expense</td>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>EBITDA</td>
<td>1,250</td>
<td>800</td>
</tr>
</tbody>
</table>

12. Based on the information provided, was Northern Cities in compliance with its EBITDA ratio at the end of 2009? By how much will it need to increase EBIT to be in compliance with its EBIT ratio for 2010 based on Kind’s projections?

<table>
<thead>
<tr>
<th>2009 EBITDA ratio compliance</th>
<th>$ needed for 2010E EBIT ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. No</td>
<td>$200</td>
</tr>
<tr>
<td>B. Yes</td>
<td>$200</td>
</tr>
<tr>
<td>C. Yes</td>
<td>$550</td>
</tr>
</tbody>
</table>
13. Assume that as of year-end 2010, Northern is in compliance with the EBIT ratio coverage covenant with a ratio of 2.10 and carries a rating of AAA. The company’s debt-to-capitalization ratio is 0.75, and the acid-test ratio is 0.60.

Industry guidelines for issues rated AAA are as follows:
- Acid-test ratio: 0.75 to 1.25.
- Debt-to-capitalization: 0.45 to 0.80.
- EBIT coverage ratio: 3.5 times.

Is Northern most likely a candidate for a downgrade? What action would most likely increase the acid-test ratio?

<table>
<thead>
<tr>
<th>Candidate for a downgrade?</th>
<th>Increase acid-test ratio?</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. No</td>
<td>Selling inventory for cash</td>
</tr>
<tr>
<td>B. No</td>
<td>Using cash to reduce accounts payable</td>
</tr>
<tr>
<td>C. Yes</td>
<td>Selling inventory for cash</td>
</tr>
</tbody>
</table>

Use the following information to answer Questions 14 and 15.

Kathy Tomasen, CFA Level 2 candidate, is a fixed-income analyst at Parker Bowden, LLC. She is reviewing the financial statements of Radomsky Enterprises, which currently holds the highest credit rating.

<table>
<thead>
<tr>
<th>Radomsky Enterprises Cash Flow Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year Ended December 31,</td>
</tr>
<tr>
<td>Net income</td>
</tr>
<tr>
<td>Depreciation</td>
</tr>
<tr>
<td>Funds from operations</td>
</tr>
<tr>
<td>Operating cash flow</td>
</tr>
<tr>
<td>Capital expenditures</td>
</tr>
<tr>
<td>Free operating cash flow</td>
</tr>
<tr>
<td>Cash dividends</td>
</tr>
<tr>
<td>Acquisitions</td>
</tr>
<tr>
<td>Asset disposals</td>
</tr>
<tr>
<td>Prefinancing cash flow</td>
</tr>
<tr>
<td>Memo: Interest expense</td>
</tr>
</tbody>
</table>

Tomasen calculates that the discretionary cash flow decreased from 2009 to 2010. She determines that the company is best assessed by looking at the free operating cash flow interest coverage ratio and notes that the change in the ratio indicates a weakening position. Based on her calculations, she believes all other ratios shown indicate an improved position year over year.
Radomsky Enterprises Selected Ratios

<table>
<thead>
<tr>
<th></th>
<th>Year Ended December 31,</th>
<th>2010</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Funds from operations to total debt</td>
<td></td>
<td>0.99</td>
<td>0.87</td>
</tr>
<tr>
<td>Funds from operations to capital spending</td>
<td></td>
<td>5.24</td>
<td>4.53</td>
</tr>
<tr>
<td>Free operating cash flow interest coverage</td>
<td></td>
<td>4.67</td>
<td>4.85</td>
</tr>
<tr>
<td>Total debt to discretionary cash flow</td>
<td></td>
<td>4.50</td>
<td>3.71</td>
</tr>
</tbody>
</table>

14. What is the calculation of discretionary cash flow in 2010, and is Tomasen’s statement that discretionary cash flow decreased from 2009 to 2010 correct or incorrect?

<table>
<thead>
<tr>
<th>2010 discretionary cash flow</th>
<th>Tomasen’s statement on cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $33</td>
<td>Incorrect</td>
</tr>
<tr>
<td>B. $20</td>
<td>Correct</td>
</tr>
<tr>
<td>C. $20</td>
<td>Incorrect</td>
</tr>
</tbody>
</table>

15. Indicate whether Tomasen’s use of the free operating cash flow ratios to assess Radomsky is correct or incorrect and whether her belief on the other three ratios is correct or incorrect.

A. Correct on both counts.
B. Correct in one case, but incorrect with regard to the other.
C. Incorrect on both counts.
1. C Credit risk encompasses three types of risk: default risk, credit spread risk, and downgrade risk.
   - Default risk is the risk that the borrower will not repay the obligation.
   - Credit spread risk is the risk that the credit spread will increase.
   - Downgrade risk is the risk that the issue will be downgraded.

2. A The factors considered include strategic direction of management, financial philosophy, management's track record, succession planning, and control systems.

3. B Covenants provide the limitations and restrictions on the borrower's activities. They provide a legal framework for the repayment of the debt issue, thus reducing uncertainty.
   - **Affirmative covenants** require the debtor to do certain things, such as pay interest and principal, pay taxes, comply with loan agreements, and maintain properties.
   - **Negative covenants** are those that require the borrower not to do certain things, such as limitations on the company’s ability to incur more debt, restrictions on subsidiaries’ borrowing, and limitations on dividend payments and stock repurchases.

4. B Because there are no changes in working capital requirements, the free operating cash flow is found by subtracting the estimated capital expenditures of $50 million from the $100 million funds from operations; thus, free operating cash flow is $50 million. The implication is that the company will be able to maintain its $25 million dividend without acquiring outside funds and has an additional $25 million of operating cash flow available for discretionary expenditures.

5. B With sovereign debt, the payments may be in either local or foreign currency. The historical default rate has been higher for sovereign debt denominated in foreign currency. Thus, rating agencies have assigned two ratings to reflect the possible difference in default risk.

6. B For an **asset-backed security**, the quality of the collateral is the single most important factor in the credit analysis process. The analyst must evaluate whether the underlying collateral (e.g., the mortgage pool in the case of an MBS) is of sufficient quality to be able to provide cash flows to pay principal and interest over the life of the issues in the asset-backed structure.

7. C Ledd is not a candidate for a downgrade, based on an analysis of the ratios. Liquidity, as measured by the current ratio, is in the range of A-rated issues; leverage, as measured by debt-to-capitalization, is at the low end of the range; and coverage is higher than A-rated issues. The outlook is best described as stable to up one notch in rating.

8. B Tax-backed municipal bond analysis involves four factors:
   - Issuer’s debt structure—measured as debt per resident in the tax jurisdiction.
   - Budgetary policy—indicates degree of financial and political discipline.
   - Local tax and intergovernmental revenue availability (e.g., tax collection rates).
   - Issuers’ socioeconomic environment (e.g., trends in the local employment level).

9. B **debt payback period** = \( \frac{\text{total debt}}{\text{discretionary cash flow}} \).

10. A **acid-test ratio** = \( \frac{\text{current assets} - \text{inventories}}{\text{current liabilities}} \).
11. A Statements 1 and 2 are incorrect, so Roberts was right to disagree with both.

Municipal bond credit analysis is very similar to corporate bond analysis, focusing on capacity to repay for both tax-backed and revenue bonds and willingness to repay (corporate character) for tax-backed bonds. The only important difference is that revenue bonds have rate covenants and a priority of revenue claim clause.

The two most important factors in an analysis of a corporate issuer are the capacity to pay (particularly a cash flow analysis) and the corporate governance structure. This requires an analysis of the issuer’s business and operating risks. ABS credit analysis requires an assessment of collateral credit quality, but operational risk analysis is not an important factor.

12. B

Northern Cities Power:
Selected Financial Information (in millions of dollars)

<table>
<thead>
<tr>
<th>Year Ended December 31,</th>
<th>2009</th>
<th>2010(E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBITDA</td>
<td>1,250</td>
<td>800</td>
</tr>
<tr>
<td>Dep and amort</td>
<td>250</td>
<td>300</td>
</tr>
<tr>
<td>EBIT</td>
<td>1,000</td>
<td>500</td>
</tr>
<tr>
<td>Interest</td>
<td>350</td>
<td>350</td>
</tr>
</tbody>
</table>

EBITDA 2009 coverage ratio = 1,250 / 350 = 3.57, which is greater than 3.5, so the company is in compliance.

EBIT 2010E coverage ratio = 500 / 350 = 1.43, which is less than 2.

To meet the EBIT covenant in 2010, the company needs EBIT of 2 x 350 = 700.

Shortfall = 500 – 700 = -200

13. C Northern is a candidate for a downgrade. While the debt-to-capitalization ratio is within the industry range, the other two ratios are not.

The acid-test ratio calculation excludes inventory. As a result, selling inventory for cash increases the numerator and leaves the denominator the same, resulting in an increase in the ratio. Using cash to reduce accounts payable reduces both the numerator and the denominator by the same amount. Because the beginning ratio is less than 1.0, the relative impact on the numerator is larger than on the denominator and the ratio declines.


2009 discretionary cash flow = 30 – 9 = $21

Discretionary cash flow decreased from $21 in 2009 to $20 in 2010.
15. C The use of free operating cash flow to assess the company is incorrect. Given the company's strong credit rating, the funds from operations ratios are a better measure. Her statement on the other three ratios is also incorrect. Increasing funds from operations ratios indicate an improved position. However, the increase in the total debt to discretionary cash flow ratio on its own indicates a weakening position.
The following is a review of the Fixed Income: Valuation Concepts principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**THE LIQUIDITY CONUNDRUM**

Study Session 14

**EXAM FOCUS**

The focus of this new topic review is on the state of liquidity in the United States during the Spring of 2007. Be able to contrast the traditional and "appetite for risk" views of liquidity. Understand the characteristics of a 2/28 adjustable rate subprime mortgage and be able to explain why that debt product is said to have a free at-the-money call option embedded in it at the time of issue. Know the steps or phases of the Minsky framework (hedge unit, speculative unit, and Ponzi unit) and be able to identify the characteristics of each phase.

**LOS 54.a:** Contrast the concept of liquidity as "appetite for risk" with the more traditional view that liquidity is created by the central bank.

**Traditional View of Liquidity**

Liquidity has been traditionally measured by economists in terms of monetary aggregates (M1, M2, etc). In this view, the available liquidity in an economy is determined by the central bank, which controls the supply of money. The central bank extends liquidity to commercial banks that create additional liquidity when they make loans.

Recent events in the United States have investors asking why there is suddenly a lack of liquidity, creating what may be referred to as a liquidity conundrum. The best answer requires a different view of liquidity than the traditional view.

**Liquidity as Appetite for Risk**

Instead of viewing liquidity as a function of central bank actions, liquidity may be best viewed as a function of risk aversion. In this view, liquidity requires the interaction of liquidity-providing savers and liquidity-demanding investors. The savers provide funds to levered investors, such as hedge funds. The levered investors use those funds to make risky investments. If either savers or investors do not want to exchange funds due to increased risk aversion, then the liquidity in an economy decreases.

In sum, if the "appetite for risk" in an economy declines, then there will be a consequent decline in liquidity. Thus, the assertion that liquidity is not measured best by traditional monetary aggregates, but by saver and investor risk aversion.
Traditional Banks vs Shadow Banks

Traditional banks are levered and heavily regulated. In return, banks are provided federal deposit insurance and liquidity via the Federal Reserve’s discount window. Because of this, there is no need for most depositors (i.e., those having less than $100,000 on deposit) to withdraw their funds from banks in periods of crisis.

More recently however, the marginal sources of liquidity in the U.S. economy have come from what is referred to as the shadow banking system. Examples of shadow banks include hedge funds, structured investments, real estate investment trusts (REITs), collateralized loan obligations (CLOs), and collateralized debt obligations (CDOs).

Like traditional banks, shadow banks are levered. But they have historically not had access to the discount window and cannot offer their investors deposit insurance. Furthermore, compared to traditional banks, shadow banks take more risk and do not have regulatory capital requirements. For these reasons, there is an incentive for investors to withdraw funds from shadow banks when risk increases.

Professor’s Note: The information in this topic review is based on “current events” in the United States in 2007. Some of the information is thus dated. For example, in spring 2008, investment banks were granted the ability to borrow at the discount window. For the exam, take the information in the topic as given and answer accordingly.

Instead of Federal Reserve funding, the shadow banks have used reverse repos and asset-backed commercial paper to fund their investments. In a reverse repo, the shadow bank provides a security as collateral for funding. In the asset-backed commercial paper market, the shadow banks would obtain funding through the issuance of short-term debt that was typically rolled over every 45-90 days. At the peak of the credit boom, some shadow banks were allowed to issue extendable commercial paper, which gave them the option to extend the maturity of the paper.

The only oversight faced by the shadow banks that even remotely resembled regulation was during evaluation by the credit rating agencies, such as Moody’s, Standard and Poor’s, and Fitch. Even with increasing leverage, less transparency, and more conflicts of interest, the shadow banks were readily provided funding at cheap rates so long as investor risk aversion remained low. Because they were so highly levered, the shadow banks were able to generate attractive returns on even the smallest of spreads.

This increase in shadow bank liquidity continued until the spring of 2007, at which time problems in the mortgage market became apparent. When a Bear Stearns hedge fund revealed that some of its lenders had demanded more collateral, there was a sell-off in Bear Stearns’ collateral. The ensuing withdrawal of funds from the shadow banking system resulted in a steep decline in liquidity.

The Liquidity Conundrum

A conundrum is something that cannot be solved. In the traditional banking system, runs on banks and steep declines in liquidity are less probable because of the support
and regulation provided by the federal government. If liquidity is lacking in an economy, the central bank steps in to provide it.

In the shadow banking system, investors will withdraw liquidity in times of crisis. As investor risk aversion increased in the United States, investors sold shadow bank assets and refused to fund the shadow banks. Risk aversion increased to the point that by late 2007, liquidity available to this sector had largely dried up.

In sum, to explain how liquidity can evaporate in an economy, one must understand the role of investor risk aversion in the shadow banking system. Liquidity is not determined solely by the central bank. When investor risk aversion increases, funds are withdrawn from the shadow banking system and liquidity evaporates.

The neutral rate used by the Federal Reserve has been traditionally used to determine the balance between inflation and economic growth. However, the neutral rate should also reflect investor risk aversion. As risk aversion drops, the neutral rate should rise.

LOS 54.b: Describe how Minsky’s “financial instability hypothesis” predicts a mortgage market crisis as debt creation journeys from conservative hedging activities to more speculative activities, and finally to a Ponzi scheme phase.

2/28 Adjustable Rate Subprime Mortgages

In a 2/28 adjustable subprime mortgage, the borrower was not required to put any money down. In other words, the loan funded 100% of the cost of the property, and the buyer did not have an equity investment. These mortgages had two-year teaser interest rates (i.e., below market rates) that subsequently adjusted up by about five percentage points at the end of the two-year period. Full interest payments were optional, which meant that negative amortization was possible at the discretion of the borrower. From 2004 to 2006, the majority of less creditworthy borrowers used this type of mortgage.

Steps Toward Instability

In an attempt to explain economic fluctuations, some economists conjecture that a free market economy is inherently unstable because in times of stability, investors will extrapolate the current stability out into the future. This encourages greater risk taking, especially in levered investments. The greater the magnitude and duration of the period of economic growth (i.e., stability), the greater the associated risk-taking that occurs. This results in greater future instability. This explanation for the booms and busts in a capitalist economy is the gist of Hyman Minsky’s “financial instability hypothesis.”

In the Minsky framework, a stable economy progresses in three steps towards greater instability. First, there is the hedge unit, then the speculative unit, then finally the Ponzi unit. Each step is typified by a particular type of debt investment.
Step 1: Hedge Unit

This levered investment is hedged and stable because it is backed by sufficient income-generating capability (wage income or rental income) to pay back both the principal and interest on its debt financing. An example is the traditional 30 year fixed-rate, amortizing mortgage. Because U.S. homeowners had historically shunned debt and tried to pay off their mortgages as soon as possible, these were generally safe investments.

Step 2: Speculative Unit

This levered investment is riskier than the hedge unit. Here, the borrower purchases an asset that is backed by enough cash flow to pay back the interest but not the principal. An example is an interest-only mortgage with a balloon payment equal to the original principal. This mortgage can be tempting to borrowers because of its lower monthly payment.

The speculative unit is less stable than a hedge unit because the borrower is speculating that interest rates will not rise, that other mortgage terms and conditions will not worsen, and that the property will not decline in value between the time the loan is originated and when it must be refinanced.

Step 3: Ponzi Unit

In this phase, the asset cash flow cannot cover either the principal or interest. An example is a negative amortization mortgage with a balloon payment equal to the original principal plus unpaid interest. In addition to speculating that interest rates will not rise and that mortgage terms and conditions will not worsen, the borrower is speculating that the property will rise in value. The borrower is effectively betting that if he or she buys an overvalued asset, another buyer willing to pay an even higher price will emerge in the future.

By 2006, the marginal unit of debt was a Ponzi unit in U.S. mortgage markets, as evidenced by the proliferation of the 2/28 adjustable rate subprime mortgage. The paradox of the Minsky framework is that the longer the economy has been stable, the more likely it is that the marginal unit of debt moves from hedge to speculative to Ponzi.

LOS 54.c: Explain how subprime mortgage borrowers are granted a free at-the-money call option on the value of their property.

Even up to the initial stages of the Ponzi unit phase, credit risks present in the U.S. housing market appeared relatively benign. This was because property prices were steadily increasing, which facilitated the formation of the Ponzi units. Essentially, marginal borrowers had been granted a free at-the-money call option on the property. The option was free because no down payment or origination fees were required to obtain the mortgage. It was a call because, if the property value increased above the purchase (strike) price, all of the gains accrued to the borrower (the option holder). As long as housing prices kept going up, borrowers continued to make their monthly payments because doing so allowed them to continue to hold the valuable call option.
Eventually, property values reached unsustainable levels, new buyers willing to pay ever-higher prices failed to emerge, and prices began to decline. Credit rating agencies had assumed that low default rates would continue into the future, but, by the first quarter of 2007, there was a sudden increase in early defaults on mortgages made in 2006. In essence, borrowers realized that with declining property values, their call options were now worthless and they had no incentive to make their monthly payments. The Ponzi vehicle was no longer profitable.

With the Minsky cycle now having reached the Ponzi phase, the U.S. economy is now expected to move in the opposite direction towards stability. Hedge units will become the marginal units of debt creation, the neutral fed funds rate will decline, and the economy is forecast to enter a deflationary stage.
**Key Concepts**

**LOS 54.a**
Liquidity has traditionally been measured in terms of monetary aggregates. In this view, the available liquidity in an economy is determined by the central bank, which controls the supply of money.

In the Minsky “financial instability hypothesis” framework, instead of viewing liquidity as a function of central bank actions, liquidity is viewed as a function of risk aversion. If investor risk aversion increases, liquidity declines as funds are withdrawn from shadow banks that have no government guarantees.

**LOS 54.b**
A 2/28 adjustable rate subprime mortgage is characterized by the following:
- The borrower puts no money down, i.e. it requires no equity investment;
- Interest payments are optional; and
- The interest rate is a two-year teaser rate that subsequently adjusts up.

In Minsky’s framework, a stable economy progresses in three steps towards greater instability. First there is the hedge unit, then the speculative unit, then finally the Ponzi unit. During the Ponzi phase default rates on mortgages, such as the 2/28 adjustable rate mortgages, increase because, with declining property values, borrowers realize they have no incentive to make payments on their loans.

**LOS 54.c**
A subprime mortgage borrower was effectively granted a free at-the-money call because the mortgage:
- Did not require a down payment;
- Full-interest payments were optional; and
- The borrower’s position would be valuable if home prices rose.

As long as housing prices kept going up, borrowers made their monthly payment because the call had value. When house prices fell, borrowers defaulted on their mortgages because the call option was worthless.
CONCEPT CHECKERS

1. Consider the following two statements:
   1. According to the liquidity conundrum view, the best measure of liquidity is risk aversion.
   2. According to the “appetite for risk” theory of liquidity, risk aversion should lead to a higher neutral Federal Funds Rate.

   Are these statements correct?
   A. Yes.
   B. No, because according to the “appetite for risk” theory of liquidity, risk aversion should lead to a lower neutral Federal Funds Rate.
   C. No, because according to the liquidity conundrum view, the best measure of liquidity is the money supply.

2. Which of the following is a characteristic common to both traditional and shadow banks?
   A. Leverage.
   B. Heavy regulation.
   C. Deposit insurance.

3. Which of the following choices least accurately identifies the characteristic of a 2/28 adjustable rate subprime mortgage and the phase of the Minsky process where these assets were most prevalent? A 2/28 adjustable rate subprime mortgage:
   A. did not require an equity investment and is typical of the hedge unit phase of the Minsky process.
   B. did not require principal amortization and is typical of the speculative unit phase of the Minsky process.
   C. allowed less than full interest payments and is typical of the Ponzi unit phase of the Minsky process.

4. Which of the following choices gives the correct order of the steps, from first to last, in the Minsky framework?
   A. Speculative unit, hedge unit, Ponzi unit
   B. Hedge unit, speculative unit, Ponzi unit.
   C. Speculative unit, Ponzi unit, hedge unit.

5. An interest-only mortgage with a balloon payment is most characteristic of which phase of the Minsky framework?
   A. Ponzi.
   B. Hedge.
   C. Speculative.
6. Consider the following two statements:

1. The borrower in a 2/28 adjustable rate subprime mortgage is long a call.

2. The value of the option is worth more to the borrower when home prices are falling.

Are these statements correct?
A. Yes.
B. No, because the borrower in a 2/28 adjustable rate subprime mortgage is short a call.
C. No, because the value of the option is worth more when home prices are rising.
1. B Statement 1 is correct. Statement 2 is incorrect. According to the “appetite for risk” theory of liquidity, risk aversion should lead to a lower neutral Federal Funds Rate. Risk aversion is indirectly related to the neutral rate. “Appetite for risk” is directly related to the neutral rate (i.e., the higher the appetite for risk, the higher the neutral rate).

2. A Both traditional and shadow banks use leverage. Heavy regulation and deposit insurance are characteristics of traditional banks, not shadow banks.

3. A A 2/28 adjustable rate subprime mortgage did not require an equity investment, did not require principal amortization, and allowed less than full interest. This type of funding vehicle is characteristic of both the speculative unit and the Ponzi unit phases of the Minsky framework.

4. B The steps in the Minsky framework are as follows: (1) hedge unit, (2) speculative unit, (3) Ponzi unit.

5. C An interest-only mortgage is most characteristic of the Speculative phase of the Minsky framework.

6. C Statement 1 is correct. Statement 2 is incorrect. A subprime mortgage borrower is granted a free at-the-money call option on the value of their property. He or she is long the call because if the value of the property increases, the value of the call increases. The call is free because the mortgages did not require a down payment and interest payments were optional.
The following is a review of the Fixed Income: Valuation Concepts principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**TERM STRUCTURE AND VOLATILITY OF INTEREST RATES**

**EXAM FOCUS**

This topic review provides important information on the construction, theoretical underpinnings, and implications of the term structure of interest rates. Interpreting the shape of the yield curve and implied forward rates in the context of one of the three term structure theories is a favorite exam topic. Make sure you understand all three theories and can discuss their implications. Also pay close attention to key rate duration, a useful bond portfolio management tool.

**WARM-UP: YIELD CURVE SHAPES**

Historically, the yield curve has taken on three fundamental shapes, as shown in Figure 1.

**Figure 1: Yield Curve Shapes**

A *normal* yield curve is one in which long-term rates are greater than short-term rates, so the curve has a *positive slope*. A *flat* yield curve represents the situation where the yield on all maturities is essentially the same. An *inverted* yield curve reflects the condition where long-term rates are less than short-term rates, giving the yield curve a *negative slope*. 
LOS 55.a: Illustrate and explain parallel and nonparallel shifts in the yield curve, a yield curve twist, and a change in the curvature of the yield curve (i.e., a butterfly shift).

When the yield curve undergoes a **parallel shift**, the yields on all maturities change in the same direction and by the same amount. As indicated in Figure 2, the slope of the yield curve remains unchanged following a parallel shift.

**Figure 2: Parallel Yield Curve Shift**

When the yield curve undergoes a **nonparallel shift**, the yields for the various maturities change by differing amounts. The slope of the yield curve after a nonparallel shift is not the same as it was prior to the shift. Nonparallel shifts fall into two general categories: twists and butterfly shifts.

**Yield curve twists** refer to yield curve changes when the slope becomes either flatter or steeper. A flattening of the yield curve means that the spread between short- and long-term rates has narrowed; the curve gets steeper when spreads widen.

As shown in Figure 3, the most common shifts tend to be either a downward shift and a steepened curve or an upward shift and a flattened curve.

**Figure 3: Nonparallel Yield Curve Shifts—Twists**
Yield curve butterfly shifts refer to changes in the degree of curvature. A positive butterfly means that the yield curve has become less curved. For example, if rates increase, the short and long maturity yields increase by more than the intermediate maturity yields, as shown in Figure 4. A negative butterfly means that there is more curvature to the yield curve. For example, if rates increase, intermediate term yields increase by more than the long and short maturity yields, as shown in Figure 4.

**Figure 4: Nonparallel Yield Curve Shifts—Butterfly Shifts**

<table>
<thead>
<tr>
<th>Positive Butterfly Shift</th>
<th>Negative Butterfly Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Positive Butterfly Shift" /></td>
<td><img src="image" alt="Negative Butterfly Shift" /></td>
</tr>
</tbody>
</table>

**LOS 55.b: Describe the factors that drive U.S. Treasury security returns, and evaluate the importance of each factor.**

Research studies have identified three factors that explain historical Treasury security returns. Each one corresponds to the types of yield curve shifts discussed in the previous LOS.

1. Changes in the **level** of interest rates (parallel shifts in the yield curve).
2. Changes in the **slope** of the yield curve (twists in the yield curve).
3. Changes in the **curvature** of the yield curve (butterfly shifts).

Litterman and Scheinkman (1991)\(^1\) used the R\(^2\) from regression analysis to estimate the ability of these three variables to explain the total returns on 6-month through 18-year zero-coupon Treasury securities. The results indicated that, collectively, these three factors explained more than 95% of the total return variance.

- **Factor 1:** Changes in the **level** of rates made the greatest contribution, explaining almost 90% of the observed variation in total returns for all maturity levels.
- **Factor 2:** Slope changes explained, on average, 8.5% of the total returns’ variance over all maturity levels.
- **Factor 3:** Curvature changes contributed relatively little toward the explanation of total returns, with an average proportion of total explained variance equal to 1.5%.

---

Recall from Level 1 that duration measures bond price sensitivity to parallel changes in interest rates. Because parallel changes in the level of interest rates (Factor 1) have a significant influence on Treasury returns, duration is a useful tool for quantifying interest rate risk. Later on in this topic review, we will discuss key rate duration, which is used to quantify bond price sensitivity to changes in the slope of the yield curve. Because changes in the shape of the yield curve (nonparallel shifts) also impact Treasury returns, key rate duration is useful to supplement the information provided by duration when measuring interest rate risk exposure.

WARM-UP: SPOT CURVES AND BOOTSTRAPPING

Suppose you want to know the required return on a fixed-income security with seven years to maturity in order to determine its correct price. Your first impulse may be to look at the current rate on a U.S. Treasury note (T-note) with seven years to maturity. However, this methodology ignores all other characteristics of the bonds except maturity.

For example, suppose that the 7-year Treasury security you are interested in pricing has a coupon rate of 9%. If you look at the universe of 7-year Treasury securities, you will find that there are several different issues, each with different coupon rates. Which one is correct? Unfortunately, the answer is that none of them are correct. In order to accurately price a fixed-income security, you need a spot rate for each cash flow (recall that the spot rate is the rate prevailing today for a zero-coupon bond with the same maturity as the cash flow being valued). Hence, to value your 7-year 9% bond, you will need unique spot rates for each of the 14 coupon payments and a spot rate for the return of principal. In order to obtain all the necessary spot rates, you will need to construct a theoretical spot rate curve for Treasury securities.

Bootstrapping spot rates from the yields on coupon Treasury securities was covered in the Level 1 curriculum. However, because bootstrapping is necessary for understanding the issues surrounding the construction of the spot rate curve, we will briefly review bootstrapping.

Bootstrapping is the process of sequentially calculating spot rates from securities with different maturities, using the yields on Treasury bonds from the yield curve, as shown in Figure 5.

Figure 5: Bootstrapping

For example, suppose that you know a 6-month U.S. Treasury bill has an annualized yield of 4% and a 1-year Treasury STRIP has an annualized yield of 4.5% (assume...
annual rates stated on a bond equivalent basis). Because these are both discount securities, the yields are spot rates. Given these spot rates, we can calculate the spot yield on a 1.5-year Treasury via bootstrapping. Assume that the 1.5-year Treasury is priced at $95 and carries a 4% coupon ($2 every six months). In this case, to calculate the 1.5-year spot rate, solve the following equation:

\[
\text{price} = \frac{\$2}{1 + \left(\frac{\text{6-month spot}}{2}\right)^2} + \frac{\$2}{1 + \left(\frac{\text{12-month spot}}{2}\right)^2} + \frac{\$102}{1 + \left(\frac{\text{18-month spot}}{2}\right)^3}
\]

\[
\$95 = \frac{\$2}{1.02^1} + \frac{\$2}{1.0225^2} + \frac{\$102}{1 + \left(\frac{\text{18-month spot}}{2}\right)^3} \Rightarrow \text{18-month spot rate} = 7.66\%
\]

Our abbreviated theoretical spot rate curve looks like this:

- 6-month spot rate = 4.00%.
- 12-month spot rate = 4.50%.
- 18-month spot rate = 7.66%.

Professor's Note: The U.S. Treasury currently issues Treasury bills with various maturities including 4 weeks, 13 weeks, 26 weeks, and 52 weeks; Treasury notes of 2 years, 5 years, and 10 years; and Treasury bonds with 30-year maturities.

LOS 55.c: Explain the various universes of Treasury securities that are used to construct the theoretical spot rate curve, and evaluate their advantages and disadvantages.

There are four combinations of securities that can be used to construct a theoretical Treasury spot rate curve: (1) all on-the-run Treasury securities, (2) all on-the-run and some off-the-run Treasury securities, (3) all Treasury bonds, notes, and bills, and (4) Treasury strips. After the explanation of each combination of Treasury securities is a summary of their advantages and disadvantages.

All On-the-Run Treasury Securities

On-the-run Treasury issues refer to the newest Treasury issues of a given maturity as described:

- T-bills: Zero-coupon securities with 1-month, 3-month, 6-month, and 12-month maturities.
- T-notes: Coupon instruments with 2-year, 5-year, and 10-year maturities.
- T-bonds: Coupon instruments with 30-year maturities.

The on-the-run issues have the largest trading volume and are, therefore, the most accurately priced issues. However, due to the tax effects on premium priced and discounted on-the-run coupon instruments, it is not appropriate to use the observed yield for these issues unless they are trading at par. Instead, the yield that is necessary to
make the issue trade at par must be computed. Using these adjusted yields and filling in the missing maturities using linear extrapolation, an on-the-run yield curve (called the par coupon curve) can be constructed. The bootstrapping methodology can then be used to generate a theoretical spot rate curve.

**Advantage:** Uses only the most accurately priced issues.  
**Disadvantage:** Large maturity gaps after the 5-year note.

### All On-the-Run and Some Off-the-Run Treasury Securities

Using just on-the-run issues creates problems because there aren’t on-the-run issues at every maturity, which leaves large maturity gaps between the 5-year and longest maturities. To provide additional observed points on the par coupon curve, 20- and 25-year off-the-run issues are added to the on-the-run issues. Linear interpolation is used to estimate yields for missing on-the-run maturities, and bootstrapping is employed to generate the theoretical spot rate curve.

There are two problems with the “on-the-run plus selected issues” framework. First, information is lost regarding the yield on Treasury securities not included in the construction of the curve. Second, the true yield for on-the-run issues may be distorted if any of these issues is “cheap” in the repo market.

**Advantage:** Reduces maturity gaps.  
**Disadvantages:** (1) Still doesn’t use all the rate information contained in Treasury issues, and (2) rates may be distorted by the repo market.

### All Treasury Coupon Securities and Bills

It has been argued that using only on-the-run issues, even with some off-the-run issues, ignores important information contained in other Treasury security prices. Therefore, some practitioners feel it is better to use all Treasury coupon securities and bills to construct the theoretical spot rate curve.

Bootstrapping is not useful in generating the theoretical spot rate curve when all Treasury securities and bills are used, because more than one yield may exist for each maturity. However, other statistical methodologies are available for fitting a curve to the data made available when all Treasury securities and bills are used.  

A disadvantage of using all Treasury securities and bills to develop the theoretical spot rate curve is that current information is not available for all issues.

**Advantage:** Does not ignore information from issues excluded by other approaches.  
**Disadvantages:** (1) Some maturities have more than one yield, and (2) current prices may not reflect accurate interest rates for all maturities.
Treasury Strips

Don’t lose sight of the big picture: the goal of the different yield curve construction approaches is to determine the spot rates for all maturities. Treasury coupon strips are zero-coupon securities made by stripping the coupons from normal T-bonds (i.e., coupon strips are zero-coupon T-bonds, so their rates are expressed as spot rates—no bootstrapping necessary). Hence, a seemingly simple solution to the problems associated with the other approaches is to use Treasury coupon strips, because the rates on strips are spot rates.

Unfortunately, this intuitive solution does not work well for several reasons. First, the strips market is not as liquid as the Treasury coupon market, so observed strip rates include a liquidity premium. Second, Treasury strip yields reflect a tax disadvantage, because the accrued interest on strips is taxed even though no cash flows are realized. Finally, some non-U.S. tax laws allow investors to recognize the difference between the maturity value and the price of some types of principal strips as a favorably taxed capital gain.

Advantages: (1) Provides yields at most maturities and reduces maturity gaps; and (2) intuitive approach that does not require bootstrapping to derive spot rates.

Disadvantages: (1) Liquidity premium embedded in strip rates; and (2) tax treatment affects observed rates.

LOS 55.d: Explain the swap rate curve (LIBOR curve), and discuss why market participants have used the swap rate curve rather than a government bond yield curve as a benchmark.

In the topic review of swaps in Study Session 17 we will explain how interest rate swaps are “priced,” which means how the fixed interest rate is determined. For example, two parties might enter into a 2-year interest rate swap in which counterparty A agrees to pay a fixed rate of 8% quarterly on a notional principal amount of $25 million, and counterparty B agrees to pay a floating rate equal to 3-month LIBOR on $25 million. The 8% fixed rate is called the 2-year swap rate. As we explain in Study Session 17, it is derived from the series of LIBOR rates from three months to two years.

The swap rate curve (also known as the LIBOR curve) is the series of swap rates quoted by swap dealers over maturities extending from 2 to 30 years. For U.S. dollar LIBOR, the LIBOR curve specifically refers to swap rates in which one party pays the fixed swap rate in U.S. dollars. LIBOR-based swap spreads reflect only the credit risk of the counterparty, which is usually a bank, so the swap curve is a AA-rated curve, not a default-free curve. Other currencies will have their own unique swap rate curves.

There are a number of reasons that market participants tend to prefer the swap rate curve as a benchmark interest rate curve rather than a government bond yield curve.

- The swap market is not regulated by any government, which makes swap rates in different countries more comparable.
• The supply of swaps and the equilibrium pricing that results from the interaction of supply and demand depends only on the number of participants willing to enter into a swap. It is not affected by technical market factors that can affect government bonds.

• Swap curves across countries are also more comparable because they reflect similar levels of credit risk, while government bond yield curves also reflect sovereign risk unique to each country.

• The swap curve typically has yield quotes at 11 maturities between 2 and 30 years. The U.S. government bond yield curve, however, only has on-the-run issues trading at four maturities of at least two years (2-year, 5-year, 10-year, and 30-year).

**LOS 55.e:** Illustrate the theories of the term structure of interest rates (i.e., pure expectations, liquidity, and preferred habitat), and discuss the implications of each for the shape of the yield curve.

We’ll explain each of the theories referenced in the LOS, paying particular attention to the implications of each theory for the shape of the yield curve and the interpretation of forward rates.

**Pure (Unbiased) Expectations Theory**

The pure (unbiased) expectations theory suggests that forward rates are solely a function of expected future spot rates. In other words, *long-term interest rates equal the mean of future expected short-term rates.* This implies, for example, that an investor could earn the same return by investing in a 5-year bond or by investing in a 3-year bond and then a 2-year bond after the 3-year bond expires.

For example, suppose the 1-year spot rate is 5% and the 2-year spot rate is 7%. Under the pure expectations theory, the 1-year forward rate in one year must be 9%, because investing for two years at 7% yields approximately the same annual return as investing for the first year at 5% and the second year at 9%. In other words, the 2-year rate of 7% is the average of the expected future 1-year rates of 5% and 9%, and is shown in Figure 6.

**Figure 6: Spot and Future Rates**
Notice that in this example, because short-term rates are expected to rise (from 5% to 9%), the yield curve is upward sloping, as depicted in Figure 7.

**Figure 7: Upward Sloping Yield Curve**

![Yield Curve Diagram](image)

Therefore, the implications for the shape of the yield curve under the pure expectations theory are:

- If the yield curve is upward sloping, short-term rates are expected to rise.
- If the curve is downward-sloping, short-term rates are expected to fall.
- A flat yield curve implies that the market expects short-term rates to remain constant.

The 9% rate in the previous example is called the **implied forward rate**. We call it an implied rate because it’s not a quoted rate; it’s implied by the quoted spot interest rates for one and two years. There are three interpretations of this 9% forward rate:

1. **Breakeven rate** refers to the forward rate that leaves investors indifferent between investing for two years, or investing for one year and then reinvesting at the breakeven forward rate of 9% for the second year. An investor would be indifferent between investing for two years at 7%, or investing at 5% for the first year and reinvesting in one year at the 9% breakeven rate.

2. Forward rates can also be interpreted as the **locked-in rate** for some future period. Using this interpretation, the investor in the previous example could invest in the 2-year bond instead of the 1-year bond, and essentially lock in a 9% rate for the 1-year period starting in one year.

3. How do we interpret the 9% forward rate in relation to expected future spot rates in the context of the pure expectations theory? The pure expectations theory predicts that the **expected spot rate** in one year is equal to the implied 1-year forward rate of 9%. In other words, expectations are unbiased.
The pure expectations theory has a significant shortcoming because it fails to consider the riskiness of bond investing. Specifically, the pure expectations theory fails to recognize:

- **Price risk**—the uncertainty associated with the future price of a bond that may be sold prior to its maturity.
- **Reinvestment risk**—the uncertainty associated with the rate at which bond cash flows can be reinvested over an investment horizon.

Hence, the pure (or unbiased) expectations theory does not recognize the risk difference between investing in a 1-year bond and sequentially investing in two 6-month bonds. This leads to a discussion of the next two term structure theories.

**Liquidity Preference Theory**

The liquidity preference theory of the term structure addresses the shortcomings of the pure expectations theory by proposing that forward rates reflect investors’ expectations of future spot rates plus a **liquidity premium** to compensate them for exposure to interest rate risk. Furthermore, the theory suggests that this liquidity premium is positively related to maturity: a 25-year bond has a larger liquidity premium than a 5-year bond.

The liquidity theory says that forward rates are biased estimates of the market’s expectation of future rates because they include a liquidity premium. Therefore, a positive-sloping yield curve may indicate that either: (1) the market expects future interest rates to rise; or (2) that rates are expected to remain constant (or even fall), but the addition of the liquidity premium results in a positive slope. A downward-sloping yield curve indicates falling short term rates according to the liquidity theory.

The size of the liquidity premiums need not be constant over time. They may be larger during periods of greater economic uncertainty, when risk aversion among investors is higher.

The liquidity preference theory says that the 2-year spot rate includes an extra return to compensate investors for the liquidity risk. Continuing our previous example, suppose the expected spot rate in the second year is only 8%, and the 2-year spot rate is still 7%. The 2-year spot rate must be equal to the average of the 1-year rates plus a liquidity premium or:

\[
\text{2-year spot rate} = 7.0\% = \frac{5.0\% + 8.0\%}{2} + \text{2-year liquidity premium}
\]

\[
\text{2-year liquidity premium} = 7.0\% - 6.5\% = 0.5\%
\]

Now the implied forward rate of 9% is not equal to the expected spot rate of 8%, so expectations are biased. Remember that this 0.5% extra return is a liquidity premium required by investors to induce them to hold the 2-year bond instead of two 1-year bonds. Furthermore, the liquidity theory predicts that this premium will be higher for longer-term investments, so if the 2-year premium is 0.5%, the 3-year premium might be 1%.
Preferred Habitat Theory

The preferred habitat theory also proposes that forward rates represent expected future spot rates plus a premium, but it does not support the view that this premium is directly related to maturity.

Instead, the preferred habitat theory suggests that the existence of an imbalance between the supply and demand for funds in a given maturity range will induce lenders and borrowers to shift from their preferred habitats (maturity range) to one that has the opposite imbalance. However, to get them to do so, they must be offered an incentive to compensate them for exposure to price and/or reinvestment rate risk in the "less-than-preferred" habitat. Borrowers require cost savings (i.e., lower yields) and lenders require a yield premium (i.e., higher yields) to move out of their preferred habitats.

Under this theory, premiums are related to supply and demand for funds at various maturities—not the term to maturity, as in the liquidity theory. This means that, for example, the 10-year bond might have a higher or lower risk premium than the 25-year bond. It also means this theory can be used to explain almost any yield curve shape.

The preferred habitat theory provides an interpretation of the implied forward rate that is similar to that of the liquidity theory. However, there are two subtle differences:

1. The premium is a positive or negative risk premium related to supply and demand for funds at various maturities, not necessarily a liquidity premium.
2. This risk premium is not necessarily related to maturity.

Warm-Up: Calculating Key Rate Duration

Recall that duration is an adequate measure of bond price risk only for small parallel shifts in the yield curve. The impact on nonparallel shifts can be measured using a concept known as key rate duration.

Rate duration is defined as the sensitivity of the value of a security or portfolio to changes in a single spot rate, holding all other spot rates constant. A security or portfolio will have a rate duration for every point (maturity) on the spot rate curve. Thus, a set of rate durations accompanies every security or portfolio.

To put this concept into practice, the analyst selects a certain number of maturities on the spot rate curve for which rate durations for bonds or bond portfolios are available. The set of rate durations associated with these maturities is referred to as the key rate durations. For example, the key rates might be 3 months, 1 year, 2 years, 3 years, 5 years, 7 years, 10 years, 15 years, and 20 years.

A key rate duration is defined as the approximate percentage change in the value of a bond or bond portfolio in response to a 100 basis point change in the corresponding key rate, holding all other rates constant. In other words, you can determine the key rate duration for the 5-year portion of the yield curve by changing the 5-year spot rate.
and observing the change in value of the portfolio. Keep in mind that every security or portfolio has a set of key rate durations, one for each key rate.

To keep it simple, we will use only four maturity points on the spot rate curve: the 2-year, 10-year, 20-year, and 25-year maturities, with key rate durations represented by $D_2$, $D_{10}$, $D_{20}$, and $D_{25}$, respectively. Also, assume that we have a portfolio of zero-coupon bonds with maturity and portfolio weights given in Figure 8.

Professor’s Note: The use of a portfolio of zero-coupon bonds simplifies this example in two ways: (1) their duration is approximately equal to their maturity, and (2) because they have no intermediate cash flows, they are not sensitive to spot rate changes for other maturities. This does not change the analysis much, because we can always categorize the portfolio’s cash flows this way by putting each cash flow into its appropriate maturity bucket.

Figure 8: Key Rate Duration Matrix

<table>
<thead>
<tr>
<th>Bond</th>
<th>Weight (%)</th>
<th>$D_2$</th>
<th>$D_{10}$</th>
<th>$D_{20}$</th>
<th>$D_{25}$</th>
<th>Rate Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-year</td>
<td>10</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>10-year</td>
<td>20</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>2.0</td>
</tr>
<tr>
<td>20-year</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>8.0</td>
</tr>
<tr>
<td>25-year</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>7.5</td>
</tr>
<tr>
<td>Portfolio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>17.7</td>
</tr>
</tbody>
</table>

In this example, the maturities that have been selected correspond to the various zero-coupon bond maturities. Thus, the key rate durations for the individual issues correspond with the maturity of each of the bonds (recall that the duration of a zero-coupon bond is equal to its maturity). If we used coupon bonds, there would be more than one rate duration for each of the issues.

\[ D_2 = (0.1 \times 2) + (0.2 \times 0) + (0.4 \times 0) + (0.3 \times 0) = 0.2 \]
\[ D_{10} = (0.1 \times 0) + (0.2 \times 10) + (0.4 \times 0) + (0.3 \times 0) = 2.0 \]
\[ D_{20} = (0.1 \times 0) + (0.2 \times 0) + (0.4 \times 20) + (0.3 \times 0) = 8.0 \]
\[ D_{25} = (0.1 \times 0) + (0.2 \times 0) + (0.4 \times 0) + (0.3 \times 25) = 7.5 \]

Effective portfolio duration = 17.7

The effective duration of a portfolio is the weighted average of the key rate durations of its individual security durations, where the weights are based on the market value portfolio weights. If the yield curve undergoes a parallel upward shift of 100 basis points, the value of the portfolio will decline by about 17.7 \( \times 1.00\% = 17.7\% \).
LOS 55.f: Compute and interpret the yield curve risk of a security or a portfolio by using key rate duration.

Key rate duration is particularly useful for measuring the effect of a nonparallel shift in the yield curve on a bond portfolio. We can use the key rate duration for each key rate to compute the effect on the portfolio of the rate change at that maturity. The effect on the overall portfolio is the sum of these individual effects.

Example: Computing the effects of a nonparallel shift in the yield curve

Suppose that the yield curve shifts such that 2-year rates increase by 100 basis points, 10-year rates increase by 150 basis points, 20-year rates increase by 80 basis points, and 25-year rates decline by 100 basis points. Calculate the effect of this nonparallel shift in the yield curve on the portfolio with 2-year, 10-year, 20-year, and 25-year key rate durations of 0.2, 2.0, 8.0, and 7.5, respectively (same as Figure 8).

Answer:

The change in the portfolio's value can be determined by computing the change in value associated with each key rate change.

<table>
<thead>
<tr>
<th>Change from key rate change</th>
<th>Effect on portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-year key rate increase</td>
<td>-0.2% decrease</td>
</tr>
<tr>
<td>10-year key rate increase</td>
<td>-3.0% decrease</td>
</tr>
<tr>
<td>20-year key rate increase</td>
<td>-6.4% decrease</td>
</tr>
<tr>
<td>25-year key rate decrease</td>
<td>+7.5% increase</td>
</tr>
<tr>
<td>Total</td>
<td>-2.1% decrease</td>
</tr>
</tbody>
</table>

Thus, the nonparallel shift in the spot rate curve caused a 2.1% decrease in the value of the portfolio. (The minus sign in front of each computation reminds us that prices and interest rates move in opposite directions.)

Now let's apply the concept of key rate duration to three common bond portfolio structures:

1. **Barbell portfolios** contain a relatively large percentage of long and short maturity bonds.
2. **Ladder portfolios** contain bonds that are evenly distributed throughout the maturity spectrum.
3. **Bullet portfolios** typically have a relatively high concentration of bonds at some intermediate maturity.

Consider the key rate durations provided in Figure 9 for each of these types of portfolios. The portfolio duration for each structure is 9.63, implying that for a parallel shift in the yield curve, the value of each of these portfolios will change by the same amount. Note that the sum of the portfolio's key rate durations equals the portfolio's effective duration. For example, if the yield curve experiences a parallel downward shift
of 75 basis points, the value of each portfolio will increase by approximately $9.63 \times 0.75\% = 7.22\%$.

Figure 9: Complex Portfolio Structure Duration

<table>
<thead>
<tr>
<th>Key Rate Maturity</th>
<th>Bullet</th>
<th>Ladder</th>
<th>Barbell</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-month</td>
<td>0.07</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>1-year</td>
<td>0.09</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>2-year</td>
<td>1.10</td>
<td>1.04</td>
<td>0.11</td>
</tr>
<tr>
<td>3-year</td>
<td>0.83</td>
<td>1.04</td>
<td>2.25</td>
</tr>
<tr>
<td>5-year</td>
<td>0.42</td>
<td>1.07</td>
<td>0.65</td>
</tr>
<tr>
<td>7-year</td>
<td>0.73</td>
<td>1.07</td>
<td>1.05</td>
</tr>
<tr>
<td>10-year</td>
<td>1.20</td>
<td>1.07</td>
<td>1.03</td>
</tr>
<tr>
<td>15-year</td>
<td>4.22</td>
<td>1.08</td>
<td>1.12</td>
</tr>
<tr>
<td>20-year</td>
<td>0.70</td>
<td>1.06</td>
<td>1.08</td>
</tr>
<tr>
<td>25-year</td>
<td>0.20</td>
<td>1.05</td>
<td>2.15</td>
</tr>
<tr>
<td>27-year</td>
<td>0.07</td>
<td>1.04</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Effective portfolio duration 9.63 9.63 9.63

As you can see in Figure 9, the 15-year key rate duration dominates all other key rate maturities for the bullet structure. The ladder portfolio’s key rate durations are fairly equal across all key rate maturities, and the barbell portfolio is characterized by relatively large key rate durations at the 3- and 25-year maturity levels.

To further illustrate key rate duration, now consider the impact of a 75 basis point increase in the 15-year spot rate while all other key rate maturity rates remain fairly stable. Because the bullet portfolio has the largest 15-year key rate duration, its value will decline more than the value of the ladder or barbell portfolios even though it has the same effective portfolio duration as the other two portfolios.

The concept of key rate duration allows us to assess the impact of nonparallel shifts in the yield curve on the value of our portfolio. The sensitivity of a portfolio to any type of yield curve change can be evaluated with the key rate concept.

LOS 55.g: Compute and interpret yield volatility, distinguish between historical yield volatility and implied yield volatility, and explain how yield volatility is forecasted.

Yield volatility measurement has applications when valuing callable bonds and interest rate derivatives, and when measuring interest rate risk. The standard deviation of yield changes is the common yield volatility measurement.
Using historical data, yield volatility is measured by the standard deviation of daily yield changes. Continuously compounded yield changes are computed as the natural log of the ratio of yield levels. The general formula for the variance of daily yield change (assuming continuous compounding) is:

\[
\text{variance} = \frac{1}{T-1} \sum_{t=1}^{T} (X_t - \bar{X})^2
\]

\[
\text{standard deviation} = \sqrt{\text{variance}}
\]

where:

\[
X_t = 100 \times \ln \left( \frac{y_t}{y_{t-1}} \right)
\]

\[
y_t = \text{yield on day } t
\]

\[
\bar{X} = \text{the average yield change over period } t = 1 \text{ to } t = T
\]

Professor's Note: The natural logarithm function “ln” is labeled “LN” on your TI BAII Plus. On the HP, input [g] \rightarrow LN.

Example: Calculating continuously-compounded yield changes

Assume that today’s yield is 7.56% and yesterday’s yield was 7.50%. Compute the percentage yield change from yesterday to today, assuming continuous compounding.

Answer:

\[
X_t = 100 \times \ln \left( \frac{7.56}{7.50} \right) = 0.7968\%
\]

Continuing this process over a specific period of analysis will generate a set of observations for which the variance and standard deviation can be calculated.

The choice of sample period can have a significant effect on the estimate of standard deviation. The appropriate number depends on the investment horizon of the user of the volatility measurement. For example, day traders may only be interested in volatility over the most recent week or two, whereas bond portfolio managers might be interested in the volatility of yields over the past month or longer.

Professor's Note: It is unlikely that you will be asked to calculate the standard deviation given a series of daily yields. Therefore, you should concentrate on the interpretation of yield volatility.
It is common practice to annualize the standard deviation of daily yield changes using the following formula:

\[
\sigma_{\text{annual}} = \sigma_{\text{daily}} \times (\text{number of trading days in the year})^{1/2}
\]

where:

- \(\sigma_{\text{annual}}\) = annualized standard deviation
- \(\sigma_{\text{daily}}\) = daily standard deviation

The number of days that constitute a year can be the number of calendar days (365), the number of weekdays (52 \times 5 = 260), or the number of actual trading days (260 – 10 holidays = 250). On the exam, you will most likely be given the number of days to use. If not, use 250, the actual number of trading days.

**Interpreting Historical Yield Volatility**

What does an annualized standard deviation of 10% mean? If the yield on a portfolio is currently 8%, the standard deviation is 80 basis points (8% \times 0.10). This standard deviation of yield changes in basis point form can be used to construct confidence intervals. If yield changes are normally distributed, then there is a 68.3% probability that the observed yield will be plus or minus one standard deviation from the expected (prevailing) yield. That is, if the prevailing yield is 8% and the annualized standard deviation is 80 basis points, there is a 68.3% chance that next year’s yield will be between 7.2% and 8.8% (8.0% +/- 80bp). This is referred to as the 68.3% confidence interval.

Continuing with this process, there is a 99.7% probability that the yield next year will be plus or minus 3 standard deviations from the prevailing rate. In this example, the 99.7% confidence interval is 5.6% to 10.4% [8.0% +/- (3) \times (80bp)].

*Professor’s Note: Confidence intervals are discussed in Study Session 3.*

**IMPLIED YIELD VOLATILITY**

In addition to using historical observations, yield volatility can be estimated using observed prices for interest rate derivatives and option pricing models. The method for doing this is to plug the observed price of an option into the option pricing model, along with the model’s other observable variables, then solve the model for the unknown volatility (standard deviation). When yield volatility is derived from option prices in this manner, it is referred to as implied volatility.

The use of implied volatility is often criticized because:

- It is based on the assumption that the option pricing model is correct.
- Models make the simplifying assumption that volatility is constant.
FORECASTING YIELD VOLATILITY

Typically, the standard deviation of daily yield changes is calculated using the moving average of yield changes over some appropriate time interval. For forecasting purposes, it is more appropriate to use a zero value for the expected change in yields. Thus, the general formula for the variance changes is:

\[
\text{variance} = \sum_{t=1}^{T} \frac{X_t^2}{T-1}
\]

The easiest way to compute the variance is to assign equal weights to each observation. Investors, however, often feel it is necessary to weight recent observations more heavily than distant observations. The formula for forecasting variance can be modified to incorporate any desired weighting scheme as follows:

\[
\text{variance} = \sum_{t=1}^{T} \frac{W_t X_t^2}{T-1}
\]

where:
\[
W_t = \text{the weight assigned to each period’s observation such that the sum of the weights equals 1}
\]

Yield volatility has been observed to follow patterns over time. These patterns can be used to forecast volatility using a statistical technique based on autoregressive conditional heteroskedasticity (ARCH) models.

Professor’s Note: ARCH models are covered in Study Session 3.
KEY CONCEPTS

LOS 55.a
When the yield curve undergoes a parallel shift, the yield on all maturities changes in the same direction and by the same amount. The slope remains unchanged.

When the yield curve undergoes a nonparallel shift, the yields on the various maturities do not necessarily change in the same direction, nor do they change by the same amount. The slope of the yield curve changes.
• Twists refer to nonparallel yield curve changes when the slope becomes either flatter or steeper.
• Butterfly shifts refer to nonparallel changes in the curvature of the yield curve.

LOS 55.b
Three factors that explain historical Treasury returns have been identified. In the order of relative importance, these are:
• Changes in the level of interest rates (by far the most important factor).
• Changes in the slope of the yield curve (distant second most influential factor).
• Changes in the curvature of the yield curve (slight impact).

LOS 55.c
All on-the-run Treasury issues:
Advantage: Uses only the most accurately priced issues.
Disadvantage: Large maturity gaps after the five-year note.

All on-the-run and selected off-the-run Treasury issues:
Advantage: Reduces maturity gaps.
Disadvantages: (1) Still doesn’t use all issues, and (2) rates may be distorted by the repo market.

All Treasury coupon securities and bills:
Advantage: Uses information from issues excluded by other approaches.
Disadvantages: (1) Some maturities have more than one yield, and (2) current prices may not reflect accurate interest rates for all maturities.

Treasury coupon strips:
Advantages: (1) Provides yields at most maturities and reduces maturity gaps, and (2) intuitive approach that does not require bootstrapping to derive spot rates.
Disadvantages: (1) Liquidity premium embedded in strip rates, and (2) tax treatment affects observed rates.
LOS 55.d
The swap rate curve (also known as the LIBOR curve) is the series of swap rates quoted by swap dealers over maturities extending from 2 to 30 years. Reasons why market participants may prefer to use the swap rate curve as a benchmark:

- The swap market is not regulated by any government, which makes swap rates in different countries more comparable.
- The availability of swaps and the equilibrium pricing are only driven by the interaction of supply and demand. It is not affected by technical market factors that can affect government bonds.
- Swap curves across countries are also more comparable because they reflect similar levels of credit risk, while government bond yield curves also reflect sovereign risk unique to each country.
- The swap curve typically has yield quotes at 11 maturities between 2 and 30 years. The U.S. government bond yield curve only has on-the-run issues trading at four maturities between 2 and 30 years.

LOS 55.e
There are two versions of the expectations theory (pure expectations and biased expectations). Both are based on the premise that current rates are related to the market’s expectations regarding future rates. The difference lies in whether or not other factors also affect forward rates and, if they do, how? The pure expectations theory argues that forward rates are solely a function of expected future spot rates. This implies that if the yield curve is upward (downward) sloping, short-term rates are expected to rise (fall), and if the yield curve is flat, the market expects short-term rates to be constant. The theory’s drawback is that it fails to consider price risk and reinvestment risk.

The two forms of the biased expectations theory are the liquidity theory and the preferred habitat theory. They contend that other factors affect forward rates.

- The liquidity preference theory of the term structure proposes that forward rates reflect investors’ expectations of future rates plus a liquidity premium to compensate them for exposure to interest rate risk.
- The preferred habitat theory proposes that forward rates represent expected future spot rates plus a premium, but it suggests that this premium is related to disequilibrium between the supply and demand for funds in a given maturity range. Investors will switch to another maturity range (habitat) only if they are offered a premium to compensate them for exposure to price and/or reinvestment rate risk in the “less-than-preferred” habitat.

LOS 55.f
Key rate duration is a methodology that can be used to measure the impact of nonparallel shifts in the yield curve.

- Rate duration is defined as the sensitivity of the value of a security or portfolio to changes in a single spot rate, holding all other spot rates constant.
- In practice, a certain number of maturities on the spot rate curve are selected for which bond durations are measured. The set of rate durations associated with these key maturities are the key rate durations.
LOS 55.g

Yield volatility is measured with variance (standard deviation) of changes in daily yields.

- Continuously compounded daily yield changes are used. The formula used is:
  \[ 100 \ln(y_t / y_{t-1}) \], where \( y_t \) and \( y_{t-1} \) = the day \( t \) and day \( t-1 \) yields, respectively.

- The number of daily observations of yield changes in the sample can have a
  significant effect on the computed standard deviation. The appropriate number
  depends on the investment horizon of the user of the volatility measurement.

- It is common practice to annualize the standard deviation using the formula:
  \[ \sigma_{\text{annual}} = \sigma_{\text{daily}} \times (\text{number of trading days in the year})^{1/2} \]. Typically, the number of
  trading days is estimated as 250.

- The standard deviation of yield changes is used to construct confidence intervals and
  interpret yield volatility.

Implied volatility can be estimated using observed prices for interest rate derivatives
and option pricing models. Implied volatility is criticized because it is based on the
assumptions that the option pricing model is correct and that volatility is constant.

Yield volatility forecasts are based on the standard deviation of daily yield changes using
the moving average of yield changes over some appropriate time interval.

- It is more appropriate to use zero as the value for the expected change in yields.
- The easiest way to compute the variance is to assign equal weights to each
  observation.
- Some investors weight recent observations more heavily than distant observations.

Yield volatility has been observed to follow patterns over time. This pattern can be
modeled and used to forecast volatility using autoregressive statistical techniques.
1. Which of the following statements concerning yield curve shifts is least accurate?
   A. A twist results in a flatter or steeper yield curve.
   B. Butterfly shifts result in a change in curvature.
   C. A positive butterfly shift results in more curvature.

2. Of the three factors that have been observed to affect Treasury returns, which is the most important? Changes in the:
   A. slope of the yield curve.
   B. curvature of the yield curve.
   C. level of interest rates.

Use the following information for an equally weighted U.S. Treasury portfolio to answer Questions 3 and 4.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Key rate duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-month</td>
<td>0.06</td>
</tr>
<tr>
<td>2-year</td>
<td>0.73</td>
</tr>
<tr>
<td>5-year</td>
<td>0.34</td>
</tr>
<tr>
<td>10-year</td>
<td>3.09</td>
</tr>
<tr>
<td>15-year</td>
<td>0.63</td>
</tr>
<tr>
<td>20-year</td>
<td>1.22</td>
</tr>
<tr>
<td>25-year</td>
<td>2.19</td>
</tr>
<tr>
<td>27-year</td>
<td>3.65</td>
</tr>
</tbody>
</table>

3. The effective duration of the portfolio for a parallel shift in the yield curve is closest to:
   A. 9.09.
   B. 10.29.
   C. 11.91.

4. What is the impact on the portfolio of a 25 basis point increase in the 5-year rate and a 50 basis point increase in the 20-year rate, holding other key rates constant? The portfolio will:
   A. decrease in value by 0.695%.
   B. decrease in value by 0.372%.
   C. decrease in value by 0.816%.

5. Which of the following statements describes an inverted yield curve?
   A. Short-term rates are higher than long-term rates.
   B. Medium-term rates are higher than both short-term rates and long-term rates.
   C. Long-term rates are higher than short-term rates.
6. Which of the following statements concerning theories of the term structure is least accurate?
   A. Under the pure expectations hypothesis, if the term structure is inverted, the market expects future short-term interest rates to be lower than current short-term interest rates.
   B. Under the pure expectations hypothesis, if the term structure is normal, the market expects future short-term interest rates to be higher than current short-term interest rates.
   C. Under the liquidity theory, if the term structure is normal, the market expects future short-term interest rates to be higher than current short-term interest rates.

7. Which of the following statements is the best definition of key rate duration?
   A. The duration-weighted sensitivity of a bond to a parallel shift in the term structure.
   B. The convexity-enhanced sensitivity of a bond to a non-parallel shift in the term structure.
   C. The approximate percentage change in the value of a bond or bond portfolio in response to a 100 basis point change in a key rate, holding all other rates constant.

8. Which of the following is a major criticism of the pure expectations theory of the term structure? It ignores:
   A. the duration of the bond.
   B. price risk and reinvestment risk of an investment.
   C. convexity of an investment.

9. Today's yield is 6.10% and yesterday's yield was 6.18%. The percentage yield change from yesterday to today, assuming continuous compounding, is closest to:
   A. -1.303%.
   B. -1.137%.
   C. -1.098%.

10. If there are 250 trading days in a year and the daily historical yield volatility is 0.34%, the annual standard deviation of the yield is closest to:
    A. 5.38%.
    B. 0.85%.
    C. 6.84%.

11. The current yield on a bond is 7.19% and the bond's standard deviation is 11.3%. The 99.7% confidence interval for the yield is closest to:
    A. [6.32%, 8.04%].
    B. [5.56%, 8.81%].
    C. [4.75%, 9.63%).
### Challenge Problems

12. Suppose you observe a 1-year (zero-coupon) Treasury security trading at a yield to maturity of 5% (price of 95.2381% of par). You also observe a 2-year T-note with a 6% coupon trading at a yield to maturity of 5.5% (price of 100.9232). And, finally, you observe a 3-year T-note with a 7% coupon trading at a yield to maturity of 6.0% (price of 102.6730). Assume annual coupon payments. Use the bootstrapping method to determine the 2-year and 3-year spot rates.

<table>
<thead>
<tr>
<th>2-year spot rate</th>
<th>3-year spot rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.51%</td>
<td>5.92%</td>
</tr>
<tr>
<td>5.46%</td>
<td>5.92%</td>
</tr>
<tr>
<td>5.51%</td>
<td>6.05%</td>
</tr>
</tbody>
</table>

13. Former Treasury Secretary Robert Rubin decided to stop issuing 30-year Treasury bonds in 2001 and to replace them by borrowing more with shorter-maturity Treasury bills and notes (although the U.S. Treasury has since resumed issuing 30-year bonds). Which of the following statements concerning this decision is most accurate?

A. If the pure expectations hypothesis of the term structure is correct, this decision will reduce the government's borrowing cost.

B. If the liquidity theory of the term structure is correct, this decision will reduce the government's borrowing cost.

C. If the liquidity theory of the term structure is correct, this decision will not change the government's borrowing cost.

14. Ynot Investments currently uses the yield on all on-the-run Treasury securities to construct the theoretical spot curve the firm uses to price bonds. Frank Bristow, a fixed income analyst at Ynot, has proposed to his colleagues that the firm switch to using yields on U.S. Treasury strips. He defends his proposal by making two statements that he purports to be advantages of using Treasury strips vs. on-the-run Treasury securities:

1. On-the-run Treasury yields reflect a liquidity premium that distorts the spot rate curve derived from those yields; that liquidity premium is not embedded in strip yields.

2. Observable market yields on strips are available at a variety of maturities above 10 years; there are large gaps in on-the-run yields between 10 and 30 years.

With regards to Bristow’s statements justifying use of Treasury strip yields instead of on-the-run Treasury yields to derive the theoretical spot rate curve:

A. both statements are correct.

B. only one statement is correct.

C. both statements are incorrect.
15. Do spreads on swap rates [based on the London Interbank Offered Rate (LIBOR) curve] over comparable Treasury yields reflect credit risk and/or sovereign risk?
   A. Yes on both counts.
   B. Yes on one count.
   C. No on both counts.

16. The use of implied volatility is often criticized for the assumption that:
   A. the option pricing model provides a poor estimate, at best.
   B. volatility is constant.
   C. historical observation has more predictive power than current interest rate derivatives prices.

17. Forecasting yield volatility is based on each assumption except:
   A. a statistical technique based on autoregressive conditional heteroskedasticity (ARCH) models can be used to forecast volatility.
   B. it is often desirable to weight recent observations more heavily than distant observations.
   C. including weighting schemes when forecasting variance eliminates autoregressive conditional heteroskedasticity.
1. C A positive butterfly shift is a nonparallel shift in the yield curve that results in less curvature.

2. C The three factors that affect Treasury returns are: (1) changes in the level of yields, (2) changes in the slope of the yield curve, and (3) changes in the curvature of the yield curve. Changes in the level of yields are by far the most influential of these factors, explaining about 90% of the variation in Treasury security returns.

3. C Given the key rate durations for an equally weighted bond portfolio, effective duration for a parallel shift in the yield curve is the sum of the individual rate durations. In this case, the portfolio effective duration is 11.91.

4. A Change in Portfolio Value

\[
\begin{align*}
\text{Change from 5-year key rate increase:} & \quad -0.25\% \times 0.34 = 0.085\% \text{ decrease} \\
\text{Change from 20-year key rate increase:} & \quad -0.50\% \times 1.22 = 0.610\% \text{ decrease} \\
\text{Net change} & \quad 0.695\% \text{ decrease}
\end{align*}
\]

Thus, the portfolio value will decrease by 0.695%.

5. A An inverted yield curve is one where short-term rates are higher than long-term rates.

6. C Under the liquidity theory, there is a premium in long-term borrowing rates. Therefore, the term structure may be upward-sloping (i.e., normal) even if future short-term interest rates are expected to be equal to current short-term rates. If the term structure is inverted, even with a positive liquidity premium, the expectation for future short-term rates must be lower. Under the pure expectations theory, an upward-sloping yield curve (i.e., a normal curve) means short-term spot rates are expected to increase. An inverted yield curve implies a decrease in short-term expected spot rates, according to the pure expectations theory.

7. C Key rate duration is the approximate percentage change in the value of a bond or bond portfolio in response to a 100 basis point change in a key rate, holding all other rates constant.

8. B The major criticism of the pure expectations theory is that it fails to recognize interest rate risk; specifically, price risk and reinvestment risk. Price risk is the uncertainty associated with future bond prices as a result of interest rate changes, and reinvestment risk reflects the uncertainty associated with the rate at which the bond's cash flows can be reinvested.

9. A percentage yield change \[= 100 \times \ln \left( \frac{6.10}{6.18} \right) = -1.303\% \]

10. A annualized standard deviation \[= 0.0034 \times \sqrt{250} = 0.0538 = 5.38\% \]
11. C  The standard deviation is calculated as:

\[ \sigma = 7.19\% \times 0.113 = 0.8125\% = 81.25 \text{ basis points} \]

The 99.7% confidence interval equals the current rate (7.19%) plus or minus three times the standard deviation:

\[ 7.19\% \pm (3 \times 0.8125\%) \Rightarrow [4.75\%, 9.63\%] \]

**ANSWERS – CHALLENGE PROBLEMS**

12. C  Here are the cash flows associated with the three bonds:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-year</td>
<td>-$95.2381</td>
<td>+$100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-year</td>
<td>-$100.9232</td>
<td>+$6</td>
<td>+$106</td>
<td></td>
</tr>
<tr>
<td>3-year</td>
<td>-$102.6730</td>
<td>+$7</td>
<td>+$7</td>
<td>+$107</td>
</tr>
</tbody>
</table>

To find \( Z_2 \), the 2-year spot rate:

\[ \frac{\$100.9232}{1.05^1} + \frac{\$106}{(1+Z_2)^2} \Rightarrow Z_2 = 5.51\% \]

To find \( Z_3 \), the 3-year spot rate:

\[ \frac{\$102.6730}{1.05^2} + \frac{\$7}{1.0551^2} + \frac{\$107}{(1+Z_3)^3} \Rightarrow Z_3 = 6.05\% \]

13. B  If the pure expectations hypothesis of the term structure is correct, altering the maturity of the government’s borrowing will not affect the government’s borrowing cost (i.e., borrowing once for 30 years is the same as borrowing 30 times for one year at a time). If the liquidity theory is correct, the government’s borrowing cost will go down, as it no longer has to compensate lenders with the liquidity premium for borrowing long term.

14. B  An important disadvantage of using strip yields is that strip markets are less liquid, so strip yields contain a liquidity premium not embedded in on-the-run issues; therefore, Statement 1 is inaccurate and does not support Bristow’s proposal. The primary advantage of using strip yields is that it reduces or eliminates the maturity gaps found in on-the-run Treasury yields; therefore, Statement 2 is accurate and supports Bristow’s proposal.

15. B  Swap curves are not default-free curves because the LIBOR-based swap spreads over U.S. Treasuries reflect counterparty credit risk. However, swap spreads over U.S. Treasuries do not reflect sovereign risk. The swap market is not regulated by any government, which makes swap spreads more comparable across borders than government bond yields, which do reflect sovereign risk.
16. B The use of implied volatility is often criticized for assuming that the option pricing model is correct, and that volatility is constant. The use of implied volatility does not assume that historical observation has more predictive power than current interest rate derivatives prices.

17. C Because yield volatility has been observed to follow patterns over time, a statistical technique based on (ARCH) models can be used to forecast volatility. Weighting schemes are common in forecasting, but are not used to eliminate autoregressive conditional heteroskedasticity.
The following is a review of the Fixed Income: Valuation Concepts principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**VALUING BONDS WITH EMBEDDED OPTIONS**

*Study Session 14*

**EXAM FOCUS**

The binomial model is used to value bonds with embedded options (callable bonds and putable bonds), assess OAS, value the call feature, and calculate duration and convexity. The final section of this topic review addresses convertible bonds. Concentrate on valuation of option-free, callable, and putable bonds using interest rate trees, as well as the interpretation of a bond's OAS. You'll see these topics again in Study Session 15.

**LOS 56.a: Evaluate, using relative value analysis, whether a security is undervalued or overvalued.**

Relative value analysis of bonds involves comparing the spread on the bond (over some benchmark) to the required spread and determining whether the bond is over or undervalued relative to the benchmark. The required spread is the spread available on comparable securities. In simple terms:

- Undervalued ("cheap") bonds have spreads larger than the required spread.
- Overvalued ("rich") bonds have spreads smaller than the required spread.
- Properly valued ("fairly priced") bonds have spreads equal to the required spread.

**WARM-UP: BINOMIAL MODEL**

A binomial model is a relatively simple single factor interest rate model that, given an assumed level of volatility, suggests that interest rates have an equal probability of taking on one of two possible values in the next period. The binomial interest rate model is used throughout this review to illustrate the issues that must be considered when valuing bonds with embedded options.

An interest rate model makes assumptions about interest rate volatility, along with a set of paths that interest rates may follow over time. This set of possible interest rate paths is referred to as an interest rate tree.

**Binomial Interest Rate Trees**

The set of possible interest rate paths that are used to value bonds with a binomial model is called a binomial interest rate tree. The diagram in Figure 1 depicts a binomial interest rate tree.
To understand this two-period binomial tree, consider the nodes indicated with the bold dots (♦). A node is a point in time when interest rates can take one of two possible paths, an upper path, $U$, or a lower path, $L$. Now, consider the node on the right side of the diagram where the interest rate $i_{2,LU}$ appears. This is the rate that will occur if the initial rate, $i_0$, follows the lower path from Node 0 to Node 1 to become $i_{1,L}$, then follows the upper of the two possible paths to Node 2, where it takes on the value $i_{2,LU}$. At the risk of stating the obvious, the upper path from a given node leads to a higher rate than the lower path. Notice also that an upward move followed by a downward move gets us to the same place on the tree as a down-then-up move, so $i_{2,UL} = i_{2,LU}$.

The interest rates at each node in this interest rate tree are 1-period forward rates corresponding to the nodal period. Beyond the root of the tree, there is more than one 1-period forward rate for each nodal period (i.e., at Year 1, we have two 1-year forward rates, $i_{1,U}$ and $i_{1,L}$). The relationship among the set of rates associated with each individual nodal period is a function of the interest rate volatility assumption of the model being employed to generate the tree. An example of a model that specifies the relationship between $i_{1,U}$ and $i_{1,L}$ is:

$$ i_{1,U} = i_{1,L} \times e^{2\sigma} $$

where:

$\sigma$ = the assumed volatility of the 1-period rate

e = the natural antilogarithm, 2.71828 (the $e^x$ button on your TI BAII Plus)

Obviously, a change in the assumed interest rate volatility will affect the rates at every node in the tree. More complex interest rate models are also used.

**CONSTRUCTING AN ARBITRAGE-FREE TREE**

The construction of an interest rate tree, binomial or otherwise, is a tedious process. In practice, the interest rate tree is usually generated using specialized computer software. There is one underlying rule governing the construction of an interest rate tree: the interest rate tree should generate **arbitrage-free values** for on-the-run issues of the benchmark security (benchmark interest rates are discussed in the next LOS). This means that the value of these on-the-run issues produced by the interest rate tree must equal their market prices, which excludes arbitrage opportunities. This requirement is very important because without it, the model will not properly price more complex callable and putable securities, which is the intended purpose of the model.
The methodology for constructing an arbitrage-free interest rate tree involves the following steps:

**Step 1:** Use the yield on the current 1-year on-the-run U.S. Treasury security issue for \( i_0 \). Suppose, for example, that \( i_0 = 4.5749\% \).

**Step 2:** Make an assumption about the volatility of interest rates. Suppose, for example, we assume \( \sigma = 15\% \).

**Step 3:** Given the coupon rate and market value of the 2-year on-the-run issue, provide a guess of \( i_{1,L} \), compute \( i_{1,U} = i_{1,L}e^{2\sigma} \), and use the resulting interest rate tree to compute the value of the on-the-run issue. Suppose, for example, that the coupon rate and market price of the 2-year on-the-run U.S. treasury security issue are 7% and $102,999, respectively.

**Step 4:** If the value from the model is higher than the market price, increase the guess of \( i_{1,L} \), recompute \( i_{1,U} \), and compute the new value of the on-the-run issue. If the model value is too low, decrease the interest rates in the tree.

**Step 5:** Repeat this iterative process until the value generated by the model is equal to the market price. Suppose, for example, we determine that if \( i_{1,L} = 5.321\% \), and \( i_{1,U} = 5.321\% \times e^{(0.15)} = 7.1826\% \), then the value from the model is equal to the market price of $102,999. Then the interest rate tree (as shown in the next example) is arbitrage-free.

### VALUING AN OPTION-FREE BOND WITH THE BINOMIAL MODEL

Remember that the value of a bond at a given node in a binomial tree is the average of the present values of the two possible values from the next period, because the probabilities of an up move and a down move are both 50%. The appropriate discount rate is the forward rate associated with the node under analysis.

#### Example: Valuing an option-free bond with the binomial model

A 7% annual coupon bond has two years to maturity. The interest rate tree is shown in the figure below. Fill in the tree and calculate the value of the bond today.

**Valuing a 2-Year, 7.0% Coupon, Option-Free Bond**
Consider the value of the bond at the upper node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \left[ \frac{100 + 7}{1.071826} + \frac{100 + 7}{1.071826} \right] = 99.830$$

Similarly, the value of the bond at the lower node for Period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2} \left[ \frac{100 + 7}{1.053210} + \frac{100 + 7}{1.053210} \right] = 101.594$$

Now calculate $V_0$, the current value of the bond at Node 0.

$$V_0 = \frac{1}{2} \left[ \frac{99.830 + 7}{1.045749} + \frac{101.594 + 7}{1.045749} \right] = 102.999$$

The completed binomial tree is shown below:

**Spread Measures**

There are three important spread measures that you must be able to interpret: nominal spread, zero-volatility spread (Z-spread), and option-adjusted spread (OAS). The first two were discussed in some detail at Level 1; OAS was mentioned briefly, so we're going to spend more time on it at Level 2. We'll provide a quick review of these three spread measures now before starting our discussion of benchmark interest rates and relative value analysis. LOS 56.g includes more detail on the OAS.
The **nominal spread** is the bond's yield to maturity minus the yield on a comparable-maturity treasury benchmark security. For example, if a 5-year AA bond has a yield to maturity of 8% and the 5-year U.S. Treasury note has a yield of 5%, the bond’s nominal spread is 3%. The problem with the nominal spread is that it uses a single interest rate to discount each cash flow that makes up the bond; if the yield curve is not flat, each cash flow should instead be discounted at the appropriate spot rate for that maturity.

The **Z-spread** is the spread that when added to each spot rate on the yield curve, makes the present value of the bond’s cash flows equal to the bond’s market price. Therefore, it is a spread over the entire spot rate curve. The term zero volatility in the Z-spread refers to the fact that it assumes interest rate volatility is zero. If interest rates are volatile, the Z-spread is not appropriate to use to value bonds with embedded options because the Z-spread includes the cost of the embedded option. The nominal spread and the Z-spread are approximately equal to each other. The difference between the two is larger (1) if the yield curve is not flat, (2) for securities that repay principal over time such as mortgage-backed securities (MBS), and (3) for securities with longer maturities.

For example, suppose the 1-year spot rate is 4% and the 2-year spot rate is 5%. The market price of a 2-year bond with annual coupon payments of 8% is $104.12. The Z-spread is the spread that solves the following equation:

$$\frac{8}{(1 + 0.04 + Z)^1} + \frac{108}{(1 + 0.05 + Z)^2} = 104.12$$

In this case, the Z-spread is 0.008, or 80 basis points. If you plug $Z = 0.008$ into the equation above, you'll find that the present value of the bond's cash flows (the right-hand side) will equal $104.12$.

The **OAS** is the spread on a bond with an embedded option after the embedded option cost has been removed. It’s equal to the Z-spread minus the option cost. The OAS for a corporate bond must be calculated using a binomial interest rate model. Because the model is created using a spot rate curve, the OAS is also a spread over the spot rate curve. The relationship between the Z-spread, the OAS, and the option cost are shown in Figure 2. The line between the spot rate curve and the corporate yield curve is not usually identified separately, but technically it represents the spot rate curve plus the OAS.
LOS 56.b: Evaluate the importance of benchmark interest rates in interpreting spread measures.

A nominal yield spread is the difference between the yield on a security and a benchmark yield, usually the comparable U.S. treasury yield of the same maturity. If we use a different benchmark (e.g., the AAA corporate yield), the spread and our interpretation of the spread will be different. The same concept applies to spreads measured from binomial interest rate trees. For example, in LOS 56.g we'll discuss the OAS that is derived from a binomial model, and our interpretation of the OAS will depend on the benchmark rates used to create the tree.

In the previous example in which we valued the 7% coupon option-free bond, we used U.S. Treasury securities as benchmark rates to construct the interest rate tree. There are, in fact, three different benchmark interest rates that can be used to calculate spreads:

1. Treasury securities.
2. A specific sector of the bond market with a credit rating higher than the issue being valued.
3. A specific issuer.

Once again, our interpretation of a spread calculated for a specific security will depend on the benchmark rates we used.
LOS 56.c: Illustrate the backward induction valuation methodology within the binomial interest rate tree framework.

Backward induction refers to the process of valuing a bond using a binomial interest rate tree. The term “backward” is used because in order to determine the value of a bond at Node 0, you need to know the values that the bond can take on at Node 1. But to determine the values of the bond at the Year 1 nodes, you need to know the possible values of the bond at the Year 2 nodes. Thus, for a bond that has $N$ compounding periods, the current value of the bond is determined by computing the bond’s possible values at Period $N$ and working backwards to Node 0.

LOS 56.d: Compute the value of a callable bond from an interest rate tree.

The basic process for valuing a callable bond from an interest rate tree is similar to the process for a noncallable bond. When valuing a callable bond, however, the value used at any node corresponding to the call date and beyond must be either the price at which the issuer will call the bond at that date or the computed value if the bond is not called, whichever is less. The price at which the bond will be called is determined using a call rule (e.g., the issue will be called if the computed price exceeds 105% of the call price).

**Example: Valuing a callable bond**

Continuing with our example, assume that the 2-year bond can be called in one year at 100. The issuer will call the bond if the computed bond price exceeds 100 one year from today (this is the call rule). Calculate the value of the callable bond today.

**Answer:**

The call rule (call the bond if the price exceeds $100) is reflected in the boxes in the completed binomial tree, where the second line of the boxes at the 1-year node is the lesser of the call price or the computed value. For example, the value of the bond in one year at the lower node is $101.594. However, in this case, the bond will be called, and the investor will only receive $100. Therefore, for valuation purposes, the value of the bond in one year at this node is $100.

The calculation for the current value of the bond at Node 0 (today), assuming the simplified call rules of this example, is:

$$V_0 = \frac{1}{2} \times \left[ \frac{\$99.830 + \$7}{1.045749} + \frac{\$100.00 + \$7}{1.045749} \right] = \$102.238$$
The completed binomial tree is shown below:

Valuing a 2-Year, 7.0% Coupon, Callable Bond, Callable in One Year at 100

It should be noted that this example has been simplified for illustrative purposes. It is common for a callable bond to have a call schedule that specifies a different call price at different points in time. In this situation, the process described remains essentially the same, but the call prices at each node may not be equal.

In summary, the existence of an embedded option alters the cash flows that must be discounted when valuing a bond using the backward induction methodology with a binomial interest rate model.

**LOS 56.e: Illustrate the relations among the values of a callable (putable) bond, the corresponding option-free bond, and the embedded option.**

In essence, the holder of a callable bond owns a noncallable bond on which a call option has been written. The value of the embedded call option ($V_{\text{call}}$) is, therefore, simply the difference between the value of a noncallable ($V_{\text{noncallable}}$) bond and the value of the comparable callable bond ($V_{\text{callable}}$):

$$V_{\text{call}} = V_{\text{noncallable}} - V_{\text{callable}}$$
Example: Valuing an embedded call option

Calculate the value of the embedded call option from the previous example.

Answer:

The value of the call option is the difference between the noncallable bond and the callable bond:

$$102.999 - 102.238 = 0.761$$

Similarly, investors are willing to pay a premium for a putable bond, since its holder owns an option-free bond plus a put option. The value of a putable bond can be expressed as:

$$V_{\text{putable}} = V_{\text{nonputable}} + V_{\text{put}}$$

Rearranging, the value of the embedded put option can be stated as:

$$V_{\text{put}} = V_{\text{putable}} - V_{\text{nonputable}}$$

LOS 56.f: Explain the effect of volatility on the arbitrage-free value of an option.

Like ordinary options, the value of an embedded call option, $V_{\text{call}}$, increases as volatility increases.

We can explain this relationship in the context of the callable bond by recalling that the upside price of a callable bond is capped at the call price. As volatility increases, the upside prices in the binomial tree will not rise above the call price, but the downside prices will fall. That means the callable bond value ($V_{\text{callable}}$) will fall as volatility rises. However, the arbitrage-free value of the noncallable bond ($V_{\text{noncallable}}$) will be unaffected by the increased volatility. Therefore, as volatility increases, the value of the call ($V_{\text{call}}$), which is the difference between the callable and noncallable bond values, will also increase. From the investor’s perspective, increased volatility decreases the value of their callable bond. The issuer of the bond holds the call and benefits from the increased volatility.
WARM-UP: HOW OPTION-ADJUSTED SPREAD (OAS) IS CALCULATED

The interest rates used to value the callable bond in the previous example were derived to yield arbitrage-free values for on-the-run Treasury securities (i.e., the interest rates produced a theoretical value equal to the market price for Treasury securities). This does not mean that the interest rate tree will produce an arbitrage-free value for the callable bond. In order to produce an arbitrage-free value for a callable bond, interest rates must be adjusted for the option characteristics of the bond. The adjustment is called the option-adjusted spread (OAS).

The OAS is the interest rate spread that must be added to all of the 1-year forward rates in a binomial tree so that the theoretical value of a callable bond generated with the tree is equal to its market price (i.e., the OAS is the spread that forces the theoretical price to be arbitrage-free). Thus, the option-adjusted spread is based on the same assumptions as those used to construct the binomial tree from which it is derived, particularly the interest rate volatility assumption. The only way to calculate the OAS from a binomial model is by trial and error, so you won’t have to do it on the exam. However, the following example will give you a better understanding of what the OAS is and how to interpret it.

Example: Calculating the OAS

In the previous example, the value of the 2-year, 7% bond, callable in one year, was calculated as $102,238. If the market price of this bond is $101,531, the bond is selling at a discount relative to its theoretical value computed from the binomial model. Verify that if a spread of 50 basis points is added to each of the 1-year rates in the tree, the theoretical value of this bond will equal its market price of $101,531.

Answer:

Consider the value of the bond at the upper node for Period 1, \( V_{1,U} \):

\[
V_{1,U} = \frac{1}{2} \times \left[ \frac{100 + 7}{1.071826 + 0.005} + \frac{100 + 7}{1.071826 + 0.005} \right] = \$99.366
\]

Similarly, the value of the bond at the lower node for Period 1, \( V_{1,L} \) is:

\[
V_{1,L} = \frac{1}{2} \times \left[ \frac{100 + 7}{1.053210 + 0.005} + \frac{100 + 7}{1.053210 + 0.005} \right] = \$101.114
\]

Now calculate \( V_0 \), the current value of the bond at Node 0:

\[
V_0 = \frac{1}{2} \times \left[ \frac{99.366 + 7}{1.045749 + 0.005} + \frac{100.000 + 7}{1.045749 + 0.005} \right] = \$101.531
\]
The completed binomial tree is shown below:

### Verification of 50 Basis Point OAS

<table>
<thead>
<tr>
<th>Value</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>$99.366</td>
<td>$7.0</td>
</tr>
<tr>
<td>$101.114</td>
<td>$100,000</td>
</tr>
<tr>
<td>$101.531</td>
<td>$100,000</td>
</tr>
<tr>
<td>$99.366</td>
<td>$7.0</td>
</tr>
<tr>
<td>4.5749% + 0.50%</td>
<td>$100,000</td>
</tr>
<tr>
<td>7.1826% + 0.50%</td>
<td>$7.0</td>
</tr>
<tr>
<td>5.3210% + 0.50%</td>
<td>$100,000</td>
</tr>
</tbody>
</table>

### LOS 56.g: Interpret an option-adjusted spread with respect to a nominal spread and to benchmark interest rates.

Recall that:
- Undervalued (“cheap”) bonds have spreads larger than the required spread.
- Overvalued (“rich”) bonds have spreads smaller than the required spread.
- Properly valued (“fairly priced”) bonds have spreads equal to the required spread.

In order to determine the appropriate required spread and interpret the bond’s actual spread, however, we first have to determine what the spread is measuring, given the benchmark we used to calculate the spread. In general, a spread measures compensation for credit risk relative to the benchmark, liquidity risk relative to the benchmark, and option risk. Because the OAS removes the cost of the embedded option, the OAS measures only credit risk and liquidity risk relative to the benchmark.

### Treasury Benchmark

If we use a Treasury benchmark, the nominal spread and Z-spread reflect:
- Credit risk relative to Treasuries.
- Liquidity risk relative to Treasuries.
- Option risk.

The OAS reflects:
- Credit risk relative to Treasuries.
- Liquidity risk relative to Treasuries.
**Bond Sector Benchmark**

If we use a higher-rated bond sector as the benchmark, the nominal spread and Z-spread reflect:

- Credit risk relative to the bond sector.
- Liquidity risk relative to the bond sector.
- Option risk.

The OAS reflects:

- Credit risk relative to the bond sector.
- Liquidity risk relative to the bond sector.

**Issuer-Specific Benchmark**

If we use the issuer’s yield curve or spot curve as the benchmark, the nominal spread and Z-spread will not reflect any credit risk because the credit risk of the issue and the benchmark are assumed to be the same. Therefore, the nominal spread and Z-spread will reflect:

- Liquidity risk relative to the specific issuer’s other securities (which is assumed to be very small).
- Option risk.

The OAS will reflect only:

- Liquidity risk relative to the specific issuer’s other securities (which is assumed to be very small).

**Relative OAS Valuation**

In general, the interpretation of the OAS (i.e., whether the bond is over or undervalued) depends on the benchmark and, in some cases, the required OAS.

For example, if the benchmark is Treasuries or a bond sector (with a credit rating higher than the bond we’re valuing), any corporate bond with an OAS less than or equal to zero is overvalued relative to the benchmark, because it must have more credit risk, and most likely more liquidity risk, than the benchmark. If the OAS is positive, the bond is undervalued relative to the benchmark only if the OAS is greater than the required OAS.

If we use an issuer-specific benchmark (assuming relative liquidity risk is zero), the bond is undervalued relative to the benchmark if the OAS is positive, fairly valued if the OAS is zero, and overvalued if the OAS is negative.

These concepts are summarized in Figure 3.
Study Session 14
Cross-Reference to CFA Institute Assigned Reading #56 – Valuing Bonds With Embedded Options

Figure 3: Relative OAS Valuation

<table>
<thead>
<tr>
<th></th>
<th>Treasury Benchmark</th>
<th>Sector Benchmark</th>
<th>Issuer-Specific Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>OAS &gt; 0</td>
<td>Overvalued (“rich”) if actual OAS &lt; required OAS; undervalued (“cheap”) if actual OAS &gt; required OAS</td>
<td>Overvalued (“rich”) if actual OAS &lt; required OAS; undervalued (“cheap”) if actual OAS &gt; required OAS</td>
<td>Undervalued (“cheap”)</td>
</tr>
<tr>
<td>OAS = 0</td>
<td>Overvalued (“rich”)</td>
<td>Overvalued (“rich”)</td>
<td>Fairly priced</td>
</tr>
<tr>
<td>OAS &lt; 0</td>
<td>Overvalued (“rich”)</td>
<td>Overvalued (“rich”)</td>
<td>Overvalued (“rich”)</td>
</tr>
</tbody>
</table>

Example: Relative OAS valuation

An analyst makes the following spread estimates relative to U.S. Treasuries for a callable corporate bond:

- Nominal spread relative to the Treasury yield curve: 240 basis points.
- Z-spread relative to the Treasury spot curve: 225 basis points.
- OAS relative to the Treasury spot curve: 190 basis points.

The analyst also determines that the Z-spread over Treasuries on comparable option-free bonds (i.e., bonds with the same credit rating, maturity, and liquidity) in the market is 210 basis points. Determine whether the bond is overvalued, undervalued, or properly valued.

**Answer:**

The required OAS in this case is the Z-spread on comparable option-free bonds (because Z-spread is equal to OAS for option-free bonds), which is 210 basis points. This bond is overvalued, because its OAS of 190 basis points is less than the required OAS. It is not appropriate to compare the bond’s Z-spread or nominal spread to the required spread because the embedded option cost is not reflected in those spread measures.

**LOS 56.h: Illustrate how effective duration and effective convexity are calculated using the binomial model.**

Recall from Level 1 that:

- Modified duration measures a bond’s price sensitivity to interest rate changes, *assuming that the bond’s cash flows do not change as interest rates change*.
- The standard measure of convexity can be used to improve price changes estimated from modified duration.

Modified duration and convexity are not useful for bonds with embedded options, however, because the cash flows from these bonds may change if the option is exercised.
To overcome this problem, effective duration and convexity should be used because these measures take into account how changes in interest rates may alter cash flows.

The following expressions can be used to compute effective duration and convexity for any bond:

\[
effective
duration = ED = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}
\]

\[
effective
covexity = EC = \frac{BV_{-\Delta y} + BV_{+\Delta y} - (2 \times BV_0)}{2 \times BV_0 \times \Delta y^2}
\]

where:
\( \Delta y \) = change in required yield, in decimal form
\( BV_{-\Delta y} \) = estimated price if yield decreases by \( \Delta y \)
\( BV_{+\Delta y} \) = estimated price if yield increases by \( \Delta y \)
\( BV_0 \) = initial observed bond price

Calculating effective duration and effective convexity for bonds with embedded options is a complicated undertaking because you must calculate values of \( BV_{+\Delta y} \) and \( BV_{-\Delta y} \). Here's how it's done:

**Step 1:** Given assumptions about benchmark interest rates, interest rate volatility, and call and/or put rule, calculate the OAS for the issue, using the binomial model.

**Step 2:** Impose a small parallel shift in the on-the-run yield curve by an amount equal to \( +\Delta y \).

**Step 3:** Build a new binomial interest rate tree using the new yield curve.

**Step 4:** Add the OAS to each of the 1-year rates in the interest rate tree to get a “modified” tree. (We assume that the OAS does not change when interest rates change.)

**Step 5:** Compute \( BV_{+\Delta y} \) using this modified interest rate tree.

**Step 6:** Repeat steps 2 through 5 using a parallel rate shift of \(-\Delta y \) to obtain a value of \( BV_{-\Delta y} \).

Note that the values for duration and convexity that result from this procedure are based on the computed values for \( BV_{+\Delta y} \) and \( BV_{-\Delta y} \), which are functions of: (1) the interest rate volatility assumption, (2) the call and/or put rule, and (3) the benchmark interest rates used to generate the binominal interest rate tree. Changes in any of these factors will likely alter the computed values for effective duration and convexity.

*Professor’s Note: Exam questions on effective duration and effective convexity are likely to deal with interpretation or calculation given values for \( BV_{+\Delta y} \) and \( BV_{-\Delta y} \). It is unlikely that you will be asked to calculate effective duration or effective convexity with the interest rate trees, although you may be asked to recognize the formula. Notice that LOS 56.h says only to “illustrate how … are calculated.”*
LOS 56.i: Calculate the value of a putable bond, using an interest rate tree.

A putable bond gives the holder the right to sell (put) the bond to the issuer at a predetermined price at some time prior to the bond's maturity. Putable bonds can be valued using the same procedure as for a callable bond, except that the relevant cash flows are dictated by the rules governing the exercise of the embedded put option.

Example: Valuing a putable bond

Consider a 2-year, 7% coupon, putable bond that is putable in one year at a price of 100. Further, assume that the put option will be exercised if the value of the bond is less than 100. Calculate the value of the putable bond.

Answer:

This situation is illustrated in the binomial tree shown in the following figure:
For the tree shown, the second line in the boxes at the 1-year node reflects the greater of the exercise price or the computed value. When valuing this putable bond, the value used at any node corresponding to the put date and beyond must be either the exercise price at that date or the computed value, whichever is greater.

Consider the value of the bond at the upper node for Period 1, $V_{1,U}$.

$$V_{1,U} = \frac{1}{2} \left[ \frac{100 + 7}{1.071826} + \frac{100 + 7}{1.071826} \right] = 99.830$$

Similarly, the value of the bond at the lower node for Period 1, $V_{1,L}$ is:

$$V_{1,L} = \frac{1}{2} \left[ \frac{100 + 7}{1.053210} + \frac{100 + 7}{1.053210} \right] = 101.594$$

Given our put rule, the current value of the bond at Node 0 (today) is:

$$V_0 = \frac{1}{2} \left[ \frac{100,000 + 7}{1.045749} + \frac{101.594 + 7}{1.045749} \right] = 103.081$$

Recall that the value of a putable bond is given by: $V_{\text{putable}} = V_{\text{nonputable}} + V_{\text{put}}$. Thus, the value of the embedded put option is: $V_{\text{put}} = V_{\text{putable}} - V_{\text{nonputable}}$.

**Example: Valuing an embedded put option**

The $103.0810 value here is greater than the $102.999 value that was computed earlier for the option-free bond. Compute the value of the embedded put option.

**Answer:**

The value of the embedded put option (to the bondholder) is:

$$103.081 - 102.999 = 0.082$$

As with all options, the value of the embedded put option increases as volatility increases. As such, it can be seen in the expression that the value of a putable bond will also increase as volatility increases. Intuitively, this makes sense; investors are willing to pay more for a bond that gives them the right to sell it at a price greater than the market value.

**LOS 56.j: Describe and evaluate a convertible bond and its various component values.**

The owner of a convertible bond has the right to convert the bond into a fixed number of common shares of the issuer. Hence, a convertible bond includes an embedded call.
option. The option is slightly different from the embedded option in a callable bond. First, the convertible bondholder owns the call option; the issuer owns the call option in a callable bond. Secondly, the holder has the right to buy shares with a bond that changes in value, not with cash at a fixed exercise price (the call price in a callable bond).

The conversion ratio is the number of common shares for which a convertible bond can be exchanged. For example, a convertible bond issued at par with a conversion ratio of 10 allows its holder to convert one $1,000 par bond into 10 shares of common stock. Equivalently, the conversion price of the bond is $1,000 / 10 shares = $100. For bonds not issued at par, the conversion price is the issue price divided by the conversion ratio.

Almost all convertible bonds are callable, which gives the issuer the ability to force conversion. If the bond is worth more than the call price and the issuer calls the bond, it’s optimal for the holder to convert the bond into shares rather than sell back to the issuer at the lower call price.

Some convertible issues are putable. If the embedded put feature requires the issuer to redeem the bond with cash, the put is referred to as a hard put. If the issuer has a payment choice (cash, common stock, and/or subordinated notes), the embedded put is called a soft put.

Professor's Note: Exchangeable bonds are convertible at the option of the holder into shares of stock other than those of the issuer. The analysis of exchangeable bonds is the same as for convertible bonds.

The conversion value of a convertible bond is the value of the common stock into which the bond can be converted. The conversion ratio is the number of shares the holder receives from conversion for each bond. Conversion value is calculated as:

\[ \text{conversion value} = \text{market price of stock} \times \text{conversion ratio} \]

The straight value, or investment value, of a convertible bond is the value of the bond if it were not convertible—the present value of the bond’s cash flows discounted at the required return on a comparable option-free issue.

The minimum value of a convertible bond must be the greater of its conversion value or its straight value. This must be the case, or arbitrage opportunities would be possible. For example, if a convertible bond were to sell for less than its conversion value, it could be purchased, immediately converted into common stock, and the stock could be sold for more than the cost of the bond.

Example: Calculating the minimum value of a convertible bond

Consider a BSC convertible bond with a 7% coupon that is currently selling at $985 with a conversion ratio of 25 and a straight value of $950. Assume that the value of BSC’s common stock is currently $35 per share, and that it pays $1 per share in dividends annually. Compute the bond’s minimum value.
The market conversion price, or conversion parity price, is the price that the convertible bondholder would effectively pay for the stock if she bought the bond and immediately converted it. The market conversion price is given as:

\[
\text{market conversion price} = \frac{\text{market price of convertible bond}}{\text{conversion ratio}}
\]

Example: Calculating market conversion price

Compute and interpret the market conversion price of the BSC bond.

Answer:

The market conversion price is: $985 / 25 = $39.40. This can be viewed as the stock price at which an investor is indifferent between selling the bond or converting it.

The market conversion premium per share is the difference between the market conversion price and the stock's current market price:

\[
\text{market conversion premium per share} = \text{market conversion price} - \text{market price}
\]

Example: Calculating market conversion premium per share

Compute and interpret the market conversion premium per share of the BSC bond.

Answer:

Since BSC is selling for $35 per share, the market conversion premium per share for the BSC bond is: $4.40 = $39.40 – $35. This can be interpreted as the premium that investors are willing to pay for the chance that the market price of the stock will rise above the market conversion price. This is done with the assurance that even if the stock price declines, the value of the convertible bond will not fall below its straight value.

Market conversion premium per share is usually expressed as a ratio, appropriately called the market conversion premium ratio. Its formula is:

\[
\text{market conversion premium ratio} = \frac{\text{market conversion premium per share}}{\text{market price of common stock}}
\]
Example: Calculating market conversion premium ratio

Compute the market conversion premium ratio of the BSC bond.

Answer:

The BSC bond market conversion premium ratio is:

\[
\frac{4.40}{35} = 12.57\%
\]

Typically, the coupon income from a convertible bond exceeds the dividend income that would have been realized if the stock were owned directly. On a per-share basis, this tends to offset the market conversion premium. The time it takes to recoup the per-share premium is known as the premium payback period or the breakeven time and is expressed as:

premium payback period = \( \frac{\text{market conversion premium per share}}{\text{favorable income difference per share}} \)

where the favorable income difference per share is the annual per share difference in the cash flows from the convertible bond and the stock:

\[
\text{favorable income difference per share} = \frac{\text{coupon interest} - (\text{conversion ratio} \times \text{dividends per share})}{\text{conversion ratio}}
\]

Example: Calculating premium payback period

Compute and interpret the premium payback period of the BSC bond.

Answer:

For the BSC bond:

- coupon interest = 0.07 \times $1,000.00 = $70.00
- conversion ratio \times dividends per share = 25 \times $1.00 = $25.00
- favorable income difference per share is \( \frac{70.00 - 25.00}{25} = $1.80 \)
- premium payback period is: \( \frac{4.40}{1.80} = 2.44 \) years

The convertible bond investor's downside risk is limited by the bond's underlying straight value because the price of a convertible bond will not fall below this value regardless of what happens to the price of the issuer's common stock.
This downside risk is measured by the **premium over straight value**, which is calculated as:

\[
\text{premium over straight value} = \left( \frac{\text{market price of convertible bond}}{\text{straight value}} \right) - 1
\]

**Example: Calculating premium over straight value**

Compute and interpret the premium over straight value of the BSC bond.

Answer:

The premium over straight value for the BSC bond is:

\[
\left( \frac{\$985.00}{\$950.00} \right) - 1 = 3.68\%
\]

Holding all other factors constant, the greater the premium over straight value, the less attractive the convertible bond.

**Valuing Convertible Bonds Using an Option-Based Valuation Approach**

Investing in a noncallable/nonputable convertible bond is equivalent to buying:

- An option-free bond.
- A call option on an amount of the common stock equal to the conversion ratio.

The value of a noncallable/nonputable convertible bond can be expressed as:

\[
\text{convertible, noncallable bond value} = \text{straight value} + \text{value of call option on stock}
\]

The Black-Scholes-Merton option pricing model can be used to establish the value of the call option. (See Study Session 17 for more information on the Black-Scholes-Merton option pricing model.) A key variable in this model is stock price volatility, which is positively related to the value of the call option. Therefore, as price volatility increases, so does the value of the convertible.

Most convertible bonds are callable, giving the issuer the right to call the issue prior to maturity. Incorporating this feature into the valuation of a convertible bond results in the following expression:

\[
\text{callable convertible bond value} = \text{straight value of bond} + \text{value of call option on stock} - \text{value of call option on bond}
\]

Obviously, the valuation of a callable convertible bond involves the valuation of the call feature, which is a function of interest rate volatility and the economic conditions that can trigger the call feature. The Black-Scholes-Merton option pricing model *cannot* be used in this situation.
To further complicate the situation (just for fun), consider a convertible bond that is both callable and putable. The expression for value then becomes:

\[
\text{callable and putable convertible bond value} = \text{straight value of bond} + \text{value of call option on stock} - \text{value of call option on bond} + \text{value of put option on bond}
\]

Here again, the Black-Scholes-Merton model is not appropriate to value options that are dependent on future interest rates.

From this discussion, it should be apparent that valuing convertible bonds can be challenging. The valuation of convertible bonds with embedded call and/or put options requires a model that links the movement of interest rates and stock prices. You will not be asked to deal with these complex models on the exam.

However, you should know the effects of changes in volatilities on the convertible bond value. For a callable convertible bond:

- An increase in stock price volatility will increase the value of the call on the stock and increase the value of the callable convertible bond.
- An increase in interest rate volatility will increase the value of the call on the bond and reduce the value of the callable convertible bond.

**LOS 56.k:** Compare and contrast the risk-return characteristics of a convertible bond with the risk-return characteristics of ownership of the underlying common stock.

Buying convertible bonds in lieu of stocks limits downside risk. The price floor set by the straight bond value causes this downside protection. The cost of the downside protection is reduced upside potential due to the conversion premium. Keep in mind though, that just like investing in nonconvertible bonds, convertible bond investors must be concerned with credit risk, call risk, interest rate risk, and liquidity risk. The following example illustrates this point.

**Illustration:** Consider the following two scenarios based on our BSC example:

**Scenario 1 – The market price of BSC common stock increases to $45 per share.**

The return from investing in the convertible bond is about:

\[
\left(\frac{45.00}{39.40}\right) - 1 = 14.21\%
\]

The return from investing directly in the stock is about:

\[
\left(\frac{45.00}{35.00}\right) - 1 = 0.2857 = 28.57\%
\]

The lower return from the convertible bond investment is attributable to the fact that the investor effectively bought the stock at the market conversion price of $39.40 per share.
Scenario 2 – The market price of BSC common stock falls to $30 per share.

Recall that the bond will trade at the greater of its straight value or its conversion value. The conversion value in this scenario is $25 \times \$30.00 = \$750.00. Assuming the straight value of the bond does not change, the bond will trade at $950.00. So, the return from investing in the convertible bond is:

\[
\left( \frac{\$950}{\$985} \right) - 1 = -3.55\%
\]

The return from investing directly in the stock is:

\[
\left( \frac{\$30}{\$35} \right) - 1 = -14.29\%
\]

The loss is less for the convertible bond investment because we assumed that the straight value of the bond did not change. Even if it had changed, the loss would probably still be less than the loss on the straight stock investment, thus emphasizing how the straight value serves as a floor to cushion a decline, even if it is a moving floor.

The following comparisons can be made between ownership of the underlying stock and the risk-return characteristics of the convertible bond:

- When the stock’s price falls, the returns on convertible bonds exceed those of the stock, because the convertible bond’s price has a floor equal to its straight bond value.
- When the stock’s price rises, the bond will underperform because of the conversion premium. This is the main drawback of investing in convertible bonds versus investing directly in the stock.
- If the stock’s price remains stable, the return on a convertible bond may exceed the stock return due to the coupon payments received from the bond, assuming no change in interest rates or the yield or credit risk of the issuer.

Sometimes the price of the common stock associated with a convertible issue is so low that it has little or no effect on the convertible’s market price, and it trades as though it is a straight bond. When this happens, the convertible security is referred to as a fixed-income equivalent or busted convertible.

Other times, the price of the stock can be high enough that the price of the convertible behaves as though it were an equity security. When this happens, the convertible issue is referred to as a common stock equivalent. Most of the time, however, it is a hybrid security with the characteristics of equity and a fixed-income security.
**Key Concepts**

**LOS 56.a**  
Relative value analysis of bonds involves comparing the spread on the bond (over some benchmark) to the required spread and determining whether the bond is over or undervalued relative to the benchmark. The required spread is the spread available on comparable securities. Undervalued bonds (“cheap”) have spreads larger than the required spread; overvalued bonds (“rich”) have spreads smaller than the required spread; and properly valued bonds (“fairly priced”) have spreads equal to the required spread.

**LOS 56.b**  
There are three different benchmark interest rates that can be used to calculate spreads: Treasury securities, a specific sector of the bond market with a certain credit rating higher than the issue being valued, or a specific issuer. Our interpretation of a spread calculated for a specific security will depend on the benchmark rates used to create the interest rate tree.

**LOS 56.c**  
Backward induction methodology is a discounting process for valuing bonds with a binomial interest rate tree. “Backward” refers to the process of discounting distant values in a binomial tree, one node at a time, backwards through time to generate a current value.

**LOS 56.d**  
Callable bonds can be valued by modifying the cash flows at each node in the interest rate tree to reflect the cash flow prescribed by the embedded call option according to the call rule.

**LOS 56.e**  
The value of the embedded call option is the difference between the value of a noncallable bond and the value of a callable bond (i.e., $V_{\text{call}} = V_{\text{noncallable}} - V_{\text{callable}}$). Similarly, investors are willing to pay a premium for a putable bond since its holder owns an option-free bond plus a put option. The value of the embedded put option can be stated as $V_{\text{put}} = V_{\text{putable}} - V_{\text{nonputable}}$.

**LOS 56.f**  
The value of an embedded call option increases as volatility increases. The upside price of a callable bond is capped at the call price. Therefore, more volatility decreases the value of the callable bond to the investor, but increases the (call) value to the issuer.

**LOS 56.g**  
If the benchmark is Treasuries or a bond sector (with a credit rating higher than the bond we’re valuing), any corporate bond with an OAS less than or equal to zero is overvalued relative to the benchmark, because it must have more credit risk, and most likely more liquidity risk, than the benchmark. If the OAS is positive, the bond is undervalued relative to the benchmark only if the OAS is greater than the required OAS. If we use an issuer-specific benchmark (and assuming relative liquidity risk is zero) the bond is undervalued relative to the benchmark if the OAS is positive, fairly valued if the OAS is zero, and overvalued if the OAS is negative.
LOS 56.h
The binomial model can be used to compute the value of bonds with embedded options in the equations for effective duration and convexity. The general procedure for calculating \( BV_{+\Delta y} \) (and \( BV_{-\Delta y} \)) is as follows:

**Step 1:** Given assumptions about benchmark interest rates, interest rate volatility, and a call and/or put rule, calculate the OAS for the issue using the binomial model.

**Step 2:** Impose a small parallel shift in the on-the-run yield curve by an amount equal to \( +\Delta y \).

**Step 3:** Build a new binomial interest rate tree using the new yield curve.

**Step 4:** Add the OAS to each of the 1-year rates in the interest rate tree to get a modified tree. (We assume that the OAS does not change when interest rates change.)

**Step 5:** Compute \( BV_{+\Delta y} \) using this modified interest rate tree.

**Step 6:** Repeat steps 2 through 5 using a parallel rate shift of \( -\Delta y \) to obtain a value of \( BV_{-\Delta y} \).

LOS 56.i
Putable bonds are valued using the same procedure as for a callable bond, except that the relevant cash flows are dictated by the rules governing the exercise of the embedded put option. The value of a putable bond is given by: \( V_{\text{putable}} = V_{\text{nonputable}} + V_{\text{put}} \). The value of the embedded put option is: \( V_{\text{put}} = V_{\text{putable}} - V_{\text{nonputable}} \).

LOS 56.j
The owner of a convertible bond can exchange the bond for the common shares of the issuer. A convertible bond includes an embedded call option giving the bondholder the right to buy the common stock of the issuer. Almost all convertible bonds are callable, and some convertible issues are putable.

The conversion ratio is the number of common shares for which a convertible bond can be exchanged.

The conversion price is the issue price divided by the conversion ratio. Conversion value is the value of the stock into which the bond can be converted. Conversion value = market price of stock \( \times \) conversion ratio.

Straight value is the value of the bond if it were not convertible.

Market conversion price is the price that a convertible bondholder would effectively pay if the bond were purchased and immediately converted. Market conversion price = market price of convertible bond/conversion ratio.

Market conversion premium per share is the difference between the market conversion price and the current market price. It can also be expressed as the ratio of conversion price to market price, called the conversion premium ratio:

\[
\text{market conversion premium per share} = \frac{\text{market conversion price}}{\text{market price}} - \text{market price}
\]
The coupon income from a convertible bond usually exceeds the dividend income that would have been realized if the stock were owned directly. The time it takes to recoup the per-share premium via this extra income is known as the premium payback period.

The minimum value at which a convertible bond trades is its straight value or its conversion value, whichever is greater.

Straight value is the usual measure of the downside risk for a convertible bond, because it sets a bond price floor that is independent of stock price.
- Downside risk is often measured using the premium over straight value.
- All other factors held constant, the greater the premium over straight value, the less attractive the convertible bond.

**LOS 56.k**

The major reason for investing in convertible bonds is the price appreciation resulting from an increase in the value of the common stock.
- The main drawback of investing in a convertible bond versus investing directly in the stock is that when the stock price rises, the bond will underperform because of the conversion premium of the bond.
- If the stock price remains stable, the return on the bond may exceed the stock returns due to the coupon payments received from the bond.
- If the stock price falls, the straight value of the bond limits downside risk. This is based on the assumption that bond yields remain stable.
1. Which of the following statements concerning the calculation of value at a node in a binomial interest rate tree is most accurate? The value at each node is the:
   A. present value of the two possible values from the next period.
   B. average of the present values of the two possible values from the next period.
   C. sum of the present values of the two possible values from the next period.

2. An increase in interest rate volatility:
   A. increases the value of bonds with embedded call options.
   B. increases the value of bonds with embedded put options.
   C. increases the value of low-coupon bonds with embedded options, but decreases the value of high-coupon bonds with embedded options.

3. The option-adjusted spread (OAS) on a callable corporate bond is 73 basis points using on-the-run Treasuries as the benchmark rates in the construction of the binomial tree. The best interpretation of this OAS is the:
   A. cost of the embedded option is 73 basis points.
   B. cost of the option is 73 basis points over Treasury.
   C. spread that reflects the credit risk and liquidity risk is 73 basis points over Treasury.

4. An analyst has gathered the following information on a convertible bond and the common equity of the issuer.
   • Market price of bond: $925.00
   • Annual coupon: 7.5%
   • Conversion ratio: 30
   • Market price of stock: $28.50
   • Annual stock dividend: $2.15 per share

   The premium payback period for the convertible bond is closest to:
   A. 4.85 years.
   B. 5.29 years.
   C. 6.67 years.

5. Which of the following statements concerning the comparison between the risk and return of convertible bond investing and common stock investing is least accurate, assuming interest rates are stable?
   A. When stock prices fall, the returns on convertible bonds may exceed those of the stock because the convertible bond’s price has a floor equal to the straight bond value.
   B. The main drawback of investing in convertible bonds versus direct stock purchases is that when stock prices rise, the convertible bond will likely underperform due to the conversion premium.
   C. Buying convertible bonds in lieu of direct stock investing limits upside potential to that of buying a straight bond at the cost of increased downside risk due to the conversion premium.
6. Which of the following statements concerning the option-adjusted spread (OAS) is least accurate?
   A. The OAS is the interest rate spread that must be added to all of the 1-year forward rates in the binomial tree so that the arbitrage-free value of a bond generated by the tree is equal to its market price.
   B. The OAS reflects credit and/or liquidity risk differences between the bond and the benchmark securities used to create the interest rate tree.
   C. The OAS is equal to the Z-spread plus the cost of the embedded option.

7. A convertible bond with a 9% annual coupon is currently selling for $1,073 with a conversion ratio of 30 and a straight value of $1,031. Assume that the common stock pays a $1.25 dividend and is currently selling for $32. The premium payback period is closest to:
   A. 2.64 years.
   B. 3.09 years.
   C. 2.15 years.

8. The difference between the value of a callable convertible bond and the value of an otherwise comparable option-free bond is closest to the value of the:
   A. call option on the stock minus value of the call option on the bond.
   B. put option on the stock plus value of the call option on the bond.
   C. call option on the stock plus value of call option on the bond.

9. With respect to the value of a callable convertible bond, what are the most likely effects of a decrease in interest rate volatility or a decrease in the underlying stock price volatility?
   A. Both will result in an increase in value.
   B. One will result in an increase in value, the other a decrease.
   C. Both will result in a decrease in value.

**CHALLENGE PROBLEMS**

10. Data on two convertible bonds are shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Convertible Bond ABC</th>
<th>Convertible Bond XYZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conversion price</td>
<td>$40</td>
<td>$50</td>
</tr>
<tr>
<td>Current stock price</td>
<td>$123</td>
<td>$8</td>
</tr>
</tbody>
</table>

Which factors are more likely to influence the market prices of ABC and XYZ: factors that affect equity prices, or factors that affect option-free bond prices?
   A. Both will be more affected by equity factors.
   B. One will be more affected by equity factors, the other by bond factors.
   C. Both will be more affected by bond factors.
11. Ron Hyatt has been asked to do a presentation on how effective duration (ED) and effective convexity (EC) are calculated with a binomial model. His presentation includes the following formulas:

\[
\text{effective duration} = ED = \frac{BV_{+\Delta y} - BV_{-\Delta y}}{2 \times BV_0 \times \Delta y} \\
\text{effective convexity} = EC = \frac{BV_{+\Delta y} + BV_{-\Delta y} - (2 \times BV_0)}{2 \times BV_0 \times \Delta y^2}
\]

where:

\( \Delta y \) = change in required yield, in decimal form
\( BV_{-\Delta y} \) = estimated price if yield decreases by \( \Delta y \)
\( BV_{+\Delta y} \) = estimated price if yield increases by \( \Delta y \)
\( BV_0 \) = initial observed bond price

Are Hyatt’s formulas for effective duration and effective convexity correctly presented?
A. The formulas are both correct.
B. One formula is correct, the other incorrect.
C. Both formulas are incorrect.

Use the following binomial interest rate tree to answer Questions 12 through 14.

12. The value today of an option-free, 12% annual coupon bond with two years remaining until maturity is closest to:
A. 111.485.
B. 112.282.
C. 113.394.
13. The value of the bond and the value of the embedded call option, assuming the bond in Question 12 is callable at $105 at the end of Year 1, are closest to:

<table>
<thead>
<tr>
<th>Callable bond value</th>
<th>Embedded call option value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 110.573</td>
<td>1.709</td>
</tr>
<tr>
<td>B. 110.573</td>
<td>0.642</td>
</tr>
<tr>
<td>C. 111.640</td>
<td>0.642</td>
</tr>
</tbody>
</table>

14. The value of the bond and the value of the embedded put option, assuming the bond in Question 12 is putable at $105 at the end of Year 1, are closest to:

<table>
<thead>
<tr>
<th>Putable bond value</th>
<th>Embedded put option value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. 112.523</td>
<td>0.241</td>
</tr>
<tr>
<td>B. 112.523</td>
<td>1.646</td>
</tr>
<tr>
<td>C. 113.928</td>
<td>1.646</td>
</tr>
</tbody>
</table>

15. Spamaloan, Inc., has issued a bond with an embedded call option that has been rated single A. Using the AA-rated bond sector as the benchmark, the bond’s nominal spread is 60 basis points, the Z-spread is 45 points, and the option-adjusted spread (OAS) is 30 points. The nominal spread relative to the same benchmark on otherwise comparable option-free single A bonds with the same liquidity and maturity is 50 basis points. The OAS relative to the AA benchmark on other single A corporate bonds is 20 basis points. The Spamaloan bond is:

A. undervalued because its OAS is greater than the required OAS.
B. undervalued because its nominal spread is greater than the required nominal spread.
C. overvalued because its OAS is greater than the required OAS.
1. B The value at any given node in a binomial tree is the present value of the cash flows at the two possible states immediately to the right of the given node, discounted at the 1-period rate at the node under examination.

2. B Like ordinary options, the value of an embedded option increases as volatility increases. Furthermore, the arbitrage-free value of an option-free bond (\( V_{\text{option-free}} \)) is independent of the assumed volatility. This implies that the arbitrage-free value of a callable bond (\( V_{\text{callable}} \)) decreases as volatility increases the value of the embedded call option (\( V_{\text{call}} \)). This can be seen from the expression for the value of a callable bond:

\[
\downarrow V_{\text{callable}} = V_{\text{option-free}} - \uparrow V_{\text{call}}
\]

The value of the putable bond (\( V_{\text{putable}} \)) increases as the assumed volatility increases the value of the embedded put option (\( V_{\text{put}} \)).

\[
\uparrow V_{\text{putable}} = V_{\text{option-free}} + \downarrow V_{\text{put}}
\]

3. C Let’s construct a table of the risk differences between the issuer’s callable bond and on-the-run Treasuries to help us answer this question.

<table>
<thead>
<tr>
<th>Type of Risk</th>
<th>Equal?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit</td>
<td>No</td>
</tr>
<tr>
<td>Liquidity</td>
<td>No</td>
</tr>
<tr>
<td>Option</td>
<td>Removed by OAS</td>
</tr>
</tbody>
</table>

Therefore, the OAS reflects the additional credit and liquidity risk of the corporate callable bond over Treasuries, since option risk has been removed.

4. C The premium payback period can be determined using the following formula:

\[
\text{premium payback period} = \frac{\text{market conversion premium per share}}{\text{favorable income difference per share}}
\]

The market conversion premium per share is the market conversion price per share minus the market price per share. The market conversion price per share is \( \frac{925.00}{30} = $30.833 \), so the conversion premium per share is $30.833 – $28.50 = $2.333. The per share coupon payment from the bond is the annual coupon divided by the conversion ratio, or \( \frac{\$75.00}{30} = $2.50 \) per share. Since the stock dividend is $2.15 per share, the favorable income difference per share is $2.50 – $2.15 = $0.35.

Thus, the premium payback period is \( \frac{$2.333}{$0.35} = 6.67 \) years.
5. C Buying convertible bonds in lieu of direct stock investing limits downside risk to that of straight bond investing at the cost of reduced upside potential, due to the conversion premium. Note that this analysis assumes that interest rates remain stable. Otherwise, the interest rate risk associated with the straight bond investing must be considered. When stock prices fall, the returns on convertible bonds may exceed those of the stock, because the convertible bond’s price has a floor equal to the straight bond value. The main drawback of investing in convertible bonds versus direct stock purchases is that when stock prices rise, the convertible bond will likely underperform due to the conversion premium. If the stock price remains stable, the return on the bond may exceed the stock’s return if the bond’s coupon payment exceeds the dividend income of the stock.

6. C The OAS is equal to the Z-spread less the cost of the embedded option.

7. C First determine the market conversion price, as:

\[
\frac{1,073}{30} = 35.77
\]

Next determine the market conversion premium per share, as:

\[
35.77 - 32.00 = 3.77
\]

Then determine the income difference per share, as \[
\frac{90 - (30 \times 1.25)}{30} = 1.75.
\]

The premium payback period is then \[
\frac{3.77}{1.75} = 2.15 \text{ years}.
\]

8. A A bond that is both callable and convertible contains two embedded options: (1) a call option on the stock and (2) a call option on the bond. The investor has a short position in the call option on the stock (the issuer has the right to convert the bond into shares of stock) and a long position in the call option on the bond (the investor has the right to call the bond). Therefore, the difference in value between the convertible bond and the value of the comparable option-free bond to the investor is equal to the value of the call option on the stock minus the value of the call option on the bond.

9. B A decrease in interest rate volatility will decrease the value of the embedded short call on the bond (but have no effect on the value of the embedded call on the stock) and increase the value of the convertible bond.

A decrease in stock price volatility will decrease the value of the embedded call on the stock (but have no effect on the embedded call on the bond) and decrease the value of the convertible bond.
10. B ABC has a conversion price much less than the current stock price, so the conversion option is deep-in-the-money. Bond ABC effectively trades like equity and is more likely to be influenced by the same factors that affect equity prices, in general, rather than the factors that affect bond prices.

A busted convertible like XYZ, with a stock price significantly less than the conversion price, trades like a bond (that's why a busted convertible is also called a fixed-income equivalent) and is therefore more likely to be influenced by the factors that affect bond prices.

11. B The numerator of the effective duration formula is presented incorrectly. The numerator should be the bond price if the yield decreases by $\Delta y$ ($BV_{-\Delta y}$) minus the bond price if the yield increases by $\Delta y$ ($BV_{\Delta y}$). Bond price increases when yield falls for an option-free bond, and vice versa, so $BV_{-\Delta y}$ will be larger than $BV_{\Delta y}$ and the numerator will be positive for an option-free bond. Hyatt has switched the order in his presentation of the effective duration formula, which will yield a negative effective duration measure for an option-free bond. The convexity formula is presented correctly, even though the typical order of the first two terms in the numerator is reversed; note that $BV_{-\Delta y} + BV_{\Delta y}$ is equal to $BV_{-\Delta y} + BV_{\Delta y}$. We were just trying to make sure you hadn't fallen asleep!

12. B The tree should look like this:

Consider the value of the bond at the upper node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \times \left[ \frac{100 + 12}{1.071826} + \frac{100 + 12}{1.071826} \right] = 104.495$$

Similarly, the value of the bond at the lower node for Period 1, $V_{1,L}$ is:

$$V_{1,L} = \frac{1}{2} \times \left[ \frac{100 + 12}{1.053210} + \frac{100 + 12}{1.053210} \right] = 106.342$$
Now calculate $V_0$, the current value of the bond at Node 0:

$$V_0 = \frac{1}{2} \left[ \frac{104.495 + 12}{1.045749} + \frac{106.342 + 12}{1.045749} \right] = 112.282$$

13. C The tree should look like this:

Consider the value of the bond at the upper node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \left[ \frac{100 + 12}{1.071826} + \frac{100 + 12}{1.071826} \right] = 104.495$$

Similarly, the value of the bond at the lower node for Period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2} \left[ \frac{100 + 12}{1.053210} + \frac{100 + 12}{1.053210} \right] = 106.342$$

Now calculate $V_0$, the current value of the bond at Node 0:

$$V_0 = \frac{1}{2} \left[ \frac{104.495 + 12}{1.045749} + \frac{105 + 12}{1.045749} \right] = 111.640$$

The value of the embedded call option is $112.282 - 111.640 = 0.642$. 
14. A The tree should look like this:

Consider the value of the bond at the upper node for Period 1, $V_{1,U}$:

$$V_{1,U} = \frac{1}{2} \left[ \frac{\$100 + \$12}{1.071826} + \frac{\$100 + \$12}{1.071826} \right] = \$104.495$$

Similarly, the value of the bond at the lower node for Period 1, $V_{1,L}$, is:

$$V_{1,L} = \frac{1}{2} \left[ \frac{\$100 + \$12}{1.053210} + \frac{\$100 + \$12}{1.053210} \right] = \$106.342$$

Now calculate $V_0$, the current value of the bond at Node 0:

$$V_0 = \frac{1}{2} \left[ \frac{\$105.000 + \$12}{1.045749} + \frac{\$106.342 + \$12}{1.045749} \right] = \$112.523$$

The value of the embedded put option is $112,523 - 112,282 = \$0.241$.

15. A The required OAS on the Spamaloan bond is the OAS relative to the AA benchmark on other single A corporate bonds, which is 20 basis points. The Spamaloan bond is undervalued because its OAS of 30 basis points is greater than the required OAS of 20 basis points.
The following is a review of the Fixed Income: Structured Securities principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**MORTGAGE-BACKED SECTOR OF THE BOND MARKET**

**EXAM FOCUS**

Mortgage-backed securities are securities backed by pools of residential or commercial mortgage loans. They include mortgage pass-through securities, collateralized mortgage obligations, and stripped mortgage-backed securities. Because the underlying mortgages can be prepaid, prepayment risk is a major concern for mortgage-backed security investors. Make sure you understand the relative prepayment risk exposure of the various collateralized mortgage obligation tranches (e.g., sequential, planned amortization class, and support tranches), as well as interest-only and principal-only strips. Also pay attention to the unique features and issues related to commercial mortgage-backed securities.

**LOS 57.a:** Describe a mortgage loan, and illustrate the cash flow characteristics of a fixed-rate, level payment, and fully amortized mortgage loan.

A mortgage is a loan that is collateralized with a specific piece of real property, either residential or commercial. The borrower must make a series of mortgage payments over the life of the loan, and the lender has the right to *foreclose* or lay claim against the real estate in the event of loan default. The interest rate on the loan is called the mortgage rate or contract rate.

A *conventional mortgage* is the most common residential mortgage. The loan is based on the creditworthiness of the borrower and is collateralized by the residential real estate that it is used to purchase. If a borrower’s credit quality is questionable or the borrower is lacking a sufficient down payment, the mortgage lender may require mortgage insurance to guarantee the loan. Mortgage insurance is made available by both government agencies and private insurers. The cost of the insurance is borne by the borrower and effectively raises the interest rate on the mortgage loan.

There are a wide variety of mortgage designs that specify the rates, terms, amortization, and repayment methods. All of the concepts associated with risk analysis and valuation, however, can be understood through an examination of fixed-rate, level payment, fully amortized mortgage loans. This common type of mortgage loan requires equal payments (usually monthly) over the life of the mortgage. Each of these payments consists of an interest component and a principal component.
There are four important features of fixed-rate, level payment, fully amortized mortgage loans to remember when we move on to mortgage-backed securities (MBS):

1. The amount of the principal payment increases as time passes.
2. The amount of interest decreases as time passes.
3. The servicing fee also declines as time passes.
4. The ability of the borrower to prepay results in prepayment risk. Prepayments and curtailments reduce the amount of interest the lender receives over the life of the mortgage and cause the principal to be repaid sooner.

Example: Calculating a mortgage payment

Consider a 30-year, $500,000 level payment, fully amortized mortgage with a fixed rate of 12%. Calculate the monthly payment and prepare an amortization schedule for the first three months.

Answer:

The monthly payment is $5,143.06:

\[ N = 360; I/Y = 1.0 \text{ (12/12)}; PV = -500,000; FV = 0; \text{CPT} \rightarrow \text{PMT} = 5,143.06 \]

With reference to the partial amortization schedule in the figure below, the portion of the first payment that represents interest is $5,000.00 ($0.01 \times $500,000). The remainder of the payment, $143.06 ($5,143.06 – $5,000.00), goes toward the reduction of principal. The portion of the second payment that represents interest is $4,998.57 ($0.01 \times $499,856.94). The remaining $144.49 ($5,143.06 – $4,998.57) goes toward the further reduction of principal.

| Monthly Amortization Schedule for a 30-Year, $500,000 Mortgage Loan at 12% |
|-------------------|----------------|----------------|----------------|----------------|
| Payment Number | Initial Principal | Monthly Payment | Interest Component | Reduction of Principal | Outstanding Principal |
| 1              | $500,000.00       | $5,143.06       | $5,000.00          | $143.06            | $499,856.94         |
| 2              | 499,856.94       | 5,143.06        | 4,998.57           | 144.49             | 499,712.45          |
| 3              | 499,712.45       | 5,143.06        | 4,997.12           | 145.94             | 499,566.51          |

Notice that the monthly interest charge is based on the beginning-of-period outstanding principal. As time passes, the proportion of the monthly payment that represents interest decreases, and, since the payment is level, the proportion that goes toward the repayment of principal increases. This process continues until the outstanding principal reaches zero and the loan is paid in full.
The incremental reduction of outstanding principal is referred to as scheduled amortization (or scheduled principal repayment). The figure above is a portion of what is commonly called an amortization schedule. Amortization schedules are easily constructed using an electronic spreadsheet. Also, your business calculator will compute the interest and principal components of any payment and the outstanding loan balance. The procedure is described in the calculator's guidebook.

The collection of payments and all of the other administrative activities associated with mortgage loans are paid for via a servicing fee, also known as the servicing spread, because it is usually built into the mortgage rate.

For example, if the mortgage rate is 10.5% and the servicing fee is 35 basis points, the provider of the mortgage funds will receive 10.15%. This amount is called the net interest or net coupon. The dollar amount of the servicing fee is based on the outstanding loan balance; thus, it declines as the mortgage is amortized.

**Prepayment Risk**

In the previous example, it was assumed that the borrower paid the exact amount of the monthly payment, and the interest and principal followed the amortization schedule. It is possible, however, for a borrower to pay an amount in excess of the required payment or even to pay off the loan entirely. Payments in excess of the required monthly amount are called prepayments, and prepayments for less than the outstanding principal balance are called curtailments.

Keep in mind that interest paid by the borrower (and received by the lender) is based on the outstanding principal at the beginning of each payment period. Thus, prepayments or curtailments will reduce the amount of interest the lender receives over the life of the loan. The likelihood of this situation actually occurring is very real and is known as prepayment risk. In order to reduce prepayment risk, some mortgages have prepayment penalties, which are intended to discourage prepayments when interest rates decline. However, residential mortgages in the United States typically do not contain prepayment penalties.

**LOS 57.b: Illustrate the investment characteristics, payment characteristics, and risks of mortgage passthrough securities.**

A mortgage passthrough security represents a claim against a pool of mortgages. Any number of mortgages may be used to form the pool, and any mortgage included in the pool is referred to as a securitized mortgage. The mortgages in the pool have different maturities and different mortgage rates. The weighted average maturity (WAM) of the pool is equal to the weighted average of all the mortgages in the pool, each weighted by the relative outstanding mortgage balance to the value of the entire pool. The weighted average coupon (WAC) of the pool is the weighted average of the mortgage rates in the pool. The investment characteristics of a mortgage passthrough are a function of its cash flow features and the strength of its government guarantee.
As illustrated in Figure 1, passthrough security investors receive the monthly cash flows generated by the underlying pool of mortgages, less any servicing and guarantee/insurance fees. The fees account for the fact that passthrough rates (i.e., the coupon rate on the passthrough) are less than the average coupon rate of the underlying mortgages in the pool.

Figure 1: Mortgage Passthrough Cash Flow

Since passthrough securities may be traded in the secondary market, they effectively convert illiquid mortgages into liquid securities (this process is called securitization). As we will see later, more than one class of passthrough securities may be issued against a single mortgage pool.

The timing of the cash flows to passthrough security holders does not exactly coincide with the cash flows generated by the pool. This is due to the delay between the time the mortgage service provider receives the mortgage payments and the time the cash flows are passed through to the security holders.

The three major types of agency passthrough securities issued in the United States are:

1. **Ginnie Mae.** Issued by the Government National Mortgage Association (GNMA), an agency of the U.S. government under the Department of Housing and Urban Development. Thus, its guarantee is backed by the full faith and credit of the U.S. government, and there is no credit risk.

2. **Freddie Mac.** Issued by the Federal Home Loan Mortgage Corporation (FHLMC).

3. **Fannie Mae.** Issued by the Federal National Mortgage Association (FNMA).

The securities issued by all three of these entities are referred to as agency passthrough securities. However, FHLMC and FNMA are not truly government agencies, but government-sponsored enterprises originally created by the U.S. government. Thus, a guarantee from Freddie or Fannie is not backed by the full faith and credit of the U.S. government, but they are considered to be of very high credit quality.

The most important characteristic of passthrough securities is their prepayment risk; because the mortgages used as collateral for the passthrough can be prepaid, the passthroughs themselves have significant prepayment risk. Most of this topic review deals specifically with the prepayment risk in passthroughs: how to measure prepayment speeds (LOS 57.d), the factors that affect prepayment speeds (LOS 57.f), and how to create securities collateralized by passthroughs that have different levels of prepayment risk and are therefore more attractive to investors (LOS 57.g through 57.j).
LOS 57.d: Compare and contrast the conditional prepayment rate (CPR) with the Public Securities Association (PSA) prepayment benchmark.

Prepayments cause the timing and amount of cash flows from mortgage loans and MBS to be uncertain; they speed up principal repayments and reduce the amount of interest paid over the life of the mortgage. Thus, it is necessary to make specific assumptions about the rate at which prepayment of the pooled mortgages occurs when valuing pass-through securities. Two industry conventions have been adopted as benchmarks for prepayment rates: the conditional prepayment rate (CPR) and the Public Securities Association (PSA) prepayment benchmark.

The CPR is the annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool’s CPR is a function of past prepayment rates and expected future economic conditions.

We can convert the CPR into a monthly prepayment rate called the single monthly mortality rate (SMM) using the following formula:

$$SMM = 1 - (1 - CPR)^{1/12}$$

An SMM of 10% implies that 10% of a pool’s beginning-of-month outstanding balance, less scheduled payments, will be prepaid during the month.

The PSA prepayment benchmark assumes that the monthly prepayment rate for a mortgage pool increases as it ages, or becomes seasoned. The PSA benchmark is expressed as a monthly series of CPRs.

The PSA standard benchmark is referred to as 100% PSA (or just 100 PSA). 100 PSA (see Figure 2) assumes the following graduated CPRs for 30-year mortgages:

- CPR = 0.2% for the first month after origination, increasing by 0.2% per month up to 30 months. For example, the CPR in month 14 is 14 (0.2%) = 2.8%.
- CPR = 6% for months 30 to 360.

Figure 2: 100 PSA
Remember that the CPRs are expressed as annual rates.

A particular pool of mortgages may exhibit prepayment rates faster or slower than 100% PSA, depending on the current level of interest rates and the coupon rate of the issue. A 50% PSA refers to one-half of the CPR prescribed by 100% PSA, and 200% PSA refers to two times the CPR called for by 100% PSA.

The SMM is computed from the CPR. Let’s look at an example.

**Example: Computing the SMM**

Compute the CPR and SMM for the 5th and 25th months, assuming 100 PSA and 150 PSA.

**Answer:**

**Assuming 100 PSA:**

- \( \text{CPR (month 5)} = 5 \times 0.2\% = 1\% \)
- \( 100 \text{ PSA} = 1 \times 0.01 = 0.01 \)
- \( \text{SMM} = 1 - (1 - 0.01)^{1/12} = 0.000837 \)

- \( \text{CPR (month 25)} = 25 \times 0.2\% = 5\% \)
- \( 100 \text{ PSA} = 1 \times 0.05 = 0.05 \)
- \( \text{SMM} = 1 - (1 - 0.05)^{1/12} = 0.004265 \)

**Assuming 150 PSA:**

- \( \text{CPR (month 5)} = 5 \times 0.2\% = 1\% \)
- \( 150 \text{ PSA} = 1.5 \times 0.01 = 0.015 \)
- \( \text{SMM} = 1 - (1 - 0.015)^{1/12} = 0.001259 \)

- \( \text{CPR (month 25)} = 25 \times 0.2\% = 5\% \)
- \( 150 \text{ PSA} = 1.5 \times 0.05 = 0.075 \)
- \( \text{SMM} = 1 - (1 - 0.075)^{1/12} = 0.006476 \)
It is important for you to recognize that the nonlinear relationship between CPR and SMM implies that the SMM for 150% PSA does not equal 1.5 times the SMM for 100% PSA. Also, keep in mind that the PSA standard benchmark is nothing more than a market convention. It is not a model for predicting prepayment rates for MBS. In fact, empirical studies have shown that actual CPRs differ substantially from those assumed by the PSA benchmark.

**LOS 57.c: Calculate the prepayment amount for a month, given the single monthly mortality rate.**

The estimated prepayment for any month \( m \) can be expressed as:

\[
\text{Prepayment}_m = \text{SMM}_m \times (\text{mortgage balance at beginning of month } m - \text{scheduled principal payment for month } m)
\]

**Example: Calculating prepayment amount**

Assume that you have invested in a mortgage pool with a $100,000 principal balance outstanding at the beginning of the 25th month. The scheduled monthly principal payment for month 25 is $28.61. Borrowing from the previous example, the CPR and SMM, assuming 100 PSA, are 5% and 0.4265%, respectively. Compute the prepayment for the 25th month.

**Answer:**

This means that 0.4265% of the pool balance, less scheduled payments, will be prepaid this month. So the estimated prepayment amount is:

\[
\text{Prepayment}_{25} = (0.004265)(100,000 - 28.61) = 426.38
\]
LOS 57.f: Explain the factors that affect prepayments and the types of prepayment risks.

There are three main factors that have been shown to affect prepayments: prevailing mortgage rates, housing turnover, and characteristics of the underlying mortgages.

**Prevailing mortgage rates** affect prepayments by influencing the:

- **Spread between the current mortgage rate and the original mortgage rate.** This is the most important factor. If a homeowner is holding a high interest rate mortgage and current mortgage rates fall, the incentive to refinance is large.
- **Path of mortgage rates.** The path that mortgage rates follow on their way to the current level will affect prepayments today. Consider a mortgage pool that was formed when rates were 12%, then interest rates dropped to 9%, rose to 12%, and then dropped again to 9%. Many homeowners will have refinanced when interest rates dipped the first time. On the second occurrence of 9% interest rates, most homeowners in the pool who were able to refinance would have already taken advantage of the opportunity. This tendency is called **refinancing burnout.**

**Housing turnover** increases as rates fall and housing becomes more affordable. This increases refinancings and prepayments. Housing turnover is also higher when economic growth is higher. As the level of general economic activity rises, personal income increases, and workers move to pursue career opportunities. The result is an increase in housing turnover and mortgage prepayments.

Two particular characteristics of the underlying mortgages also affect the level of prepayments: seasoning (i.e., the age of the loan) and property location. Prepayments are low for new mortgages but increase as the loan seasons (the PSA benchmark reflects this idea). Local economics also influence prepayments, which tend to be faster in some parts of the country and slower in others.

**Types of Prepayment Risk**

**Contraction risk** refers to the shortening of the expected life of the mortgage pool due to falling interest rates and higher prepayment rates. There are two undesirable consequences for passsthrough investors when interest rates decline:

1. First, MBS exhibit **negative convexity** as rates decline due to the embedded call option granting the mortgage borrower the right to prepay. Hence, the upside price potential of passsthrough securities is restricted, because investors receive principal sooner than expected (like a callable bond).

2. The second undesirable outcome is **reinvestment rate risk.** Declining interest rates stimulate prepayments resulting in the earlier-than-expected receipt of principal. This means that investors are faced with having to reinvest at relatively lower rates.

**Extension risk** is associated with interest rate increases and falling prepayment rates. Bond prices typically fall when interest rates rise. With passthroughs, the accompanying decrease in passsthroughs compounds this price decline, because the timing of the passthrough cash flows is extended further than originally expected (i.e., the duration of...
the bond is extended). This is undesirable for mortgage investors, particularly short-term investors, because they would prefer to recapture their principal as soon as possible and reinvest at the current higher rates. Essentially, investors’ capital must remain invested at the lower rate.

**LOS 57.e: Explain why the average life of a mortgage-backed security is more relevant than the security’s maturity.**

Because of contraction and extension risk, the stated maturity of a mortgage pass-through security is unlikely to equal its true life. Instead, investors calculate average life (weighted average life), which is the weighted average time until both scheduled principal payments and expected prepayments are received. It’s similar to Macaulay duration, except time is weighted by the projected principal to be received at time \( t \), rather than the present value of the projected principal.

Remember that contraction and extension risk are functions of the security’s average life:

- Contraction risk occurs as mortgage rates fall, prepayment rates increase, and the average life of the pass-through security decreases.
- Extension risk occurs as mortgage rates rise, prepayment rates slow, and the average life of the pass-through security increases.

**LOS 57.g: Illustrate how a collateralized mortgage obligation (CMO) is created and how it provides a better matching of assets and liabilities for institutional investors.**

Institutional investors have varying degrees of concern about exposure to prepayment risk. Some are primarily concerned with extension risk, while others want to minimize exposure to contraction risk. Fortunately, all of the pass-through securities issued on a pool of mortgages do not have to be the same. The ability to partition and distribute the cash flows generated by a mortgage pool into different risk packages has led to the creation of collateralized mortgage obligations (CMOs).

CMOs are securities issued against pass-through securities (i.e., they are securities secured by other securities) for which the cash flows have been reallocated to different bond classes called tranches, each having a different claim against the cash flows of the mortgage pass-throughs or pool from which they were derived. Each CMO tranche represents a different mixture of contraction and extension risk. Hence, CMO securities can be more closely matched to the unique asset/liability needs of institutional investors and investment managers.

It is important to note that the redistribution of the original pass-through securities’ cash flows does not eliminate contraction and extension risk. It merely repackages these risks and apportions them to different classes of bondholders. However, the distribution of these risks to investors most able to deal with a specific type of risk enhances the investment value of the mortgage pool.
LOS 57.h: Distinguish among the sequential pay tranche, the accrual tranche, the planned amortization class tranche, and the support tranche in a CMO.

Sequential Pay CMO

A popular arrangement for separating the cash flows from a mortgage pool is a sequential pay CMO, in which each class of bonds is retired sequentially. To illustrate, consider a simple CMO structure with two tranches in which both tranches receive interest payments at a specified coupon rate, but all principal payments are directed to tranche one until it is completely amortized (the short tranche). Principal payments would then accrue to Tranche 2 until it was fully amortized and the underlying pool was exhausted.

Contraction and extension risk still exist with this structure, but they have been redistributed to some extent between the two tranches. The short tranche, which matures first, offers investors relatively more protection against extension risk. The other tranche provides relatively more protection against contraction risk. Let’s expand this example with some specific numbers to illustrate how sequential pay structures work.

Consider the simplified CMO structure presented in Figure 3. Payments to the two sequential-pay tranches are made first to Tranche A and then to Tranche B.

Figure 3: Sequential Pay CMO Structure

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Outstanding Par Value</th>
<th>Coupon Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$200,000,000</td>
<td>8.50%</td>
</tr>
<tr>
<td>B</td>
<td>50,000,000</td>
<td>8.50%</td>
</tr>
</tbody>
</table>

Payments from the underlying collateral (which has a passthrough coupon rate of 8.5%) for the first five months, as well as months 183 through 187, are shown in Figure 4. These payments include scheduled payments plus projected prepayments based on a prepayment speed of 150 PSA. Note: Some totals might not add due to rounding.
Figure 4: Prepayment Based on 150 PSA

<table>
<thead>
<tr>
<th>Month</th>
<th>Beginning Principal Balance</th>
<th>Principal Payment</th>
<th>Interest</th>
<th>Total Cash Flow = Principal Plus Interest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$250,000,000</td>
<td>$391,128</td>
<td>$1,770,833</td>
<td>$2,161,961</td>
</tr>
<tr>
<td>2</td>
<td>249,608,872</td>
<td>454,790</td>
<td>1,768,063</td>
<td>2,222,853</td>
</tr>
<tr>
<td>3</td>
<td>249,154,082</td>
<td>518,304</td>
<td>1,764,841</td>
<td>2,283,145</td>
</tr>
<tr>
<td>4</td>
<td>248,635,778</td>
<td>581,620</td>
<td>1,761,170</td>
<td>2,342,790</td>
</tr>
<tr>
<td>5</td>
<td>248,054,157</td>
<td>644,690</td>
<td>1,757,050</td>
<td>2,401,741</td>
</tr>
</tbody>
</table>

183  | $51,491,678                | $545,153          | $364,733 | $909,886                              |
184  | 50,946,525                | 540,831           | 360,871  | 901,702                                |
185  | 50,405,694                | 536,542           | 357,040  | 893,582                                |
186  | 49,869,152                | 532,287           | 353,240  | 885,526                                |
187  | 49,336,866                | 528,065           | 349,469  | 877,534                                |

Professor's Note: This example is provided as an illustration of how a basic CMO is created. The LOS does not require you to do the calculations that underlie the numbers in Figure 4. Concentrate on how the cash flows are allocated to each tranche.

Example: Calculating principal payments on a sequential pay tranche

Calculate the principal payments, ending principal balance, and interest payments to each tranche in the first month using the data in Figure 4.

Answer:

Tranche A gets the entire principal payment as well as its share of the interest. Tranche B only receives interest.

Tranche A principal payment = $391,128
Tranche A ending principal balance = $200,000,000 - $391,128 = $199,608,872
Tranche A interest = $200,000,000 × \( \frac{0.085}{12} \) = $1,416,667

Tranche B principal payment = $0
Tranche B ending principal balance = $50,000,000 - $0 = $50,000,000
Tranche B interest = $50,000,000 × \( \frac{0.085}{12} \) = $354,167
Example: Calculating principal payments – Part 2

Calculate the principal payments, ending principal balance, and interest payments to each tranche in the 185th month, assuming the principal balance of Tranche A is now $405,694.

Answer:

From Figure 4 you can see that the total principal payment is $536,542 and the total interest payment is $357,040. Tranche A receives enough principal to pay off its balance, as well as its share of the interest. Tranche B receives the remaining principal as well as its interest.

\[
\begin{align*}
\text{Tranche A principal payment} &= \$405,694 \\
\text{Tranche A ending principal balance} &= \$405,694 - \$405,694 = \$0 \\
\text{Tranche A interest} &= \$405,694 \times \frac{0.085}{12} = \$2,874
\end{align*}
\]

\[
\begin{align*}
\text{Tranche B principal payment} &= \$536,542 - \$405,694 = \$130,848 \\
\text{Tranche B ending principal balance} &= \$50,000,000 - \$130,848 = \$49,869,152 \\
\text{Tranche B interest} &= \$50,000,000 \times \frac{0.085}{12} = \$354,167
\end{align*}
\]
Cash Flow to Sequential Pay Tranche: Month 185

<table>
<thead>
<tr>
<th></th>
<th>Total Cash Flow From Collateral</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Principal</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$536,542</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Interest</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$357,040</td>
</tr>
<tr>
<td>Tranche A</td>
<td></td>
<td>$405,694</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2,874</td>
</tr>
<tr>
<td>Tranche B</td>
<td></td>
<td>$130,848</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$354,167</td>
</tr>
</tbody>
</table>

The time period between the first and last principal payments on a CMO tranche is called the principal pay down window. The principal pay down window of Tranche A in the previous example is 185 months because the principal balance of Tranche A falls to zero in month 185.

For many sequential-pay CMO structures, the last tranche to receive principal also does not receive current interest until the other tranches have been paid off. This tranche is called the Z-tranche or accrual tranche, and the securities that represent a claim against its cash flows are called Z-bonds or accrual bonds. The interest that would ordinarily be paid to the accrual tranche is applied against the outstanding principal of the other tranches, in sequence. The diverted interest from the accrual tranche accrues. That is, it is added to the outstanding principal balance of the Z-tranche.

**Planned Amortization Class (PAC) CMO**

The most common type of CMO today is the planned amortization class (PAC). A PAC is a tranche that is amortized based on a sinking fund schedule that is established within a range of prepayment speeds called the initial PAC collar.

There are two principal repayment schedules associated with a PAC bond, one for the lower prepayment rate and one for the upper rate of the initial PAC collar. PAC bondholders are guaranteed a principal payment that is equal to the lesser amount prescribed by these two repayment schedules. This planned amortization schedule gives the PAC tranche a highly predictable life.

Figure 5 illustrates the planned amortization schedule for a PAC I tranche with an initial collar of 90 PSA to 300 PSA. Notice that the principal payments (which include scheduled payments and prepayments) increase up to month 30 at both prepayment
speeds and then slow down. The principal payments at 300 PSA are much higher through the 30 months because of higher prepayments, then decline much more quickly than the 90 PSA after 30 months. After approximately 90 months, principal payments on the 90 PSA begin to exceed the 300 PSA because the higher earlier prepayments under the 300 PSA significantly reduced the outstanding balance. The planned amortization schedule promised to the PAC I tranche is the minimum of the two prepayment speeds.

Figure 5: Planned Amortization Schedule for PAC Tranche

Support Tranche

What makes a PAC bond work is that it is packaged with a support, or companion, tranche created from the original mortgage pool. Support tranches are included in a structure with PAC tranches specifically to provide prepayment protection for the PAC tranches (each tranche is, of course, priced according to the timing risk of the cash flows). If the prepayment speed of the collateral stays at one level between the lower prepayment rate (90 PSA in Figure 5) and the upper prepayment rate (300 PSA in Figure 5), the principal will be received as scheduled because the support tranche will absorb excess principal or provide principal as needed. If prepayment speeds are outside the initial collar (above 300 or below 90), or even if prepayment speeds vary but stay within the collar, the PAC tranche principal amortization schedule will not necessarily be met. It should be pointed out that the extent of prepayment risk protection provided by a support tranche increases as its par value increases relative to its associated PAC tranche.

There is an inverse relationship between the prepayment risk of PAC tranches and the prepayment risk associated with the support tranches. In other words, the certainty of PAC bond cash flow comes at the expense of increased risk to the support tranches.

To understand the relatively high prepayment risk for support tranches, consider the situation where prepayments are slower than planned. Since the PAC tranches have priority claim against the cash flows, principal payments to the support tranches must
be deferred until the PAC repayment schedule is satisfied. Thus, the average life of the support tranche is extended. Similarly, when actual prepayments come faster than expected, the support tranches must absorb the amount in excess of that required to maintain the repayment schedule for the PAC. In this case, the average life of the support tranche is contracted. If these excesses continue to occur, the support tranches will eventually be paid off, and the principal will then go to the PAC holders. When this happens, the PAC is referred to as a broken or busted PAC, and any further prepayments go directly to the PAC tranche. Essentially, the PAC tranche becomes an ordinary sequential-pay structure.

Notice that the prepayment risk protection provided by the support tranches causes their average life to extend and contract. This relationship is such that as the prepayment risk protection for a PAC tranche increases, its average life variability decreases, and the average life variability of the support tranche increases.

For example, Figure 6 shows the average life for a hypothetical structure that includes a PAC I tranche and a support tranche at various PSA speeds, assuming the PSA speed stays at that level for the entire life of the PAC tranche.

**Figure 6: Average Life Variability of PAC I Tranche vs. Support Tranche**

<table>
<thead>
<tr>
<th>PSA Speed</th>
<th>PAC I Tranche</th>
<th>Support Tranche</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.2</td>
<td>24.0</td>
</tr>
<tr>
<td>50</td>
<td>8.8</td>
<td>21.2</td>
</tr>
<tr>
<td>100</td>
<td>6.5</td>
<td>17.1</td>
</tr>
<tr>
<td>150</td>
<td>6.5</td>
<td>13.3</td>
</tr>
<tr>
<td>200</td>
<td>6.5</td>
<td>10.4</td>
</tr>
<tr>
<td>250</td>
<td>6.5</td>
<td>5.2</td>
</tr>
<tr>
<td>300</td>
<td>6.5</td>
<td>2.9</td>
</tr>
<tr>
<td>350</td>
<td>5.9</td>
<td>2.4</td>
</tr>
<tr>
<td>400</td>
<td>5.4</td>
<td>1.8</td>
</tr>
<tr>
<td>450</td>
<td>4.6</td>
<td>1.5</td>
</tr>
<tr>
<td>500</td>
<td>4.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Figure 6 illustrates the fact that the PAC I tranche has less prepayment risk than the support tranche because the variability of its average life is significantly lower:

- When prepayment speeds fall and prepayments decrease, the support tranche average life is significantly higher than the average life of the PAC I tranche. Thus the support tranche has significantly more extension risk.
- When prepayment speeds rise and prepayments increase, the support tranche average life is much shorter than that of the PAC I tranche. Thus the support tranche also has significantly more contraction risk.
• Over a relatively wide range of prepayment speeds (100 PSA to 300 PSA), the average life of the PAC I tranche is constant at 6.5 years. This range is called the effective collar.

Support tranches are usually subdivided into other tranches. Support tranches can also be split to create support tranches that have a schedule for principal payments just like a PAC tranche. Thus, there are CMO PAC tranches with support tranches that have a PAC schedule. The following definitions may help clarify this type of structure.

• **PAC I tranche** (or level I PAC tranche): A PAC structure having a support tranche with a PAC principal repayment schedule.

• **PAC II tranche** (level II PAC tranche or scheduled tranche): The support tranche for a PAC I tranche that has a PAC schedule of principal repayments. PAC II tranches have higher prepayment risk (and average life variability) than PAC I tranches but more prepayment protection (and less average life variability) than support tranches without schedules for principal repayment.

**LOS 57.i:** Evaluate the risk characteristics and the relative performance of each type of CMO tranche, given changes in the interest rate environment.

Let's repeat something we said earlier:

• Prepayment risk encompasses contraction risk and extension risk.
  • Contraction risk occurs as mortgage rates fall, prepayment rates increase, and the average life of the passthrough security decreases.
  • Extension risk occurs as mortgage rates rise, prepayment rates slow, and the average life of the passthrough security increases.

CMO structures are designed to redistribute contraction and extension risk among the tranches. The risk characteristics and relative price performance of each CMO tranche is a function of how well protected it is against these risks.

First, let's analyze the contraction and extension risk of a simple sequential pay CMO with four sequential pay tranches (A, B, C, and D) and a one accrual tranche (Z bond).

**Figure 7: Contraction and Extension Risk**

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Contraction Risk</th>
<th>Extension Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (sequential pay)</td>
<td>HIGH</td>
<td>LOW</td>
</tr>
<tr>
<td>B (sequential pay)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C (sequential pay)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D (sequential pay)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z (accrual)</td>
<td>LOW</td>
<td>HIGH</td>
</tr>
</tbody>
</table>

The *early* tranches are protected against extension risk, while the later tranches are protected against contraction risk. The Z bond has low contraction risk, because reinvestment risk is eliminated until the other tranches have paid off.
Now let’s consider a more realistic CMO structure with four PAC I tranches (A through D), two PAC II tranches (E and F), and an unscheduled support tranche. The PAC I tranches receive principal in order (A first, then B, and so on), as do the PAC II tranches. The support tranche provides support to the PAC tranches. We can characterize the relative prepayment risk of each tranche based on our previous discussions.

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Prepayment Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (PAC I)</td>
<td>LOW</td>
</tr>
<tr>
<td>B (PAC I)</td>
<td></td>
</tr>
<tr>
<td>C (PAC I)</td>
<td></td>
</tr>
<tr>
<td>D (PAC I)</td>
<td></td>
</tr>
<tr>
<td>E (PAC II)</td>
<td></td>
</tr>
<tr>
<td>F (PAC II)</td>
<td></td>
</tr>
<tr>
<td>Support</td>
<td>HIGH</td>
</tr>
</tbody>
</table>

Notice the PAC I tranches (which have a specified prepayment collar that limits both contraction and extension risk) have lower prepayment risk than the PAC II tranches, which have lower prepayment risk than the support tranche. Within the PAC I and II tranches, the early tranches have lower prepayment risk than the later tranches. The unscheduled support tranche absorbs most of the prepayment risk.

**LOS 57.j: Explain the investment characteristics of stripped mortgage-backed securities.**

A distinguishing characteristic of a traditional passthrough security is that the interest and principal payments generated by the underlying mortgage pool are allocated to the bondholders on a pro rata basis. This means that each passthrough certificate holder receives the same amount of interest and the same amount of principal. Stripped mortgage-backed securities differ in that principal and interest are not allocated on a pro rata basis. The unequal allocation of principal and interest results in a price/yield relationship for the stripped securities that is significantly different from that of the underlying passsthrough. The two most common types of stripped MBS are principal-only (PO) strips and interest-only (IO) strips.

POs are a class of securities that receive only the principal payment portion of each mortgage payment. They are sold at a considerable discount to par. The PO cash flow stream starts out small and increases with the passage of time as the principal component of the mortgage payments grows. The entire par value of a PO is ultimately paid to the PO investor. The only question is whether realized prepayment rates will cause it to be paid sooner or later than expected.
**IOs** are a class that receives only the interest component of each payment. IO strip cash flow starts out big and gets smaller over time. Thus, IOs have shorter effective lives than POs. The major risk associated with IO strips is that the value of the cash flow investors receive over the life of the mortgage pool may be less than initially expected and possibly less than the amount originally invested. Why? The amount of interest produced by the pool depends on its beginning-of-month balance. If market rates fall, the mortgage pool will be paid off sooner than expected, leaving IO investors with no interest cash flows. Therefore, IO investors benefit when prepayments are low.

The price/yield relationships for IO and PO securities are shown in Figure 9. Notice the following **investment characteristics of IOs and POs**:

- The investment performance of a PO is extremely sensitive to prepayment rates. Higher prepayment rates result in a faster-than-expected return of principal and, thus, a higher return. Since prepayment rates increase as mortgage rates decline, PO prices increase when interest rates fall. They also exhibit some negative convexity at low rates.

- The IO price is positively related to mortgage rates at low current rates. When market rates decline below the average mortgage rate in the pool, prepayment rates increase and the principal amount falls. Interest payments to the IO decrease because they are based on the outstanding principal on the underlying pool. The diminished cash flow usually causes the IO price to decline despite the fact that the cash flows are now being discounted at a lower rate. On the other hand, as interest rates rise above the contract rate, the expected cash flows improve. Even though the higher rate must be used to discount these improved cash flows, there is usually a range above the contract rate for which the price increases.

- Both IOs and POs exhibit greater *price volatility* than the passthrough from which they were derived. This occurs because IO and PO returns are negatively correlated (their prices respond in opposite directions to changes in interest rates), but the combined price volatility of the two strips equals the price volatility of the passthrough.

**Figure 9: Investment Characteristics of IOs and POs**

![Figure 9: Investment Characteristics of IOs and POs](image)
LOS 57.k: Compare and contrast agency and nonagency mortgage-backed securities.

Up to this point, the MBS that we have discussed have been backed by residential mortgage loans issued by agencies of the federal government. There also are MBS that are issued by private entities. These are referred to as nonagency mortgage-backed securities or, simply, nonagency securities.

The nonagency securities discussed in this section are backed by a pool of one to four single-family residential first-lien mortgages issued by private entities. However, nonagency securities are sometimes backed by second mortgage loans, manufactured housing loans, and a variety of commercial real estate loans.

The loans that form the mortgage pool for agency securities must conform to the underwriting standards of the issuing or guaranteeing agency. The loans that are used to back nonagency securities are usually those that fail to meet the agency's underwriting standards. These loans are referred to as nonconforming mortgage loans. The underwriting standards are primarily concerned with the maximum loan-to-value ratio, payment-to-income ratio, and loan amount.

In the agency market, CMOs are formed by splitting up a pool of passthrough securities. However, since it is rare for nonconforming mortgage loans to be securitized, nonagency CMOs are usually created directly from the nonconforming mortgages. Nonagency CMOs are often referred to as whole-loan CMOs, because unsecuritized loans are called whole loans. In other words, the collateral behind nonagency CMOs is a pool of loans rather than passthrough securities.

The key difference between agency issued MBS and nonagency issues is that the agency issues are backed by a pseudo-governmental guarantee and, as such, may have a relatively more certain cash flow stream. Consequently, the risk and expected return of agency issues is lower than nonagency issues. Since the cash flows from nonagency securities are affected by mortgage default rates, they require credit enhancement (i.e., additional support against default).

WARM-UP: COMMERCIAL MBS

Commercial mortgage-backed securities (CMBS) are backed by income-producing real estate, typically in the form of:

- Apartments (multi-family).
- Warehouses (industrial use property).
- Shopping centers.
- Office buildings.
- Health care facilities.
- Senior housing.
- Hotel/resort properties.

These loans are typically originated by conduit organizations (commercial mortgage companies). They negotiate and close commercial real estate loans which are then incorporated into a CMBS. Conduit-originated transactions are the most popular...
way to generate mortgages to securitize. Other, less popular, CMBS structures include liquidating trusts and multi-property single-borrower programs.

**LOS 57.1:** Distinguish credit risk analysis of commercial mortgage-backed securities (CMBS) from credit risk analysis of residential nonagency mortgage-backed securities.

The biggest difference between residential and commercial MBS loans is the obligation of the underlying borrower. Residential MBS loans are repaid by homeowners; commercial MBS loans are repaid by real estate investors who, in turn, rely on tenants and customers to provide the cash flow to repay the mortgage loan. CMBS mortgages are structured as nonrecourse loans, meaning that the lender can only look to the collateral as a means to repay a delinquent loan if the cash flows from the property are insufficient. In contrast, the residential mortgage lender can go back to the borrower personally in an attempt to collect a delinquent mortgage loan.

*For these reasons, the analysis of CMBS securities focuses on the credit risk of the property and not the credit risk of the borrower.* The analysis of CMBS structures focuses on two key ratios to assess credit risk.

1. **Debt-to-service coverage ratio** is a basic cash flow coverage ratio of the amount of cash flow from a commercial property available to make debt service payments.

\[
\text{debt-to-service coverage ratio} = \frac{\text{net operating income}}{\text{debt service}}
\]

Net operating income (NOI) is calculated after the deduction for real estate taxes but before any relevant income taxes. This ratio, which is typically between one and two, provides increased comfort to the lender as it increases. Debt service coverage ratios below one indicate that the borrower is not capable of making the debt payments and is likely to default. Remember: the *higher the better* for this ratio from the perspective of the lender and the MBS investor.

2. **Loan-to-value ratio** compares the loan amount on the property to its current fair market or appraisal value.

\[
\text{loan-to-value ratio} = \frac{\text{current mortgage amount}}{\text{current appraised value}}
\]

The lower this ratio, the more comfortable the mortgage lender is in making the loan. Loan-to-value ratios determine the amount of collateral available, above the loan amount, to provide a cushion to the lender should the property need to be foreclosed on and sold. Remember: the *lower the better* for this ratio from the perspective of the lender and the MBS investor.
LOS 57.m: Describe the basic structure of a CMBS, and illustrate the ways in which a CMBS investor may realize call protection at the loan level and by means of the CMBS structure.

The basic CMBS structure is created to meet the risk and return needs of the CMBS investor. As with residential MBS securities, rating organizations such as S&P and Moody’s assess the credit risk of each CMBS issue and determine the appropriate credit rating. Each CMBS is segregated into tranches that are repaid in a specific sequence with the highest credit quality tranche being repaid first.

As with any fixed income security, call protection for the investor is important. CMBS provide call protection in two ways: loan-level call protection provided by the structure of the individual mortgage, and call protection provided by the CMBS structure.

There are several means of creating loan-level call protection:

- **Prepayment lock out.** For a specific period of time (typically two to five years), the borrower is prohibited from prepaying the mortgage loan.
- **Defeasance.** Should the borrower insist on making payments on the mortgage loan, the mortgage loan can be defeased, which means the loan proceeds are received by the loan servicer and invested in U.S. Treasury securities, essentially creating cash collateral against the loan. Upon completion of the defeasance period, these U.S. Treasuries are liquidated and the proceeds are used to repay the mortgage. Treasuries provide higher-quality collateral than the underlying real estate, so defeased loans increase the credit quality of a CMBS loan pool.
- **Prepayment penalty points.** A penalty fee may be charged if the borrower prepays the mortgage loan. This penalty fee is typically much higher in the early years of the loan (e.g., 5% of the loan amount in the first year) and then steps down over time until it finally disappears after several years. In many cases, this penalty fee is quoted as a 5-4-3-2-1, which means the penalty fee is 5% of the principal amount of the loan in the first year, and 1% of the principal amount if repaid in the fifth year of the mortgage. Beginning in the sixth year of the mortgage, there is no prepayment penalty to the borrower.
- **Yield maintenance charges.** The borrower is charged the amount of interest lost by the lender should the loan be prepaid. This *make whole* charge makes the lenders indifferent to prepayment, as they are in the same economic position whether the loan is prepaid or not.

With all loan call protection programs, any prepayment penalties received are distributed to the CMBS investors in a manner determined by the structure of the CMBS issue.

To create CMBS-level call protection, CMBS loan pools are segregated into tranches with a specific sequence of repayment. Those tranches with a higher priority for prepayment or collateral position will have a higher credit rating than lower priority tranches because loan defaults will first affect the lower tranches. A wide variety of features can be used to provide call protection to the more senior tranches of the CMBS.
Key Concepts

LOS 57.a
A mortgage is a loan that is collateralized with a specific piece of real estate, either residential or commercial. The interest rate on the loan is called the mortgage rate, or contract rate.

A fixed-rate, level payment, fully amortized mortgage loan requires equal monthly payments, each consisting of an interest component and a principal component.
• The monthly interest component is based on the amount of outstanding principal at the beginning of the month.
• The incremental reduction of outstanding principal is referred to as scheduled amortization (or scheduled principal repayment).
• A servicing fee is built into the mortgage rate to cover the cost of payment collection and other administrative activities.

Payments in excess of the required monthly amount are called prepayments, and prepayments for less than the outstanding principal balance are called curtailments.
• Prepayments or curtailments will reduce the amount of interest the lender receives over the life of the loan.
• Prepayment risk refers to the likelihood that prepayments or curtailments will actually occur.

LOS 57.b
A mortgage passthrough security represents a claim against a pool of mortgages. Any number of mortgages may be used to form the pool. A mortgage that is included in the pool is called a securitized mortgage.
• Passthrough investors receive the monthly cash flows generated by the underlying pool.
• More than one class of passthrough security may be issued against a single mortgage pool, each representing a unique claim on the pool's cash flows.

LOS 57.c
The single monthly mortality rate (SMM) is derived from the conditional prepayment rate (CPR) and is used to estimate monthly prepayments for a mortgage pool.
• $\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$

The estimated prepayment for any month $m$ can be expressed as:
• $\text{Prepayment}_{m} = \text{SMM} \times (\text{mortgage balance at beginning of month } m - \text{scheduled principal payment for month } m)$

LOS 57.d
CPR and the PSA prepayment benchmark are industry benchmarks for assumed prepayment rates. CPR is the annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool's CPR is a function of past prepayment rates and economic conditions.
The PSA prepayment benchmark assumes that the monthly prepayment rate for a mortgage pool increases as the mortgage pool ages (becomes seasoned).

- PSA is expressed as a monthly series of CPRs.
- For 30-year mortgages, the 100% PSA standard benchmark is:
  - CPR = 0.2% for the first month, increasing by 0.2% per month up to 30 months.
  - CPR = a flat 6% for months 30–360.

**LOS 57.e**
Investors calculate average life or weighted average life for pass-through securities since prepayment risk usually results in the stated maturity of a pass-through being different than its actual life.

- As mortgage rates fall, prepayment rates increase, and the average life of a pass-through security decreases.
- As mortgage rates rise, prepayment rates slow, and the average life of a pass-through security increases.

**LOS 57.f**
Three factors affect prepayments:

- Prevailing mortgage rate.
  - The spread between the current mortgage rate and the original mortgage rate is the most important factor. Historically, if mortgage interest rates fall more than 2%, refinancing activity increases dramatically.
  - The path that mortgage rates follow on their way to the current level will affect prepayments today. When mortgage rates drop, rebound, and drop again, most homeowners have already refinanced. This tendency is called refinancing burnout.
- Housing turnover increases as mortgage rates fall and housing becomes more affordable, and as the general level of economic activity increases. The result is higher prepayments.
- Prepayments are also affected by seasoning and property location.

Contraction risk for an MBS refers to undesirable consequences of declining interest rates: (1) MBS exhibit negative convexity, and (2) cash flows must be reinvested at a lower rate.

Extension risk refers to the drop in bond prices and the slowing of prepayments as interest rates increase. Investors would prefer to recapture their principal without a capital loss and reinvest at the current higher rates.

**LOS 57.g**
CMOs are securities issued against a pool of mortgages for which the cash flows have been allocated to different classes called tranches. Each tranche has a different claim against the assets of the pool and a different mixture of contraction and extension risk. CMOs can be matched to the unique asset/liability needs of investors.
LOS 57.h
Sequential-pay tranches are a common arrangement for separating mortgage cash flows into classes to create CMOs where each class of bond is retired sequentially.

PAC tranches, the most common type of CMO, have a payment schedule that is established within a range of prepayment speeds called the initial PAC collar.

Each PAC has a companion or support tranche that has a second priority claim to the cash flows. If prepayments are too high, the support tranche is paid off faster. If too slow, the support tranche provides the funds needed to keep the PAC on schedule.

LOS 57.i
The following table shows the contraction and extension risk of a simple sequential pay CMO containing four sequential pay tranches and one accrual tranche.

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Contraction Risk</th>
<th>Extension Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (sequential pay)</td>
<td>HIGH</td>
<td>LOW</td>
</tr>
<tr>
<td>B (sequential pay)</td>
<td>↓</td>
<td>↓</td>
</tr>
<tr>
<td>C (sequential pay)</td>
<td>LOW</td>
<td>HIGH</td>
</tr>
<tr>
<td>D (sequential pay)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z (accrual)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The early tranches are protected against extension risk, while the later tranches are protected against contraction risk. The Z bond has low contraction risk because reinvestment risk is eliminated until the other tranches have been paid off.

LOS 57.j
Stripped MBS differ from traditional passthroughs in that the principal and interest are not allocated on a pro rata basis.
- PO strips are a class of securities that receive only the principal payment portion of each mortgage payment. The PO exhibits some negative convexity at low rates.
- IO strips are classes that receive only the interest component of each payment. The IO price is positively related to mortgage rates at low current rates.
- PO and IO prices are more volatile than the underlying passthroughs.

LOS 57.k
Nonagency MBS (nonagency securities) are issued by private entities and are usually backed with nonconforming mortgage loans (loans that fail to meet the agency’s underwriting standards). Nonagency security cash flows are affected by mortgage default rates and thus require credit enhancement (i.e., additional support against default).

LOS 57.l
CMBS are structured as nonrecourse loans, meaning that the lender can only look to the collateral as a means to repay the loan. In contrast, the residential mortgage lender can go back to the borrower, personally, in an attempt to repay a delinquent mortgage loan. The analysis of CMBS structures focuses on two key ratios to assess the credit risk of the property: debt-to-service coverage ratio and loan-to-value ratio.
LOS 57.m
Methods of call protection for the CMBS at the loan level include:
• Prepayment lock outs prevent the borrower from repaying the loan for a set period of time.
• Defeasance increases the quality of a CMBS loan pool by reinvesting any prepayments in Treasury securities.
• Penalty fees may be assessed against a borrower for prepayment.
• Yield maintenance charges require the borrower to \textit{make whole} the amount of interest that would be paid to the lender upon prepayment.

At the CMBS pool level, cash prepayments are assigned to tranches, which mitigates the amount of prepayment risk of the more senior tranches.
CONCEPT CHECKERS

1. Which of the following statements concerning interest-only (IO) and principal-only (PO) passthrough securities is least accurate?
   A. IO passthroughs generally increase in value when interest rates rise.
   B. PO passthroughs generally increase in value when interest rates fall.
   C. If interest rates have fallen, risen, and then fallen again, the values of IOs and POs will be more affected by the second rate decrease.

2. Which of the following statements concerning the role of a support tranche in a planned amortization class collateralized mortgage obligation (PAC CMO) is least accurate?
   A. The purpose of a support tranche is to provide prepayment protection for one or more PAC tranches.
   B. The support tranches are exposed to extremely high levels of credit risk.
   C. If prepayments are too low to maintain the PAC schedule, the shortfall is provided by the support tranche.

Use the following information to answer Questions 3 through 5.

Assume that an investor has invested in a mortgage pool with a $100,000 principal balance outstanding. The scheduled monthly principal payment is $28.61.

3. The mortgage pool has a conditional prepayment rate (CPR) of 6% and the pool is seasoned. The single monthly mortality rate is closest to:
   A. 0.005098.
   B. 0.005113.
   C. 0.005143.

4. Using the information from Question 3, the estimated prepayment for the month is closest to:
   A. $509.92.
   B. $514.15.
   C. $529.25.

5. Using the Public Securities Association (PSA) standard prepayment benchmark, the single monthly mortality rate (SMM) in month 10, assuming 175% PSA, is closest to:
   A. 0.002363.
   B. 0.002793.
   C. 0.002965.

6. Consider a collateralized mortgage obligation (CMO) structure with one planned amortization class (PAC) class and one support tranche outstanding. Also, assume that the prepayment speed is higher than the upper collar on the PAC. Which of the following statements is most accurate? The:
   A. PAC tranche has no risk of prepayments.
   B. average life of the support tranche will contract.
   C. average life of the PAC tranche will extend.
7. Which of the following is least likely an important factor affecting the prepayment rate of a pool of mortgages?
   A. The location of the underlying real estate.
   B. Seasonal factors (i.e., the time of year when the mortgages were initiated).
   C. General economic conditions.

8. Which of the following scenarios is most likely to lead to the highest risk of prepayments on a mortgage pool?
   A. The mortgage pool has an average mortgage rate of 6.25% and an average life remaining of 84 payments. The current mortgage rate is 6.0%.
   B. A strong economy has caused gross domestic product to rise and unemployment to decline.
   C. Mortgage rates are currently at 7.5% and are now declining after varying between 7 and 10% for the last three years.

9. Which of the following statements regarding a planned amortization class (PAC) is least accurate?
   A. The average life of a PAC bond can remain stable even if prepayment rates go outside of the initial PAC collar for short time periods.
   B. A broken PAC is one where the support tranches have been fully repaid.
   C. A PAC II tranche gives investors in the PAC II equal prepayment risk.

10. The average life of a mortgage-backed security (MBS) is more relevant than the security’s final maturity because it represents the average time to the receipt of:
    A. scheduled principal payments.
    B. expected prepayments.
    C. both expected prepayments and scheduled principal payments.

11. Commercial mortgage-backed securities (CMBS) loans typically have greater call protection than residential MBS loans because:
    A. CMBS typically receive higher credit ratings from credit agencies than residential MBS.
    B. commercial mortgages may have yield maintenance charges.
    C. smaller dollar-sized mortgages typically are not refinanced if interest rates fall.

12. Commercial mortgage-backed securities (CMBS) mortgages are structured as nonrecourse loans, which means the:
    A. borrower cannot be forced into bankruptcy.
    B. lender cannot litigate the loan.
    C. lender can only look to the collateral as a means to repay a delinquent loan.
CHALLENGE PROBLEMS

13. What is most likely to happen to the prepayment rate and the average life of a typical passthrough security if mortgage rates rise?
   A. Both will increase.
   B. One will increase, one will decrease.
   C. Both will decrease.

14. Tiffany Childers is reviewing various mortgage-backed securities (MBS) and is interested in the calculation of single monthly mortality (SMM) rates. Childers is using the Public Securities Association (PSA) standard prepayment benchmark. She calculates the SMM for month 22, assuming a 140 PSA, to be 0.37%. She calculates the SMM for month 200, assuming a 90 PSA, to be 0.46%. Childers is:
   A. correct for both months.
   B. correct for one month, but incorrect for the other month.
   C. incorrect for both months.

15. Two different structures of collateralized mortgage obligations (CMO) are being considered for issuance:
   • Structure 1: $400 million of passthroughs will be used as collateral for two sequential pay tranches: $325 million worth of bonds of Tranche X and $75 million of bonds of Tranche Y. The principal for Tranche X must be completely paid off before any payments are made to Tranche Y.
   • Structure 2: $400 million of passthroughs will be used as collateral for $325 million of E bonds in a planned amortization class (PAC) tranche and $75 million of F bonds in a support tranche.

Which of the following statements is least accurate? The:
   A. X bonds have less contraction risk than the Y bonds.
   B. X bonds have less extension risk than the Y bonds.
   C. E bonds have less contraction risk than the F bonds.

16. Nonagency issues have all of the following characteristics except:
   A. they are usually created using nonconforming mortgages.
   B. they include 1 to 4 family residential homes but exclude second mortgage loans and manufactured housing loans.
   C. the collateral is a pool of loans rather than passthrough securities.
1. C  The values of IOs and POs will be less affected by the second rate decrease. This occurs because of refinancing burnout, wherein many borrowers will have refinanced at the first rate decrease.

2. B  The support tranches are exposed to extremely high levels of prepayment risk, not credit risk.

3. C  Seasoned means that the pool is older than 30 months.

4. B  0.005143 x ($100,000 - $28.61) = $514.15

5. C  

\[
SMM = 1 - (1 - 0.06)^{1/12} = 0.005143 
\]

6. B  If the prepayment speed is faster than the PAC collar, the support tranche receives a higher level of prepayments (so that the PAC tranche remains at the upper collar of the PAC). The average life of the support tranche will contract (shorten). The PAC tranche could receive higher prepayments if eventually the support tranche is fully repaid its principal (i.e., a busted PAC). However, the question says that the support tranche is still outstanding, which means that hasn't happened yet.

7. B  The four main factors that have been shown to affect prepayments are (1) prevailing mortgage rates; (2) the characteristics of the underlying mortgage pool (such as the location of the real estate); (3) seasonal factors (i.e., the time of year in which the prepayment rate is measured, not the time of year when the mortgages were initiated); and (4) general economic activity.

8. B  As the economy grows, personal income rises and workers transfer to new or better paying jobs. This leads to an increase in housing turnover, which results in mortgage payoffs and prepayments. The other scenarios do not carry a large risk of prepayments. The incentive to refinance is not great for only a 0.25% spread in mortgage rates and a relatively short time left on the mortgage; a mortgage rate that follows a down, up, and back down path can lead to refinancing burnout; and home-buying and prepayments are typically strongest in the spring.

9. C  A PAC II tranche does not require equal distribution of prepayment risk. The PAC II tranche can be divided into classes of investors with different prepayment risk through a PAC schedule. The other statements are accurate. The average life of a PAC bond can remain stable in certain circumstances even if the prepayment speed temporarily goes outside the initial collar. Figure 5 in the topic review illustrates the case in which the prepayment speed jumps immediately to a speed outside the initial collar and stays at that speed for the life of the tranche.

10. C  The average life is more relevant for mortgage backed securities because it represents the average time to receipt of both principal payments and expected prepayments.
11. B Any type of call protection structured into the loan itself (in this case, yield maintenance charges) increases the overall call protection of the CMBS. Residential mortgages can be prepaid without penalty at any time and do not provide any call protection at the individual loan level.

12. C Nonrecourse loans mean that the lender can only look to the collateral as a means to repay a delinquent loan, not the borrower’s personal assets.

**Answers – Challenge Problems**

13. B Prepayment rates will most likely decrease if mortgage rates rise. This will be associated with an increase in the average life.

14. B Under PSA, the conditional prepayment rate rises 0.2% per month for months 1–30 and levels off at a rate of 6%. Under the first scenario,

\[
\text{CPR} = 6\% \times \frac{22}{30} = 4.4\%
\]

\[
140\text{PSA} = 1.40 \times 4.4\% = 6.16\%
\]

\[
\text{SMM} = 1 - \left(1 - 0.0616\right)^{1/12} = 0.0053 = 0.53\%
\]

Under the second scenario, the CPR is after month 30, so CPR = 6%.

\[
\text{CPR} = 6\%
\]

\[
90\text{PSA} = 0.9 \times 6\% = 5.4\%
\]

\[
\text{SMM} = 1 - \left(1 - 0.054\right)^{1/12} = 0.0046 = 0.46\%
\]

Childers is incorrect in her estimate of the month 22 SMM, but correct in her estimate of the month 200 SMM.

15. A In Structure 1, we have two sequential pay tranches. If prepayments slow, it will take longer for cash flows to get to the Y bonds, so the X bonds have less extension risk. The X bonds have more contraction risk than the Y bonds because they will get cash flows more quickly if prepayments accelerate. The X bonds protect the Y bonds against contraction risk. In the case of Structure 2 where there are two PAC tranches, the F support tranche will absorb the impact of both accelerated and slower than expected prepayments, resulting in the E bonds having both less contraction risk and less extension risk than the F bonds.

16. B The nonagency securities are backed by a pool of one to four single-family residential first-lien mortgages issued by private entities. However, nonagency securities are sometimes backed by second mortgage loans, manufactured housing loans, and a variety of commercial real estate loans.
The following is a review of the Fixed Income: Structured Securities principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**ASSET-BACKED SECTOR OF THE BOND MARKET**

**EXAM FOCUS**

Asset-backed securities (ABS) are securities created from the pooling of non-mortgage assets (e.g., auto loans, credit card receivables, and corporate bonds). You should have a solid understanding of the basic structure of each of these different types of ABS. Also focus on the similarities and differences between mortgage-backed securities (MBS) and ABS; both are exposed to varying degrees of prepayment risk, but credit risk is a more important consideration for ABS investors. Remember that you’re being tested on your skills as an analyst, not as an investment banker; concentrate on the risks faced by investors who hold ABS, not on the details of how various ABS are created.

**LOS 58.a: Illustrate the basic structural features of and parties to a securitization transaction.**

Let’s illustrate the basic structure of a securitization transaction with a simplified, fictitious example of Fred Motor Company.

Fred Motor Company manufactures and sells automobiles in a wide range of styles and prices. Most of the company’s sales are done on retail sales installment contracts (i.e., auto loans). The customer buys the automobile, and Fred loans the customer the proceeds for the purchase (i.e., Fred originates the loan) using the auto as collateral and receives principal and interest payments on the loan until it matures. The loans have maturities of 48 to 60 months at varying interest rates. Fred is also the servicer of the loan: the company collects principal and interest payments, sends out delinquent notices, and repossesses and disposes of the auto if the customer doesn’t make timely payments.

Fred has 50,000 auto loans totaling $1 billion that it would like to remove from its balance sheet. It accomplishes this by selling the loans to a special purpose vehicle (SPV) called Auto Owner Trust for $1 billion (which is why Fred is called the **seller**). The SPV, which is set up for the specific purpose of buying these auto loans, is referred to as the **trust** or the **issuer**. The SPV then issues asset-backed securities (ABS) to investors using the portfolio of auto loans as collateral.

Let’s review the parties to this transaction and their functions:

- The **seller** (Fred Motor Company) originates the auto loans and sells the portfolio of loans to Auto Owner Trust, the SPV.
- The **issuer/trust** (Auto Owner Trust) is the SPV that buys the loans from the seller and issues ABS to investors.
- The **servicer** (Fred Motor Company) services the loans.
- In this case, the seller and the servicer are the same entity (Fred Motor Company), but that is not always the case in asset securitizations.
The structure of this securitization transaction is shown in Figure 1.

**Figure 1: Structure of Fred Motor Company Asset Securitization**

Subsequent to the initial transaction, the principal and interest payments on the original loans are paid by the customers to the servicer. This cash flow is then allocated to pay servicing fees to the servicer and principal and interest payments to the investors in the various tranches of the ABS according to the priority rules set out in the prospectus. This flow of funds structure is called the waterfall.

ABS are most commonly backed by automobile loans, credit card receivables, home equity loans, manufactured housing loans, student loans, Small Business Administration (SBA) loans, corporate loans, corporate bonds, emerging market bonds, and structured financial products.

**LOS 58.b: Explain and contrast prepayment tranching and credit tranching.**

Recall from the previous topic review that agency mortgage-backed securities (MBS) structures are divided into different tranches to distribute the prepayment risk to various investors using, for example, sequential-pay or planned amortization class (PAC) structures. ABS are also structured to distribute the prepayment risk; this is called prepayment tranching, or time tranching.

ABS can have credit risk in addition to prepayment risk. The credit risk of ABS is reduced by various forms of credit enhancement. The most common form of credit enhancement is a senior-subordinated structure in which the subordinated bonds absorb all losses first up to their par value, after which any additional losses are absorbed by the senior bonds, so that credit risk is shifted from the senior bonds to the subordinated bonds. This type of structure is called credit tranching.

Often ABS are structured with both prepayment and credit tranching, as illustrated in Figure 2.
The structure represented in Figure 2 has senior bonds, which are divided so that some have relatively short and some have relatively long target maturities. Other senior (in terms of credit risk) bonds serve as the support tranches for the short and long target maturities of the senior bonds in the first tier. The bonds in the lowest tier are the junior or subordinated bonds, which absorb defaults up to their par values, increasing the credit rating of both the senior target maturity and senior support tranches. These bonds may also be structured to provide support for the target maturity of the senior bonds as well, and therefore may have the highest levels of both prepayment and credit risk.

**Figure 2: Prepayment and Credit Tranching of an ABS**

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LOS 58.c: Distinguish between the payment structure and collateral structure of a securitization backed by amortizing assets and non-amortizing assets.
```

**Amortizing** assets are loans for which the borrower makes periodic scheduled payments that include both principal and interest. The interest amount is subtracted from the total payment, and the balance is applied toward the principal, reducing the outstanding loan. Amounts in excess of the scheduled periodic payment are applied to a further reduction of principal. Such additional payments are called prepayments. A residential mortgage is an example of an amortizing loan.

**Non-amortizing** assets are loans that do not have a scheduled payment amount. Instead, a minimum payment, which is applied against accrued interest, is required. If the minimum payment exceeds the accrued interest, the excess is applied toward reducing the outstanding principal. If the payment falls short of the accrued interest, the outstanding loan balance is increased by the amount of the shortfall. A revolving credit card loan is an example of a non-amortizing asset.

The structure of the ABS transaction is affected by whether the assets backing the bonds are amortizing or non-amortizing. For amortizing assets like auto loans, once the assets are securitized, the composition of the loans in the pool doesn’t change. Loans disappear from the pool as they are paid off or default, but no new loans are added to the pool to replace them. Principal payments and prepayments on the remaining loans are distributed to the bondholders according to the distribution rules of the structure.
For non-amortizing assets like credit card receivables, however, the composition of the loans in the pool can and does change. During the lockout period (e.g., the first 18 months), principal payments and prepayments are not distributed to the bondholders as is the case with amortizing assets. Instead, the cash flow from these principal payments is used to invest in new loans to replace the amounts paid off. This type of structure is called a revolving structure.

The bonds issued in a revolving structure can be retired early under certain conditions by requiring cash flows to be directed to the reduction of principal rather than the purchase of new loans. The call provision can be triggered by several different types of events:

- A specific date is reached (e.g., a call on or after a specified date, much like a typical callable corporate bond).
- The collateral value falls below a certain level (e.g., 10% of the original outstanding balance); this is referred to as an optional clean-up call provision.
- Either a specific date is reached or the collateral balances reach a specific level; this is called latter of percent or date call.
- The collateral’s cumulative loss history reaches a certain level; this is referred to as an insurer call.

**LOS 58.d: Distinguish among the various types of external and internal credit enhancements.**

Credit enhancements accompany all ABS. The level of credit enhancement is directly proportional to the level of rating desired by the issuer. Rating agencies determine the exact amount of credit enhancement necessary for an issue to hold a specific rating. There are two types of credit enhancements: internal and external.

**External Credit Enhancements**

External credit enhancements are financial guarantees from third parties that support the performance of the bond. They are used to supplement other forms of credit enhancements. Third-party guarantees impose a limit on the guarantor’s liability for losses at a specified level. They protect against losses before internal credit enhancements are used. External credit enhancements include the following:

- **Corporate guarantees.** The sponsor (effectively the seller of the securities) agrees to guarantee a portion of the offer.
- **Letter of credit.** A bank letter of credit provides a guarantee against loss up to a certain level.
- **Bond insurance.** Bond insurance provides for protection against losses through the purchase of insurance against nonperformance.

The problem with third-party guarantees is the “weak link” philosophy adopted by rating agencies: the credit quality of an issue cannot be higher than the credit rating of the third-party guarantor. If the guarantor is downgraded, the issue itself may also be downgraded, even if there has not been a decline in the credit quality of the underlying collateral.
Internal Credit Enhancements

Internal credit enhancements include reserve funds, overcollateralization, and structures that contain senior and subordinated debt. Internal credit enhancements do not rely on a third-party guarantee.

There are two types of reserve funds—cash reserve funds and excess servicing spread funds.

1. **Cash reserve funds** are cash deposits that come from issuance proceeds. This excess cash provides for the establishment of a reserve account to pay for future losses.

2. **Excess servicing spread funds** consist of reserve funds in the form of excess spread or cash after paying for servicing and other expenses. The excess servicing spread funds can be used to fund credit losses on the collateral. However, if defaults exceed those initially projected, the effectiveness of the excess servicing spread diminishes. Thus, default-related assumptions should be examined when assessing the extent of the default protection provided by an excess servicing spread account.

**Overcollateralization** occurs when the ABS is issued with a face value less than the value of the underlying collateral. For example, if the liability structure is $100 million and the collateral's value is $105 million, the issue is overcollateralized by $5 million. The overcollateralization can be used to absorb losses.

A senior/subordinated structure contains at least two tranches—a senior tranche and a junior, or subordinated, tranche. The subordinated tranches absorb the first losses up to their limits. The level of protection for the senior tranches increases with the percentage of subordinated bonds in the structure.

Let’s look at an example to illustrate how overcollateralization and a senior/subordinated structure enhance the creditworthiness of an ABS. Consider the following ABS structure:

<table>
<thead>
<tr>
<th>Tranche</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior tranche</td>
<td>$300,000,000</td>
</tr>
<tr>
<td>Subordinated tranche A</td>
<td>$80,000,000</td>
</tr>
<tr>
<td>Subordinated tranche B</td>
<td>$30,000,000</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$410,000,000</strong></td>
</tr>
</tbody>
</table>

The collateral value for the structure is $450,000,000, and Tranche B is first to absorb losses (the first loss tranche). The amount of overcollateralization for this structure is the difference between the value of the collateral and the combined value of all the tranches. That is:

\[
\text{overcollateralization} = \$450,000,000 - \$410,000,000 = \$40,000,000
\]

This means that losses up to $40 million will be absorbed by the overcollateralization, and none of the tranches will experience a loss. Losses between $40 million and $70 million will be absorbed by Tranche B. Losses between $70 million and $150 million will be absorbed by Tranche A. Losses greater than $150 million will be absorbed by the senior tranche.
The shifting interest mechanism is a mechanism for addressing the change in the level of credit protection provided by junior tranches as prepayments occur in a senior/subordinated structure. The key point to remember is that the shifting interest mechanism reduces the credit risk of the senior tranches, but the trade-off is greater prepayment risk.

Junior tranches are designed to provide loss protection for senior tranches by absorbing prepayments first. The percentage share of the junior or subordinate tranches to the total outstanding balance is called as the subordinate interest. As the principal on junior tranches is reduced by prepayments, subordinate interest declines, and the level of protection for senior tranches is reduced. In other words, the subordinate interest shifts, hence, the term “shifting interest.” To maintain subordinate interest at a desirable level, prepayments are allocated among the senior tranches at a relatively higher proportion in the early years.

The bond prospectus contains the schedule for the shifting interest percentage required to calculate the senior prepayment percentage (i.e., the proportion of prepayments that are applied to the senior tranche). A commonly used shifting interest schedule is shown in Figure 3.

**Figure 3: Example of Senior Prepayment Percentage Schedule**

<table>
<thead>
<tr>
<th>Years After Issuance</th>
<th>Senior Prepayment Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>70</td>
</tr>
<tr>
<td>7</td>
<td>60</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>after 9</td>
<td>0</td>
</tr>
</tbody>
</table>

Suppose, for example, that if prepayments of $80,000 occur in the 46th month (the fourth year), all of the prepayments will be allocated to the senior tranches. If there are prepayments of $80,000 in the seventh year, 60%, or $48,000, will be allocated to the senior tranches, and 40%, or $32,000, will go to the subordinated tranches. If there are prepayments of $80,000 in the tenth year, all of the prepayments will be allocated to the subordinated tranches.

Keep in mind that the shifting interest schedule that is stated in the prospectus is not fixed. The issue’s trustees may change the initial schedule if credit losses cause the credit risk of the senior tranches to increase. The decision to change the shifting interest schedule is based on performance tests that are specified in the prospectus.

It is important to realize that while the shifting interest mechanism can be effectively used to maintain the desired level of credit risk protection, it comes at the expense of increased contraction risk for the senior tranches. This is a result of the increase in
the proportion of the prepayments allocated to the senior tranches. Shifting interest mechanism is commonly used for real-estate-related ABS and non-agency MBS.

**LOS 58.e:** Describe the cash flow and prepayment characteristics for securities backed by home equity loans, manufactured housing loans, automobile loans, student loans, SBA loans, and credit card receivables.

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**HOME EQUITY LOANS**

As the name implies, a home equity loan (HEL) is a loan backed by residential property. HEL used to be a second lien on a property with an existing first lien. Lately, a HEL is frequently a first lien on property owned by a borrower that has a marginal credit history or a loan that does not meet agency requirements for a qualified loan. HELs are also commonly used to consolidate consumer debt.

There are two basic types of HELs: closed-end and open-end. We will focus on closed-end HELs, which are structured just like a standard fixed-rate, fully amortizing mortgage. As such, a closed-end HEL is a one-time lump sum loan with a fixed maturity and a payment structure such that the loan is fully amortized at maturity. An ABS issued on a pool of closed-end HELs is very similar to a standard mortgage-backed security (MBS).

In the absence of tranching, each HEL-backed certificate holder receives a proportional share of the principal and interest paid on the underlying HELs. Thus, a prepayment model, such as the conditional prepayment rate model used for an MBS, must be employed to estimate cash flows to the HEL pool.

**Prepayments**

The pattern of prepayments from HELs differs from MBS prepayment patterns, primarily because of differences in the credit traits of the borrowers. Therefore, analysts must consider the credit of the borrowers when analyzing HEL-backed securities.

In the prospectus for an HEL-backed issue, an assumption is made regarding the initial speed of prepayments and time until the issue is expected to become seasoned (e.g., the point where prepayments stabilize). This is known as the *base case prepayment assumption*. The benchmark speed stated in the prospectus for HEL-backed securities is called the *prospectus prepayment curve* (PPC). PPCs are used in a manner similar to the Public Securities Association (PSA) curves. If the speed of seasoning for an issue is faster or slower than that stated in the prospectus, an analyst would employ a multiplication factor to adjust the PPC to reflect the actual seasoning behavior. As with the PSA benchmark, the speed of prepayment measured using a PPC benchmark is measured as the *conditional prepayment rate* (CPR). Unlike the PSA benchmark, however, the PPC is not generic; it is an issuer-specific benchmark for prepayment rates associated with HELs.

**Payment Structure**

HEL-backed securities that are collateralized with variable rate HELs are known as HEL floaters. While the individual variable rate HELs commonly use the 6-month London
Interbank Offered Rate (LIBOR) as their reference rate, 1-month LIBOR is used as the reference rate for HEL floaters. This practice is a response to investor preferences. The mismatch between the reference rates for HEL floaters and the variable rate HELs used to collateralize them may result in cash shortfalls over time. As a consequence of this potential shortfall and the periodic and lifetime caps on the underlying variable rate loans, HEL floaters must have coupon rate caps. Unlike most floating-rate securities, which have fixed caps over the term of the security, the effective periodic and lifetime caps for HEL floaters are variable. This effective cap is called the available funds cap. Its level is determined on the basis of net coupon-generated funds, less all applicable fees. You can think of the available funds cap as a variable cap on interest rate adjustments in the HEL floaters.

HEL structures frequently include non-accelerating senior tranches and planned amortization class (PAC) tranches. The characteristics of these tranches include:

- **Non-accelerating senior tranches** (NAS). An NAS tranche receives principal payments on the basis of a predetermined schedule. This is a schedule of principal payments that shows the proportion of principal that is to be distributed to the NAS tranche for a given month. Typically, the schedule is such that an NAS tranche receives no prepayments in the early years—its share is paid to the other senior tranches. This structure reduces contraction risk for the NAS tranche(s). In the latter years, an NAS tranche receives a relatively high percentage of prepayments, thus reducing extension risk.

- **PAC tranche.** The prepayments to PACs are stable if prepayments fall within the specified PAC collar. The concept of a PAC as it applies to HEL-backed securities is similar to that of standard MBSs.

**MANUFACTURED HOUSING-BACKED SECURITIES**

As the name implies, manufactured housing-backed securities are backed by loans for manufactured homes (e.g., mobile homes). These loans are similar to standard mortgage loans in that they fully amortize over the life of the loan. Ginnie Mae, as well as private organizations, issues manufactured housing-backed securities.

**Cash Flow and Prepayments**

The *cash flows* for manufactured housing-backed securities include interest, scheduled principal payments, and prepayments, much like residential mortgage loans and HELs. Prepayments are also typically measured in terms of a conditional prepayment rate. In contrast to MBSs and HEL-backed assets, however, prepayments for manufactured housing-backed securities are less significant, because the underlying loans are not as sensitive to refinancing, because:

- Loan balances are usually small, reducing the extent of savings resulting from refinancing.
- The depreciation of mobile homes during the earlier years may be greater than the reduction of the loan principal, resulting in the value of the asset being less than the outstanding loan amount.
- Borrowers are likely to have relatively low credit ratings, which makes it difficult for them to refinance.
Payment Structure

The payment structure for manufactured housing-backed loans is like that of nonagency MBS and HEL-backed securities. Each issue is divided into different classes, each with a different claim against the cash flow components of the underlying collateral pool.

AUTO LOAN ABS

Auto loan-backed securities are backed by loans for automobiles. Auto loans have maturities from 36 to 72 months. Issuers include the financial subsidiaries of auto manufacturers, commercial banks, credit unions, S&Ls, finance companies, and other small financial institutions.

The cash flow components of auto loan-backed securities include scheduled monthly interest and principal payments and prepayments. Auto loans prepay if the cars are sold, traded in, or repossessed. Prepayments also occur if the car is stolen or wrecked and the loan is paid off from insurance proceeds. Finally, the borrower may simply use excess cash to reduce or pay off the loan balance.

Refinancing, however, is not a major factor contributing to auto loan prepayments because:

- Loan balances are usually small, reducing the extent of savings resulting from refinancing, especially because used-car refinancing rates are significantly higher than new car rates. (How many people do you know who refinance their cars when rates drop?)
- The automobile's value may depreciate faster than the loan balance in the early years, resulting in the value of the asset being less than the outstanding loan amount, particularly if the loans were originally done at below-market rates because of sales promotions.

That means prepayments from a pool of auto-loans are much more predictable and much less dependent on interest rate changes than prepayments on mortgage loans. This significantly reduces the prepayment risk of auto-loan ABS.

Absolute prepayment speed (ABS) is the measure of prepayments associated with securities backed by auto loans. It is calculated as the monthly prepayment expressed as a percentage of the value of the initial collateral.

Professor’s Note: The absolute prepayment speed is denoted as “ABS,” which is confusing since “ABS” is also used for “asset-backed security.” You will have to examine the context of the question to determine which “ABS” is being discussed if you see it on the exam.
The absolute prepayment speed is conceptually similar to the CPR discussed in the previous topic review. The relationship between the absolute prepayment speed and the single monthly mortality rate (SMM) is shown in the following equation.

\[ SMM = \frac{\text{ABS}}{1 - [\text{ABS} \times (m - 1)]} \]

where:
- \( \text{ABS} \) = absolute prepayment speed
- \( m \) = number of months since loan origination

Example: SMM calculation

If the absolute prepayment speed ten months after origination is 1.8\% (0.018), compute the SMM.

Answer:

\[ SMM = \frac{0.018}{1 - [0.018 \times (10 - 1)]} = 0.0215 = 2.15\% \]

**STUDENT LOAN-BACKED SECURITIES**

Student loan asset-backed securities (SLABS) have structural features similar to the other asset-backed securities we have discussed. The student loans that are most often securitized are those made by lending institutions under the U.S. Government’s Federal Family Education Loan Program (FFELP). Under FFELP, the U.S. government guarantees loans made by private lenders to students. If an FFELP loan goes into default, the U.S. government guarantees up to 98\% of the loan principal and accrued interest on the condition that the loan has been serviced properly. Student loans that are not part of the FFELP program (known as alternative loans) have been securitized but are not guaranteed by the U.S. Government.

The cash flows associated with SLABSs occur during three periods:

1. The deferment period, when the borrower makes no payments and the loan accrues no interest.
2. The grace period, when the borrower makes no payments, but interest does accrue.
3. The loan repayment period, when the borrower makes principal and interest payments based on a reference rate plus a margin.

Prepayments may occur because of defaults (inflows from the Government guarantee process) or loan consolidation.
SMALL BUSINESS ADMINISTRATION LOAN-BACKED SECURITIES

The Small Business Administration (SBA) is a U.S. government agency that guarantees loans made by private lenders to borrowers who meet specified guidelines. The private lenders must be approved by the SBA for their loans to be eligible for the guarantee. Because the SBA is a U.S. government agency, its guarantees are backed by the full faith and credit of the U.S. government.

Pooled SBA loans must have similar terms and features. Most SBA loans are variable-rate loans, based on the prime rate, and are reset either monthly or quarterly. The monthly payment for individual variable-rate SBA loans includes an interest component and a repayment-of-principal component. Level amortizing loan payments are calculated based on a reference rate at the beginning of each reset period.

An SBA-backed security investor receives the following cash flows:
- Interest based on the coupon rate set at the beginning of the reset period.
- The principal repayment that is based on the amortization schedule developed at the time of loan origination.
- Prepayments received by the lender that are applied to the outstanding loan.

CREDIT CARD RECEIVABLE-BACKED SECURITIES

Credit card receivable-backed securities are ABS backed by the pools of receivables owed to banks, retailers, travel and entertainment companies, and other credit card issuers.

Credit card receivable-backed securities use a structure that enables the issuer to sell more than one series from the same pool of receivables. This means more receivables are added each time a new series is issued. Thus, the balance may never be reduced to zero.

The cash flow to a pool of credit card receivables includes finance charges, annual fees, and principal repayments. Credit cards have periodic payment schedules, but because their balances are revolving, the principal is not amortized. Because of this characteristic, interest on credit card ABS is paid periodically, but no principal is paid to the ABS holders during the lockout period, which may last from 18 months to 10 years.

If the underlying credit card holders make principal payments during the lockout period, these payments are used to purchase additional underlying assets or receivables, keeping the overall value of the receivables pool relatively constant. Once the lockout period ends, principal payments are passed on to the security holders. This post-lockout period is known as the principal amortization period.

The distribution of payments usually follows one of three amortization payment structures:
- Passthrough structure. Principal payments received from credit card holders are distributed pro rata to investors.
- **Controlled-amortization structure.** This structure relies on the mechanism of a “principal window” similar to a PAC bond. To protect against the impact of cash shortfalls due to inadequate principal repayments or slow payments by cardholders, the ABS is designed with a relatively low principal payment in the schedule. If there is cash flow shortfall, the ABS investor receives the lower of the scheduled principal payment or a pro rata portion of the principal repayment.

- **Bullet-payment structure.** As “bullet” implies, investors receive the total principal amount in a single payment. The uncertain nature of principal payments by credit card holders, however, allows for no guarantee that the total principal amount will be available when the bullet payment is due. To overcome this problem, a soft bullet structure is often used. Under this structure, the ABS trustees place the monthly principal payments in an interest-bearing account that is expected to generate enough interest over time to fund the payment of the bullet. The period over which interest is earned, called the “accumulation period,” usually begins a few months prior to the scheduled bullet payment. Despite the lack of a guaranteed maturity, the scheduled bullet payment is seldom missed in practice.

Several metrics are typically used to assess the performance of the credit card receivable portfolio and the issuer’s ability to make the interest and principal payments required by the various tranches in a credit card ABS, as shown in Figure 4.

**Figure 4: Assessing the Performance of the Receivables Portfolio**

<table>
<thead>
<tr>
<th>Performance Measure</th>
<th>Definition</th>
<th>Warning Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net portfolio yield</td>
<td>Gross portfolio yield minus charge offs.</td>
<td>If the weighted average coupon promised to the ABS tranches is greater than the net portfolio yield, there is risk that tranches will not get paid off as promised.</td>
</tr>
<tr>
<td>Delinquencies</td>
<td>Percentage of past due receivables.</td>
<td>High delinquencies signal potential future charge offs and lower net portfolio yield.</td>
</tr>
</tbody>
</table>
| Monthly repayment rate (MPR) | Monthly payments (interest, fees, principal) as percent of outstanding receivables at previous month end. | Low MPR signals:
  - increased extension risk of the ABS tranches and,
  - insufficient cash flow to pay off tranches. |

Gross portfolio yield equals finance charges and fees collected as percent of outstanding receivables. Charge offs equal percent of uncollectible accounts that are charged off.

Early amortization of the principal in credit card receivable-backed securities can be triggered by certain events. This early amortization trigger protects the investor against declines in the credit quality of the underlying receivables. The most common trigger is when the 3-month average excess spread earned on the receivables declines to zero. Hence, even though there is no principal repayment schedule for credit card borrowers,
an early amortization trigger provision creates the potential for contraction risk in a receivables-backed structure.

LOS 58.f: Describe collateralized debt obligations (CDOs), including cash and synthetic CDOs.

COLLATERALIZED DEBT OBLIGATIONS

A collateralized debt obligation (CDO) is an ABS that is collateralized by a pool of debt obligations. Examples include:

- Corporate bonds with ratings below investment grade.
- MBS and ABS (called structured financial products).
- Bond issues in emerging markets.
- Corporate loans advanced by commercial banks.
- Special situation loans and distressed debt.

A CDO has the following structure:

- One or more senior tranches.
- Several levels of mezzanine tranches.
- A subordinate tranche, also known as the equity tranche, to provide prepayment and credit protection to the other tranches.

The senior tranche, which typically comprises about 70% to 80% of the entire deal, is assigned a floating-rate payment to attract investors who are looking for a floating-rate investment. The mezzanine tranches are assigned a fixed-coupon payment. As with other ABS, payments received from the collateral are paid to the security holders associated with the different tranches.

A CDO’s collateral pool typically contains a mix of floating-rate and fixed-rate debt instruments. However, payments made to a majority of the tranche holders (the senior tranche holders) are based on a floating rate. This creates a potential cash flow mismatch. In order to control for the interest rate risk imposed by this mismatch, asset managers often use interest rate swaps. Interest rate swaps are derivative instruments that can be used to convert fixed-rate interest receipts into floating-rate payments. The inclusion of swaps in a CDO deal is almost always mandated by the rating agencies.

Cash Flow CDO

The objective of a cash flow CDO is for the portfolio manager to generate sufficient cash flow (from interest and principal payments) to repay the senior and mezzanine tranches. A cash flow CDO has three phases:

1. Ramp up phase. In this phase, which usually lasts one to two months, the portfolio manager puts together a portfolio financed with the help of the sale of different tranches to investors.
2. **Reinvestment phase.** After the portfolio has been assembled, the asset manager monitors its performance and reinvests prepayments and cash flows from calls and loan default recoveries.

3. **Pay down phase.** During this phase, which may last from three to five years, principal payments are made to junior and senior tranche holders. As the name implies, this is a winding-down period of the CDO.

Income from the portfolio is used first to pay administration and management fees, then to pay interest on the senior tranches. If certain coverage tests are met, interest is paid to the mezzanine tranches, and any remaining cash flow is paid to the equity tranche. If the coverage tests are not met, the cash flow is used to retire senior tranche principal until the coverage tests are met. Coverage tests include par value tests and interest coverage tests.

During the reinvestment period, principal proceeds from the portfolio are reinvested in new securities (assuming the coverage tests are met). After the reinvestment period, principal proceeds are used first to pay down the senior tranches, then the mezzanine tranches, and finally the equity tranche.

The portfolio manager actively manages the portfolio, but does not try to generate trading profits to meet the cash flow obligations of the tranches. Instead, the manager structures the portfolio so that the interest and principal repayments are sufficient to meet the cash flow obligations, and rebalances as necessary.

**Market Value CDO**

In a market value CDO the manager actively manages the portfolio and sells assets to generate cash flows to meet the CDO tranches’ obligations. Because of this, the manager of a market value CDO has more flexibility than the manager of a cash flow CDO.

**Synthetic CDO**

In a synthetic CDO, the bondholders take on the economic risks of the underlying assets but do not take legal ownership of them. This is accomplished by linking certain contingent payments to a reference asset (e.g., a bond index, or a portfolio of loans held by a bank).

The CDO is divided into a senior “section” and a junior “section”; debt obligations are only issued to fund the junior section, in the same manner as a cash CDO, but no securities are issued to fund the senior section. The junior section absorbs losses up to a certain level before the senior section is forced to absorb any losses. The proceeds from the junior section are invested by the portfolio manager in high-quality debt securities.

For example, a synthetic arbitrage CDO might have a notional amount of credit exposure to the reference asset of $100 million. The $100 million is referred to as notional exposure because it is not fully funded. The senior section has $90 million in exposure and the junior section has $10 million.
The note holders also sell a $90 million credit default swap. In a credit default swap the seller receives a premium in return for the obligation to pay the buyer a specific amount if a credit event occurs on the reference asset (e.g., the issuer of one of the bonds in the bond index declares bankruptcy or fails to pay interest). In the case of a credit event the seller is required to pay the difference between the par value and the fair market value of the bond. The senior note holders receive a small premium in return for the obligation to fund any losses in excess of $10 million. Because the deal is structured so that the chance of losses exceeding $10 million is very small, the senior notes usually carry a AAA rating.

Professor’s Note: Credit default swaps are discussed in LOS 65.a in the Study Session on derivative investments.

The bottom line is that the junior bondholders receive income from the high-quality debt securities in the portfolio, as well as the insurance premium on the credit default swap. However, they are also exposed to credit losses in the reference asset. Therefore, they are in a similar position to junior bondholders in a cash CDO.

The question you’re probably asking yourself is, “Why construct the complicated synthetic CDO when you can just do the simple cash CDO?” There are several advantages to using a synthetic structure instead of a cash structure for an arbitrage CDO:

• The senior section doesn’t require funding.
• The ramp-up period is shorter.
• It is cheaper to acquire an exposure to the reference asset through the credit default swap instead of buying the asset directly.

LOS 58.g: Distinguish among the primary motivations for creating a collateralized debt obligation (arbitrage and balance sheet transactions).

The motivations for creating CDOs fall into two basic categories:

• CDOs can be arbitrage-driven, in which the motivation is to generate an arbitrage return on the spread between return on the collateral and the funding costs.
• CDOs can be balance sheet-driven, in which the motivation is to remove assets (and the associated funding) from the balance sheet. For example, a bank can use a synthetic balance sheet CDO to remove the credit risk of a loan portfolio from its balance sheet and reduce its regulatory capital requirements. The advantage to the bank of using the synthetic structure versus a cash CDO is they don’t need to obtain the consent of the borrowers to move the credit risk off the balance sheet.

Arbitrage-driven cash CDOs make up the majority of cash CDO deals, so an example of a typical transaction is presented next.

Arbitrage-Driven Cash CDO

Although this LOS asks you to “describe,” but not “construct,” this is an excellent tie-in to the swap material coming up in Study Session 17. Therefore, let’s examine how an equity tranche gets paid in an arbitrage-driven CDO deal.
Example: CDO cash flows

Assume the following:

- The CDO is a $200 million structure—the collateral will have an initial value of $200 million.
- The collateral consists entirely of bonds with 15 years remaining until maturity and a coupon rate equal to the 15-year Treasury rate plus 350 basis points.
- The senior tranche represents $150 million (75% of the structure) and carries a floating-coupon rate equal to LIBOR plus 150 basis points. There is one $20 million mezzanine tranche, and it carries a fixed coupon equal to the Treasury rate at origination plus 175 basis points.
- The manager of the trust has entered into an interest rate swap under which the trust will pay an annual fixed rate equal to the Treasury rate plus 125 basis points and receive LIBOR. The notional amount for this swap is $150 million.
- The 15-year Treasury rate is 7.5% at the time of origination for this CDO.

Calculate the interest received by the CDO from the collateral and the swap counterparty; the total interest paid by the CDO to the senior and mezzanine tranches and the swap counterparty; and the net cash flow to the equity tranche.

Answer:

Because the senior tranche and mezzanine tranche are initially valued at $150 million and $20 million, respectively, the equity tranche has an initial par amount of $30 million.

Let's look at the various cash flows.

- Assuming no defaults, the collateral will pay annual interest equal to the Treasury rate plus 350 basis points. Because the Treasury rate is 7.5%, the collateral will pay:

  \[
  \text{interest from collateral} = (0.075 + 0.035) \times 200,000,000 = 22,000,000
  \]

- The senior tranche will receive LIBOR plus 150 basis points. This amounts to:

  \[
  \text{interest to senior tranche} = 150,000,000 \times (\text{LIBOR} + 0.015)
  \]

- The mezzanine tranche will receive the Treasury rate plus 175 basis points, fixed over the term of the issue. This amounts to:

  \[
  \begin{align*}
  \text{interest to mezzanine tranche} & = 20,000,000 \times (0.075 + 0.0175) \\
  & = 1,850,000
  \end{align*}
  \]
Now let’s look at the swap. The amount that must be paid to the counterparty in the swap agreement is the Treasury rate (assumed here to be 7.5%) plus 125 basis points, or 8.75%. The dollar amount of this payment is based on the notional amount of the swap. In this case, the notional amount of the swap equals the par value of the senior tranche, or $150 million. So, the trust must pay the following:

interest to swap counterparty: $0.0875 \times 150,000,000 = 13,125,000$

Similarly, the floating rate that the trust will receive under the swap agreement is:

interest from swap counterparty = $150,000,000 \times \text{LIBOR}$

Putting it all together we have:

**Interest received by the CDO:**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest from collateral</td>
<td>$22,000,000</td>
</tr>
<tr>
<td>Interest from swap counterparty</td>
<td>150,000,000 x LIBOR</td>
</tr>
<tr>
<td><strong>Total incoming interest</strong></td>
<td><strong>$22,000,000 + (150,000,000 \times \text{LIBOR})</strong></td>
</tr>
</tbody>
</table>

**Interest paid to senior and mezzanine tranches:**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest to senior tranche</td>
<td>150,000,000 x (LIBOR + 0.015)</td>
</tr>
<tr>
<td>Interest to mezzanine tranche</td>
<td>1,850,000</td>
</tr>
<tr>
<td>Interest to swap counterparty</td>
<td>$13,125,000</td>
</tr>
<tr>
<td><strong>Total interest paid</strong></td>
<td><strong>$14,975,000 + 150,000,000 \times (LIBOR + 0.015)</strong></td>
</tr>
</tbody>
</table>

**Netting payment inflows and outflows we have:**

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total interest received</td>
<td>$22,000,000 + (150,000,000 \times \text{LIBOR})</td>
</tr>
<tr>
<td>– Total interest paid</td>
<td>$14,975,000 + 150,000,000 \times (LIBOR + 0.015)</td>
</tr>
<tr>
<td><strong>Net interest</strong></td>
<td><strong>$7,025,000 – 150,000,000 \times 0.015</strong></td>
</tr>
<tr>
<td></td>
<td>= $7,025,000 – 2,250,000</td>
</tr>
<tr>
<td></td>
<td>= $4,775,000</td>
</tr>
</tbody>
</table>

So, the cash flow each year available to pay the equity tranche is $4,775,000, less management and other fees (and assuming no defaults). The cash flow risk from the interest rate mismatch has been eliminated, and the structure has created an equity tranche with a guaranteed return of $4,775,000/$30,000,000 = 15.9%, which is an arbitrage profit.
Key Concepts

LOS 58.a
The key parties to a securitization transaction are the:
• Seller, who originates the loans and sells them to the issuer/trust.
• Issuer/trust, who buys the loans from the seller and issues the ABS.
• Servicer, who services the original loans.

The flow of funds structure in a securitization transaction is known as the waterfall.

LOS 58.b
ABS structures are divided into different tranches to distribute the prepayment risk to various investors using, for example, sequential-pay or PAC structures; this is called prepayment tranching or time tranching. In a senior-subordinated structure, the subordinated bonds absorb losses first up to their par value, after which losses are absorbed by the senior bonds. The result is to transfer some of the credit risk from the senior bonds to the subordinated bonds. This type of structure is called credit tranching.

LOS 58.c
The structure of the ABS transaction is affected by whether the assets backing the bonds are amortizing or non-amortizing. For amortizing assets like auto loans, once the assets are securitized, the composition of the loans in the pool doesn’t change. Loans disappear from the pool as they are paid off or default, but no new loans are added to the pool to replace them. For non-amortizing assets like credit card receivables, the composition of the loans in the pool can and does change. During the lockout period, cash flow from principal payments is used to invest in new loans to replace the amounts paid off.

LOS 58.d
External credit enhancements are financial guarantees from a third party that are used to supplement other forms of credit enhancements. The third-party guarantee effectively links the ABS to the credit risk of the third-party guarantor. Third-party guarantees include: corporate guarantee, letter of credit, and bond insurance.

Internal credit enhancements are “internal” to the issue—they do not rely on a third-party guarantee. Internal credit enhancements include reserve funds, overcollateralization, and senior/subordinated structure.

LOS 58.e
Closed-end HELs are secondary mortgages that are structured just like a standard fixed-rate, fully amortizing mortgage. The pattern of prepayments from HELs differs from MBS prepayment patterns, primarily because of differences in the credit traits of the borrowers. Therefore, analysts must consider the credit of the borrowers when analyzing HEL-backed securities. HEL floaters have a variable coupon rate cap called the available funds cap. HEL structures frequently include non-accelerating senior tranches and planned amortization class (PAC) tranches.
Manufactured housing ABS are backed by loans for manufactured homes. Prepayments for manufactured housing ABS are relatively stable because the underlying loans are not as sensitive to refinancing for the following reasons:

• Small loan balances reduce the extent of savings resulting from refinancing.
• Initial depreciation of mobile homes may be such that the loan principal exceeds the asset value.
• Borrowers often have relatively low credit ratings, making it difficult to refinance.

Auto loan-backed securities are backed by loans for automobiles. Auto loans have 36- to 72-month maturities and are issued by the financial subsidiaries of auto manufacturers, commercial banks, credit unions, etc. Prepayments for auto loan-backed securities are caused by sales and trade-ins, the repossession/resale process, insurance payoffs due to thefts and accidents, borrower payoffs, and refinancing. Refinancing is of minor importance, because many auto loans are frequently below market rates due to sales promotions.

Student loan ABS are most often securitized by loans made under the U.S. government’s FFELP. Qualifying FFELP loans carry a U.S. government guarantee. Prepayments may occur because of defaults (inflows from the Government guarantee process) or loan consolidation.

SBA loan-backed securities are backed by pools of SBA loans with similar terms and features. Most SBA loans are variable-rate loans, reset quarterly or monthly, and based on the prime rate.

Credit-card receivables ABS are backed by pools of receivables owed by banks, retailers, travel and entertainment companies, and other credit card issuers. The cash flow to a pool of credit card receivables includes finance charges, annual fees, and principal repayments. Credit cards have periodic payment schedules, but because their balances are revolving, the principal is not amortized. Because of this characteristic, interest on credit card ABS is paid periodically, but no principal is paid to the ABS holders during the lockout period, which may last from 18 months to 10 years.

LOS 58.f
CDOs are collateralized by a pool of debt obligations comprised of one or more of the following assets:

• Corporate bonds with ratings below investment grade.
• MBS and ABS.
• Bond issues in emerging markets.
• Corporate loans advanced by commercial banks.
• Special situations and distressed debt.

In a synthetic CDO the bondholders take on the economic risks of the underlying assets but do not take legal ownership of them. This is accomplished by linking certain contingent payments to a reference asset (e.g., a bond index). There are three advantages to a synthetic CDO versus a cash CDO:

• The senior section doesn’t require funding.
• The ramp-up period is shorter.
• It’s cheaper to acquire an exposure to the reference asset through the credit default swap instead of buying the asset directly.
LOS 58.g

The securities that back a cash CDO are cash market debt securities, such as corporate bonds, that were described previously. Cash CDOs can be arbitrage-driven or balance sheet-driven. An arbitrage-driven CDO is created to generate an arbitrage return on the spread between return on the collateral and the funding costs. A balance sheet-driven CDO is created when the lender wants to remove assets (and the associated funding) from its balance sheet.
CONCEPT CHECKERS

1. Which of the following statements is most accurate concerning non-amortizing assets? They do not:
   A. allow principal prepayments.
   B. have scheduled principal payment amounts.
   C. have scheduled interest payments.

2. Which of the following is least likely to be used as an external credit enhancement for an asset-backed security (ABS)?
   A. Crossover agreements.
   B. Corporate guarantees.
   C. Bond insurance.

3. Which of the following is a general problem associated with external credit enhancements? External credit enhancements:
   A. can only be used after internal credit enhancements have been exhausted.
   B. are expensive because of the long-term nature of the agreements.
   C. are subject to the credit risk of the third party guarantor.

4. Daren Lea, JD, is a Level 2 CFA candidate. He recently joined the securitization group of RokStarr Innovative Investments. The firm's two most recent deals were the securitization of a stock of rough and polished diamonds and the securitization of $25 million in loans originated and serviced by First One Financing to finance rock concerts in the United States. As part of the deal structure, RokStarr created a special purpose vehicle called Red Heads Rule. Which of the following choices most accurately identifies the parties to the loan securitization?
   First One  Red Heads Rule
   A. Seller  Servicer
   B. Seller  Issues ABS
   C. Trust  Servicer

5. Rating agencies require interest rate swaps in collateralized debt obligation (CDO) deals because:
   A. cash flows are mismatched.
   B. the subordinated tranche investors have credit risk.
   C. the equity tranche investors have credit risk.

6. Which of the following is not considered to be an external credit enhancement?
   A. Insurer call.
   B. Bond insurance.
   C. Corporate guarantee.

7. If a credit card receivables asset backed security (ABS) has a lock-out feature:
   A. no payments are made to the ABS investor for a certain time period.
   B. no principal payments are made to the ABS investor for a certain time period.
   C. no investors may sell the ABS for a certain time period.
8. Rashid Miller is seeking to purchase an asset-backed security that is backed by automobile loans. However, Miller is extremely concerned about prepayment risk. Which of the following factors should least concern Miller?
   A. Loan refinancing.
   B. Trade-ins.
   C. Insurance payoffs due to thefts or accidents.

9. Which of the following is the least likely reason why an asset-backed security (ABS) generally requires overcollateralization while a mortgage-backed security (MBS) does not?
   A. Some ABS do not have any tangible property as collateral.
   B. Principal recovery in the event of a default is likely to be lower in an ABS.
   C. The interest rates on the underlying assets in an ABS are likely to be lower than the rates on the underlying assets in an MBS.

10. Which of the following is the most likely reason why asset-backed securities (ABS) are often assumed to have a larger degree of default risk than mortgage-backed securities (MBS)?
    A. Most ABS are secured by variable-rate loans while most MBSs are secured by fixed-rate loans.
    B. Most ABS are backed by non-amortizing securities, which have more risk of default than amortizing securities.
    C. MBS are secured by loans on traditional real property, which has greater relative stability in value.

11. Which of the following statements concerning arbitrage-driven cash collateralized debt obligations (CDO) is least accurate?
    A. CDOs pool only fixed-rate bonds but issue both fixed- and floating-rate tranches from the pool.
    B. Rating agencies typically require the CDO to have an interest rate swap.
    C. The senior tranche in the CDO is generally about 70%–80% of the deal.

**CHALLENGE PROBLEMS**

12. Consider the following asset backed security (ABS) structure:
   
<table>
<thead>
<tr>
<th>Structure</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior tranche</td>
<td>$150,000,000</td>
</tr>
<tr>
<td>Subordinated tranche A</td>
<td>$60,000,000</td>
</tr>
<tr>
<td>Subordinated tranche B</td>
<td>$20,000,000</td>
</tr>
<tr>
<td>Total</td>
<td>$230,000,000</td>
</tr>
</tbody>
</table>

   If the assets in the pool are worth $250,000,000, what is the amount of overcollateralization and at what amount of losses will senior tranche investors begin to lose money?

<table>
<thead>
<tr>
<th>Overcollateralization</th>
<th>Senior tranche investors’ losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $20,000,000</td>
<td>$100,000,000</td>
</tr>
<tr>
<td>B. $40,000,000</td>
<td>$80,000,000</td>
</tr>
<tr>
<td>C. $20,000,000</td>
<td>$80,000,000</td>
</tr>
</tbody>
</table>
13. Marg Kingston and Albert Loo, both Level 2 CFA candidates, are discussing asset-backed securities. Kingston states that prepayment risk is more of a concern for an investor in a traditional mortgage-backed security than an auto loan asset-backed security (auto loan ABS) because auto loans typically have smaller loan balances and greater depreciation than mortgages. Loo adds that there are two other reasons—the underlying assets are less liquid in an auto loan ABS and often the loans are initially made at below-market rates as part of sales promotions. With regard to the overall statements made by Loo and Kingston, these are most likely:
A. both correct.
B. correct in one instance, but incorrect in the other.
C. both incorrect.

14. Angelique Uttaro, CFA, is reviewing a proposal for a collateralized debt obligation (CDO) from Pilot Investors. The CDO will be collateralized by a pool of emerging market bonds. The report offers two alternatives: a simple cash CDO and a synthetic CDO. The report contains the following statements:

1. In the synthetic CDO, the junior bondholders receive income from the high-quality debt securities in the portfolio and pay the insurance premium on a credit default swap.

2. Disadvantages of a cash CDO include a longer ramp-up period and the need to fund the senior section.

These statements are:
A. both correct.
B. correct in one instance, but incorrect in the other.
C. both incorrect.

15. Consider the following scenarios regarding parties analyzing the potential use of collateralized debt obligations (CDOs).

- Half-Pass Investments structures a deal to add value by repackaging bonds into tranches. Half-Pass plans to capture for equity investors the spread between relatively high yielding assets and lower yielding liabilities.
- Piaffe First Bank recently acquired Pirouette Financial. Adding Pirouette's portfolio of loans will result in Piaffe's not being in compliance with internal asset composition targets because its concentration of sub-prime loans will be too high.
- Canter Consulting has been asked to advise a U.S. commercial bank on ways to reduce the risk-based capital requirement for the commercial loan portfolio. Currently, the bank must reserve 100% capital against the loan balances.
- Renvers Holdings plans to put together a CDO that it believes can generate a profit from the spread between the return on the collateral and the funding costs.

Which of the choices below most accurately reflects the motivations for the parties in the previous scenarios?

<table>
<thead>
<tr>
<th>Arbitrage-driven CDO</th>
<th>Balance sheet-driven CDO</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Piaffe &amp; Canter</td>
<td>Half-Pass &amp; Renvers</td>
</tr>
<tr>
<td>B. Piaffe &amp; Renvers</td>
<td>Half-Pass &amp; Canter</td>
</tr>
<tr>
<td>C. Half-Pass &amp; Renvers</td>
<td>Piaffe &amp; Canter</td>
</tr>
</tbody>
</table>

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ANSWERS – CONCEPT CHECKERS

1. B Non-amortizing assets do not have scheduled principal payment amounts. Principal prepayments and curtailments are allowed, and regular interest payments based on the unpaid principal are required.

2. A Crossover agreements are not a type of external credit enhancement. Corporate guarantees, letters of credit, and bond insurance are all examples of external credit enhancement.

3. C A general problem with external credit enhancements is that they are subject to the credit risk of the third party guarantor.

4. B First One is the seller – it originates the loans and sells them to Red Heads Rule, which is the issuer/trust for the ABS.

5. A Bond rating agencies require interest rate swaps because the cash inflows from floating-or fixed-rate assets are mismatched with the cash outflows to floating-rate tranches.

6. A An insurer call is a type of call provision. The other choices are external credit enhancements to an ABS.

7. B During the lock-out period on a credit card receivables backed ABS, no principal payments are made to the ABS investor for a certain period of time.

8. A Refinancing of automobile loans is a low probability event due to the short maturity of the loans and the fact that the loans are frequently set below market rates due to promotional events by the manufacturers. The other items listed could all cause prepayments on auto loan-backed securities.

9. C The interest rates on the underlying assets for some ABSs may be lower than for MBSs (e.g., auto loans), and some rates on underlying assets will be higher (e.g., credit card receivables). In any case, this is not a reason for overcollateralization of ABS.

10. C Mortgage backed securities are secured by loans on traditional real property. In the event of a default on a given mortgage loan, it is unlikely that a substantial loss will be incurred because real property is relatively stable in value.

11. A CDOs pool both fixed- and floating-rate assets.

ANSWERS – CHALLENGE PROBLEMS

12. A The overcollateralization in the pool is the difference between the amount of the assets and the claims against the pool: $250,000,000 - $230,000,000 = $20,000,000.

Senior tranche investors begin to lose when the overcollateralization is gone and when the subordinated A and B tranches have defaulted, so losses must be $20,000,000 + $20,000,000 + $60,000,000 = $100,000,000 before the senior tranche suffers any losses.
13. B Loo is incorrect and Kingston is correct. The market for cars is probably more liquid than the market for houses. In any case, it is not a major factor contributing to auto loan prepayments. Prepayments for auto-loan ABS are less frequent because the underlying loans are not as sensitive to refinancing. This is because loan balances are usually small, reducing the extent of savings resulting from refinancing, especially if used car refinancing rates are significantly higher than new car rates.

The auto’s value may depreciate faster than the loan balance in the early years, resulting in the value of the asset being less than the outstanding loan amount, particularly if the loans are done at below-market rates because of sales promotions.

14. B Statement 1 is incorrect; junior bondholders receive income from the insurance premium on the credit default swap. Statement 2 is correct.

15. C Both Half-Pass and Renvers are in situations in which there is a motivation to create an arbitrage-driven CDO, where the motivation is to generate an arbitrage return on the spread between return on the collateral and funding costs. Piaffe and Canter are both in situations that lend themselves to a balance-sheet driven CDO, where the motivation is to remove assets (and the associated funding) from the balance sheet.
The following is a review of the Fixed Income: Structured Securities principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

**VALUING MORTGAGE-BACKED AND ASSET-BACKED SECURITIES**

**Exam Focus**

This topic review is the Level 2 fixed-income grand finale. It brings together concepts from the prior fixed-income material, where we discussed mortgage-backed securities (MBS) and asset-backed securities (ABS) and illustrated how cash flows could be distributed to different tranches to alter exposure to prepayment and credit risk. Here we address the important issue of how to value these securities and quantify their exposure to interest rate risk. Make certain you (1) understand the difference between nominal spreads, Z-spreads, and option-adjusted spread (OAS), (2) are able to apply OAS analysis to value MBS and ABS, (3) can determine which model is appropriate for valuing any specific type of fixed-income security, and (4) can analyze the interest rate risk of MBS and ABS.

**LOS 59.a: Illustrate the computation, use, and limitations of the cash flow yield, nominal spread, and zero-volatility spread for a mortgage-backed security and an asset-backed security.**

**Cash Flow Yield and Nominal Spread**

The cash flow yield is the discount rate that makes the price of a mortgage-backed security (MBS) or asset-backed security (ABS) equal to the present value of its cash flows. To compute the cash flow yield:

- Estimate the future monthly cash flows.
- Calculate the monthly rate of return that makes the present value of these future cash flows equal to the security’s current market price.

The monthly cash flow yield is usually converted to a bond-equivalent basis for comparison to yield-to-maturity:

\[
\text{bond-equivalent yield} = 2\left\{\left[1 + \text{monthly cash flow yield}\right]^6 - 1\right\}
\]

The challenge in applying this concept is that the cash flows from the MBS or ABS are uncertain because we don’t know what future prepayment rates will be. In order to compute a cash flow yield for an MBS and ABS, we must make a prepayment assumption. Furthermore, if the security is not an agency issue, we also need assumptions about default and recovery rates.
The cash flow yield has three major deficiencies. When we use cash flow yield as our 
estimate of the bond’s expected return, we assume:

1. The cash flows will be reinvested at the cash flow yield prevailing when the MBS or 
ABS is priced. In past topic reviews, we’ve called this reinvestment risk.

2. The MBS or ABS will be held until maturity. If the security is sold prior to maturity, 
uncertainty is introduced regarding terminal cash flows. This is called price risk.

3. The cash flows will be realized as expected. This assumption is less appropriate for 
MBS and ABS than many other fixed-income securities because of the risk of prepay¬
ment.

Nominal spread is the difference between the cash flow yield on an MBS and the YTM 
on a Treasury security with a maturity equal to the average life of the MBS. A portion of 
the nominal spread represents compensation to the investor for exposure to prepayment 
risk.

The limitation of using nominal spread to analyze MBS is that we don’t know how much 
of the nominal spread reflects the significant prepayment risk associated with MBS. This is 
particularly true for support (companion) collateralized mortgage obligation (CMO) 
tranches.

Let’s look at an application of nominal spread and explore its limitations. Suppose the 
yield curve for U.S. Treasury bonds (T-bonds) is as shown in Figure 1.

**Figure 1: U.S. Treasury Yields**

<table>
<thead>
<tr>
<th>Maturity</th>
<th>5-year</th>
<th>7-year</th>
<th>9-year</th>
<th>12-year</th>
<th>20-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>6.0%</td>
<td>6.2%</td>
<td>6.5%</td>
<td>6.9%</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

Now consider a Ginnie Mae passthrough certificate with a stated maturity of 20 years 
and an average life of 12 years. Assume that the bond equivalent cash flow yield for this 
MBS is 7.75% based on a prepayment assumption of 150 Public Securities Association 
(PSA). Suppose a 12-year AA corporate bond with a YTM of 7.50% is also available.

The nominal spread for an MBS is traditionally measured relative to a Treasury security 
with a maturity equal to the average life of the MBS:

\[
7.75\% - 6.9\% = 85 \text{ basis points}
\]

The nominal spread for the corporate is also computed relative to the 12-year Treasury:

\[
7.50\% - 6.9\% = 60 \text{ basis points}
\]

We can’t necessarily conclude from this relative value analysis that the MBS is a better 
investment, despite the fact that it has a higher spread and slightly lower credit risk than 
the AA bond. The reason is that some or perhaps most of that 85 basis point spread
reflects the significant prepayment risk of the MBS. What we need is a spread measure that explicitly accounts for the prepayment option embedded in MBS.

Zero Volatility Spread

Zero-volatility spread is another commonly used measure of relative value for MBS and ABS. The zero-volatility spread (also known as the Z-spread or static spread) is the spread that must be added to each Treasury spot rate that will cause the discounted value of the cash flows for an MBS or ABS to equal its price, assuming that the security is held until maturity. Note that there is a Treasury spot rate associated with each of these cash flows but only one value for the Z-spread that must be simultaneously added to each of the rates. A computer-assisted, iterative process can be used to determine the zero-volatility spread.

The zero-volatility spread and the nominal spread converge as the average life of the MBS decreases. Also, the difference between the zero-volatility spread and nominal spread increases as the slope of the yield curve increases.

The key limitation of the Z-spread is that it only considers one path of interest rates: the current Treasury spot rate curve. In contrast, the option-adjusted spread, which we discuss next, is added to the spot rates along each and every path in an interest rate tree. The key is to use the option-adjusted spread (OAS) for bonds with embedded options that are sensitive to changes in interest rate volatility (such as MBS).

LOS 59.b: Describe the Monte Carlo simulation model for valuing a mortgage-backed security.

There are five steps in the valuation of an MBS using the Monte Carlo simulation model:

Step 1: Simulate interest rate paths (e.g., 1,000 different paths) and cash flows using assumptions concerning benchmark rates, rate volatility, refinancing spreads, and prepayment rates. Non-agency MBS also require assumptions regarding default and recovery rates.

Step 2: Calculate the present value of the cash flows along each of the 1,000 interest rate paths.

Step 3: Calculate the theoretical value of the MBS as the average of the present values along each path.

Step 4: Calculate the OAS as the spread that makes the theoretical value equal to the market price.

Step 5: Calculate the option cost as the zero-volatility spread minus the OAS.

LOS 59.c: Describe path dependency in pass-through securities and the implications for valuation models.

Due to the path dependence of MBS cash flows, the Monte Carlo simulation technique is used to value these securities instead of the binomial model. Recall that when valuing a callable bond using backward induction methodology with the binomial model, the
relevant cash flow to be discounted at any given node is either the call price or the theoretical value, whichever is less. An important assumption of the binomial valuation process is that the value of the cash flows at a given point in time is independent of the path that interest rates followed up to that point. Thus, the decision to use the call price or the theoretical value at a node in the binomial tree is determined by the assumed interest rate at the time the decision must be made, not past interest rates.

In contrast to the typical callable bond, the cash flows for MBS are dependent upon the path that interest rates follow and, therefore, cannot be properly valued with the binomial model or any other model that employs the backward induction methodology. The cash flows for passthrough securities are a function of prepayment rates, and prepayment rates in any given month are affected by interest rates in the past. There are two sources of path dependency:

1. If mortgage rates trend downward over a period of time, prepayment rates will increase at the beginning of the trend as homeowners refinance their mortgages, but prepayments will slow as the trend continues because many of the homeowners that can refinance will have already done so. This prepayment pattern is called prepayment burnout, and it applies to MBS and other types of passthrough security cash flows as well as CMO tranches.

2. The cash flows that a particular CMO tranche receives in any one month depend on the outstanding principal balances of the other tranches in the structure, which in turn depend on the prepayment history and the interest rate path.

LOS 59.d: Illustrate how the option-adjusted spread is computed using the Monte Carlo simulation model and how this spread measure is interpreted.

The option-adjusted spread (OAS) estimated from a Monte Carlo simulation model is computed using the same general principle as the OAS from a binomial model. We want to determine the spread that makes the MBS value derived from the model (the present value of the projected cash flows) equal to the current market price. This process is a little more complicated with Monte Carlo models because the OAS is the spread that we have to add to every spot rate along every interest rate path.

We can interpret the OAS as the MBS spread after the “optionality” of the cash flows is taken into account. It can be used to express the dollar difference between price and theoretical value as a spread.

We can also use the relationship between the Z-spread and the OAS to estimate the cost of the embedded prepayment option inherent in MBS and ABS. The implied cost of the embedded option can be expressed as:

\[
\text{option cost} = \text{zero-volatility spread} - \text{option-adjusted spread}
\]

Note that here the option cost is derived from OAS analysis as opposed to an option pricing model.
To see how OAS is used in practice, consider a manager who computes the theoretical value of an MBS with the Monte Carlo model using Treasury rates as the benchmark. To compute this theoretical value, the manager must add a “risk-appropriate” spread to the spot rates along the model’s interest rate paths. Recall that the monthly rates along the paths generated with the Monte Carlo simulation model using the Treasury yield curve as a benchmark are Treasury spot rates that have been adjusted to be arbitrage-free. Thus, the OAS measures the average spread over Treasury spot rates, not the comparable Treasury yield.

In general, you want the OAS to be large, all else equal. A wider OAS indicates a larger risk-adjusted spread, which leads to a lower relative price.

In our examples, the benchmark rates used to create the Monte Carlo Model are Treasury rates, so we can interpret the OAS for an MBS as the additional compensation for credit risk, liquidity risk, and modeling risk after the cost of the embedded prepayment option has been removed. We’ve already discussed credit risk and liquidity risk in previous topic reviews, so let’s discuss modeling risk here.

**Modeling risk** is the uncertainty in the MBS value that results from the use of assumptions in the complicated Monte Carlo model framework. The MBS value derived from a Monte Carlo model is very sensitive to the interest rate volatility assumption and the prepayment assumption, for example.

The interpretation of the OAS depends on the security’s credit risk, liquidity risk, and modeling risk relative to the benchmark. The appropriate interpretation of an OAS using Treasury securities as a benchmark for agency and non-agency passthrough and CMO issues is shown in Figure 2.

**Figure 2: Interpretation of OAS Using Treasury Securities as a Benchmark**

<table>
<thead>
<tr>
<th>Security</th>
<th>Credit Risk?</th>
<th>Liquidity Risk?</th>
<th>Modeling Risk?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ginnie Mae passthroughs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Ginnie Mae CMOs</td>
<td>No</td>
<td>Yes (Support tranches have more than PAC I.)</td>
<td>Yes (CMOs have more than passthroughs. CMO support tranches have more than PAC I.)</td>
</tr>
<tr>
<td>Freddie Mac/ Fannie Mae passthroughs</td>
<td>Yes, but small.</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Freddie Mac/ Fannie Mae CMOs</td>
<td>Yes, but small.</td>
<td>Yes (CMO support tranches have more than PAC I.)</td>
<td>Yes (CMOs have more than passthroughs. CMO support tranches have more than PAC I.)</td>
</tr>
<tr>
<td>Non-agency MBS and real estate backed ABS</td>
<td>Yes</td>
<td>Yes, more than agency issues.</td>
<td>Yes</td>
</tr>
</tbody>
</table>
From a statistical perspective, as the number of paths generated by a Monte Carlo model increases, the "better" is the resulting estimate. Vendors, however, often employ computational procedures that reduce the full number of sample paths while maintaining the accuracy of the Monte Carlo analysis. The reduced set of paths is referred to as a set of representative paths. The theoretical value of the MBS is then the weighted average of the present values of each representative path, weighed by "path weights."

Example: Estimating OAS from a Monte Carlo model

The following figure shows the present values for six representative paths and corresponding path weights resulting from the Monte Carlo simulation analysis of a CMO tranche. Three present values for each path are shown because they were calculated using three different discount rates. Each discount rate is the short-term rate on the path plus the spread indicated at the top of the columns.

### OAS Simulations

<table>
<thead>
<tr>
<th>Representative Path</th>
<th>Path Weight</th>
<th>If Spread Is 65 bps</th>
<th>If Spread Is 70 bps</th>
<th>If Spread Is 75 bps</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18%</td>
<td>$80.00</td>
<td>$78.00</td>
<td>$76.00</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>82.00</td>
<td>80.00</td>
<td>77.00</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>78.00</td>
<td>75.00</td>
<td>74.00</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>81.00</td>
<td>78.00</td>
<td>77.00</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
<td>88.00</td>
<td>86.00</td>
<td>85.00</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>85.00</td>
<td>84.00</td>
<td>81.00</td>
</tr>
</tbody>
</table>

Theoretical value = \( \Sigma (weight \times PV) \) = $82.58 $80.44 $78.66

The last row in this figure represents the theoretical value for the CMO tranche based on the different discount rates. As indicated, this value is the weighted average of the present values of the cash flow for each representative path. Suppose that the actual market price of this tranche is $80.44. Determine the OAS of the CMO tranche.

**Answer:**

The OAS is the spread that, when added to the interest rates along an interest rate path, makes the theoretical value equal to the market price. When the spread is 70 basis points, the figure indicates the theoretical value from the model is $80.44, which is equal to the market price. Therefore, the OAS is 70 basis points.
LOS 59.e: Evaluate a mortgage-backed security using option-adjusted spread analysis.

We can identify rich and cheap securities by comparing the OAS and option costs of the various tranches in a CMO deal. The OAS is the spread adjusted for the embedded options in the security, and the option cost is the difference between the Z-spread and the OAS. Typically, securities with longer effective durations have larger OAS and option costs because of their higher interest rate exposure. Therefore, for a given Z-spread and effective duration:

- **Cheap** securities will have high OAS relative to the required OAS and low option costs.
- **Rich** securities will have low OAS relative to the required OAS and high option costs.

Cheap securities are undervalued on a relative basis, and we want to buy them; rich securities are overvalued, which means we should sell them.

**Example: OAS analysis of an MBS**

Let’s apply some of this by evaluating the following two tranches for a hypothetical sequential-pay CMO structure. Identify which CMO tranche is less expensive (“cheap”) on a relative basis and justify your answer.

### Pricing Data on CMO Tranches

<table>
<thead>
<tr>
<th>Tranche</th>
<th>OAS (bps)</th>
<th>Z-spread (bps)</th>
<th>Effective Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>92</td>
<td>142</td>
<td>4.25</td>
</tr>
<tr>
<td>II</td>
<td>118</td>
<td>134</td>
<td>4.25</td>
</tr>
</tbody>
</table>

**Answer:**

The first thing we have to do is compute the option cost of each tranche:

- Tranche I option cost = 142 - 92 = 50 basis points
- Tranche II option cost = 134 - 118 = 16 basis points

Tranche II has a higher OAS and lower option cost than Tranche I, and the effective durations of the two tranches are equal. Therefore:

- Tranche II is undervalued on a relative basis (“cheap”), and we should buy it.
- Tranche I is overvalued on a relative basis (“rich”), and we should sell it.
LOS 59.f: Discuss why effective durations reported by various dealers and vendors may differ.

Duration measures the sensitivity of a security’s price to changes in interest rates. It can be determined by changing the interest rate in a security pricing model up and down by a small amount and observing what happens to the price.

Recall that effective duration can then be calculated as:

\[
\text{effective duration} = ED = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}
\]

where:

- \( \Delta y \) = change in required yield, in decimal form
- \( BV_{-\Delta y} \) = estimated price if yield decreases by \( \Delta y \)
- \( BV_{+\Delta y} \) = estimated price if yield increases by \( \Delta y \)
- \( BV_0 \) = initial observed bond price

The Monte Carlo simulation model may be used to compute \( BV_{-\Delta y} \) and \( BV_{+\Delta y} \) in the previous equation when measuring effective duration for MBS.

The assumptions used to calculate effective duration are important and can have a material impact on estimates of effective duration reported by different vendors and dealers.

- **Differences in \( \Delta y \).** If \( \Delta y \), which is the incremental change in interest rate, is too large, the effects of convexity contaminate effective duration estimates.
- **Prepayment model differences.** Prepayment models vary among dealers.
- **OAS differences.** OAS is a product of the Monte Carlo simulation model. Differences in the inputs to the model will affect the measurement of OAS. This has particular relevance to the volatility assumption and the prepayment model used with a particular Monte Carlo model.
- **Differences in the spread.** Between 1-month rates and refinancing rates. The assumed spread between 1-month rates and refinancing rates affects the computed values of MBS. Thus, different assumptions about this relationship will provide different values for \( BV_{+\Delta y} \) and \( BV_{-\Delta y} \) in the effective duration computation.

LOS 59.g: Analyze the interest rate risk of a security given the security’s effective duration and effective convexity.

**Analyzing Interest Rate Risk With Effective Duration and Convexity**

Remember that interest rate risk is the risk that the price of a fixed-income security will change as yields change. Effective duration is a measure of interest rate risk: the larger the duration of a security, all else equal, the greater the interest rate risk. However, duration only gives us an estimate of the actual change in the price of a bond for small changes in yields. The convexity adjustment gives us a more precise estimate for larger
changes in yield. For a given duration, the greater the convexity, the lower the interest rate risk.

Given a bond’s duration and convexity, we can estimate the percentage price change in the bond for a given change in yield as:

\[
\% \text{ change in bond price} \approx \text{duration effect} + \text{convexity effect} \\
\approx (-ED \times \Delta y \times 100) + (EC \times \Delta y^2 \times 100)
\]

**Example: Assessing interest rate risk, part 1**

Durable, Inc., bonds have a duration of 5.6 years and a convexity of 38.2. Conversion Force, Inc., bonds have a duration of 7.3 years and a convexity of 38.2. Determine which bond is exposed to more interest rate risk.

**Answer:**

Conversion Force bonds have a larger duration than Durable bonds, but the two have equal convexities. That means Conversion Force bonds have greater interest rate risk exposure. To see this, calculate what happens to the price of both bonds if rates increase by 100 basis points:

\[
\% \text{ change in Conversion Force bond price} = (-7.3 \times 0.01 \times 100) + (38.2 \times 0.01^2 \times 100) \\
= -6.9%
\]

\[
\% \text{ change in Durable bond price} = (-5.6 \times 0.01 \times 100) + (38.2 \times 0.01^2 \times 100) \\
= -5.2%
\]

**Example: Assessing interest rate risk, part 2**

Suppose that Durable, Inc., bonds have a duration of 5.6 years and a convexity of 38.2, and Universal, Inc., bonds have a duration of 5.6 years and a convexity of 134.0. Determine which bond is exposed to more interest rate risk.

**Answer:**

Now the two have the same duration, but Universal bonds have greater convexity. That means Universal bonds are less exposed to changes in yields than Durable bonds. To see this, calculate what happens to the price of both bonds if rates increase by 100 basis points:

\[
\% \text{ change in Universal bond price} = (-5.6 \times 0.01 \times 100) + (134.0 \times 0.01^2 \times 100) = -4.3%
\]

\[
\% \text{ change in Durable bond price} = (-5.6 \times 0.01 \times 100) + (38.2 \times 0.01^2 \times 100) = -5.2%
\]
LOS 59.h: Explain other measures of duration used by practitioners in the mortgage-backed market (e.g., cash flow duration, coupon curve duration, and empirical duration), and describe the limitations of these duration measures.

Cash flow duration is a version of effective duration that allows for cash flows to change as interest rates change. Unlike valuation with the binomial or the Monte Carlo models, cash flow duration uses a static valuation procedure to determine $BV_{+\Delta y}$ and $BV_{-\Delta y}$. This procedure uses the following steps:

- Make a prepayment rate assumption (e.g., 100 PSA), and use it to estimate cash flows.
- Compute the cash flow yield based on the market price and the cash flow estimates. (Recall that cash flow yield is the discount rate that makes the present value of the cash flows equal to the price.)
- Increase the cash flow yield by $\Delta y$ and, with the use of a prepayment model, recompute prepayment rates. These rates are normally lower than those in the original prepayment rates due to the higher interest rates. For example, if the original prepayment assumption was 100 PSA, the new prepayment rate might be 90 PSA.
- Recompute the cash flows using the prepayment rates generated in the previous step, and discount these cash flows at the higher cash flow yield to get $BV_{+\Delta y}$.
- Decrease the cash flow yield by $\Delta y$, and with the use of a prepayment model, regenerate prepayment rates. These rates are normally greater than those in the original prepayment rates (e.g., 140 PSA) due to the lower yield.
- Recompute the cash flow using the prepayment rates from Step 5, and discount these cash flows at the lower cash flow yield to get $BV_{-\Delta y}$.

The values for $BV_{+\Delta y}$ or $BV_{-\Delta y}$ obtained from this procedure—the initial value $BV_0$ and the imposed cash flow yield change $\Delta y$—can be substituted into the effective duration equation to obtain the cash flow duration.

A major criticism of cash flow duration is that it is based on the unrealistic assumption that the new MBS’s prepayment rate is constant over its entire life for a given shock to interest rates. The Monte Carlo simulation does allow for changing prepayment rates, and therefore, effective duration computed using the Monte Carlo simulation is much better than cash flow duration for MBS.

Coupon curve duration is based on the relationship between coupon rates and prices for similar MBS. Let’s look at an example of the process for determining $BV_{+\Delta y}$ and $BV_{-\Delta y}$ in the computation of coupon rate duration. Note, however, that the LOS doesn’t ask you to “calculate.”

Consider the coupon curve for a hypothetical passthrough shown in Figure 3.

**Figure 3: Passthrough Coupon Curve**

<table>
<thead>
<tr>
<th>Coupon Rate:</th>
<th>7%</th>
<th>8%</th>
<th>9%</th>
<th>10%</th>
<th>11%</th>
<th>12%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price:</td>
<td>93.06</td>
<td>96.07</td>
<td>99.55</td>
<td>103.25</td>
<td>105.90</td>
<td>110.35</td>
</tr>
</tbody>
</table>
Assume that we want to compute the effective duration for a 10% coupon passthrough ($BV_0 = 103.25$). If rates decrease 100 basis points, it is assumed that prices will increase to the price of the 11% coupon security ($BV_{-Ay} = 105.90$). Similarly, for an increase of 100 basis points, $BV_{+Ay}$ will be 99.55. Plugging these values into the (familiar) duration equation yields a coupon curve duration equal to:

$$\frac{105.90 - 99.55}{2 \times 103.25 \times 0.01} = 3.075$$

The advantages of the coupon curve duration method are that it:

- Is easy to apply.
- Uses market prices that presumably reflect market expectations.

The limitations of the procedure are that it is:

- Only applicable to generic MBS.
- Not readily applicable for CMO structures and other mortgage-based derivatives.

Empirical duration (or implied duration) is determined using a regression analysis of the historical relationship between security prices (the dependent variable) and yields (the independent variable).

Empirical duration has three advantages:

- It requires few assumptions.
- The required parameters are easy to estimate with regression analysis.
- The time series data for Treasury prices and yields is readily available.

Disadvantages of empirical duration include:

- Time series price data on mortgage securities may be difficult to obtain.
- Embedded options can distort the results.
- The volatility of the spreads over Treasuries can distort the price reaction to interest rate changes.

**LOS 59.i:** Determine whether the nominal spread, zero-volatility spread, or option-adjusted spread should be used to evaluate a specific fixed income security.

The nominal spread is expressed as the spread between the cash flow yield and the yield on a Treasury security with the same maturity as the average life of the MBS or ABS under analysis. It is the spread at one point on the Treasury yield curve. We should never use the nominal spread for MBS and ABS because it masks the fact that a portion of the spread is compensation for accepting prepayment risk.

The zero-volatility spread is the spread over the entire Treasury spot rate curve if an MBS is held until maturity. It is suitable for assessing the value of option-free bonds. Z-spread analysis should not be used with bonds that have prepayment options because it does not reflect the possibility that cash flows may change as interest rates change.
The option-adjusted spread (OAS) should be used to assess the value of fixed-income securities that have embedded options that make it possible for cash flows to change as interest rates change.

- If the amounts of the cash flows are not interest rate path dependent (i.e., they may depend upon the current level of interest rates, but not the path that led to the current level), such as those associated with putable and callable bonds, OAS should be used with the binomial model.
- If the amounts of the cash flows are interest rate path dependent, like those associated with ABS and MBS, the OAS should be used with the Monte Carlo simulation model.

The appropriate valuation model for an ABS is dependent on whether a prepayment option is available on the underlying collateral and whether that option is typically exercised:

- Credit-card-receivable-backed ABS have no prepayment option. Automobile loans have a prepayment option, but it is not typically exercised as a result of changes in interest rates. In either case, the Z-spread is the appropriate measure.
- ABS backed by high-quality home equity loans have a prepayment option that is frequently exercised when rates drop and borrowers refinance. Furthermore, the amounts of the cash flows are path-dependent, so an OAS derived from a Monte Carlo model is appropriate.

Figure 4 summarizes this discussion.

**Figure 4: Appropriate Spread Measures for Fixed Income Securities**

```
Does security have embedded option that is typically exercised?

Yes

Is the amount of the cash flow interest rate path-dependent?

Yes
- MBS
- Home equity ABS

Use OAS from Monte Carlo model

No
- Callable corporate

Use OAS from binomial model

No

Plain vanilla corporate
Credit card ABS
Auto loan ABS

Use Z-spread
```
LOS 59.a
The cash flow yield is the discount rate that makes the price of an ABS or MBS equal to the present value of its cash flows. Cash flow yield for MBS and ABS has three major deficiencies: (1) it is implicitly assumed that the cash flows will be reinvested at the cash flow yield prevailing when the MBS or ABS is priced, (2) it is based on the assumption that an MBS or ABS will be held until maturity, and (3) it assumes that the cash flows will be realized as expected.

The nominal spread is the difference between the cash flow yield on an MBS and the YTM on a Treasury security with a maturity equal to the average life of the MBS. There are at least two problems with the interpretation of the nominal spread:
• Some unknown fraction of the nominal spread represents compensation for prepayment risk.
• The unknown fraction of the nominal spread that is compensation for prepayment risk may be more significant for the support (companion) tranches.

The zero-volatility spread (Z-spread or static spread) is the spread that must be added to each Treasury spot rate to cause the discounted value of an MBS’s cash flows to equal its price.
• A computer-aided, iterative process can be followed to determine the Z-spread.
• Zero-volatility spread and nominal spread converge as the average life of the MBS decreases.
• The difference between the Z-spread and nominal spread increases as the slope of the yield curve increases.

LOS 59.b
The OAS computed using the Monte Carlo model is the spread that makes the MBS value derived from the model (the present value of the projected cash flows) equal to the current market price.

LOS 59.c
The size of the cash flows and value of the MBS are interest rate path dependent, which means they must be valued with a Monte Carlo Model rather than a binomial model.

LOS 59.d
OAS estimates from a Monte Carlo simulation use the same general principle as the OAS from a binomial model—a spread is added to every spot rate along every interest rate path until the spread causes the model price to equal the current market price. We can interpret the OAS as the MBS spread after the “optionality” of the cash flows is taken into account. The interpretation of the OAS depends on the security’s credit risk, liquidity risk, and modeling risk relative to the benchmark.

The OAS of a Ginnie Mae product reflects liquidity risk and modeling risk. Freddie Mac and Fannie Mac MBS, as well as non-agency MBS and real-estate-backed ABS, have OAS that reflect credit risk, liquidity risk, and modeling risk.
LOS 59.e
For a given Z-spread and effective duration:
• Cheap securities will have high OAS relative to the required OAS and low option costs.
• Rich securities will have low OAS relative to the required OAS and high option costs.

LOS 59.f
The assumptions used to calculate effective duration are important and can have a material impact on estimates of effective duration reported by different vendors and dealers.
• Differences in Δy: If Δy, which is the incremental change in interest rate, is too large, the effects of convexity contaminate effective duration estimates.
• Prepayment model differences: Prepayment models vary among dealers.
• OAS differences: OAS is a product of the Monte Carlo simulation model. Differences in the inputs to the model will affect the measurement of OAS. This has particular relevance to the volatility assumption and the prepayment model used with a particular Monte Carlo model.
• Differences in the spread between one-month rates and refinancing rates. The assumed spread between one-month rates and refinancing rates affects the computed values of MBS. Thus, different assumptions about this relationship will provide different values for $BV_{+Δy}$ and $BV_{-Δy}$ in the effective duration computation.

LOS 59.g
The larger the effective duration of a security, all else equal, the greater the interest rate risk. For a given duration, the greater the convexity, the lower the interest rate risk.

LOS 59.h
In practice, there are several measures of duration that may be used with MBS:
• Effective duration is the appropriate measure for securities with embedded options, such as callable corporate bonds and MBS. Effective duration is computed using Monte Carlo simulation by “bumping” interest rates up and down and using the new values in the duration equation.
• Coupon curve duration is based on the relationship between coupon rates and prices for similar MBS. The values used in the duration formula are found by moving up and down the coupon curve. Coupon curve duration applicability is, however, limited to generic MBS only.
• Empirical duration (implied duration) is determined using a regression analysis of the historical relationship between security prices and yields. Limitations of empirical duration include the following: time series data are difficult to obtain; spread volatility could change; and presence of embedded options could distort results.
• Cash flow duration is a form of effective duration. It recognizes that cash flows can change as interest rates change and is based on the changes in value that occur after the initial cash flow yield is shifted up and down. This causes prepayment rates, expected cash flows, and values to change. The changed values are plugged into the duration equation. Cash flow duration is, however, based on an unrealistic assumption of single prepayment rate over the life of a MBS.
### LOS 59.i

<table>
<thead>
<tr>
<th>Embedded Option That is Typically Exercised?</th>
<th>Path-Dependent Option?</th>
<th>Example</th>
<th>Spread Measure</th>
<th>Option Valuation Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>—</td>
<td>Plain-vanilla corporate</td>
<td>Z-spread</td>
<td>—</td>
</tr>
<tr>
<td>No</td>
<td>—</td>
<td>Credit card ABS</td>
<td>Z-spread</td>
<td>—</td>
</tr>
<tr>
<td>No</td>
<td>—</td>
<td>Auto loanABS</td>
<td>Z-spread</td>
<td>—</td>
</tr>
<tr>
<td>Yes</td>
<td>No</td>
<td>Callable corporate</td>
<td>OAS</td>
<td>Binomial</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>MBS</td>
<td>OAS</td>
<td>Monte Carlo</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Home equity ABS</td>
<td>OAS</td>
<td>Monte Carlo</td>
</tr>
</tbody>
</table>
1. Which of the following is least likely to be considered a shortcoming of the cash flow yield as a process for the cash flow analysis of a mortgage or asset-backed security?
   A. All cash flows are assumed to be reinvested at the cash flow yield.
   B. All cash flows are assumed to be certain to occur.
   C. The term structure is assumed to be normal.

2. Brian Heltzel recently completed a Monte Carlo simulation analysis of a collateralized mortgage obligation (CMO) tranche. Heltzel’s analysis includes six equally weighted paths, with the present value of each calculated using four different discount rates, as shown in the following figure.

<table>
<thead>
<tr>
<th>Representative Path</th>
<th>PV if spread is 50 basis points</th>
<th>PV if spread is 60 basis points</th>
<th>PV if spread is 70 basis points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>68</td>
<td>66</td>
</tr>
<tr>
<td>2</td>
<td>73</td>
<td>70</td>
<td>68</td>
</tr>
<tr>
<td>3</td>
<td>68</td>
<td>66</td>
<td>64</td>
</tr>
<tr>
<td>4</td>
<td>71</td>
<td>69</td>
<td>68</td>
</tr>
<tr>
<td>5</td>
<td>77</td>
<td>75</td>
<td>73</td>
</tr>
<tr>
<td>6</td>
<td>75</td>
<td>73</td>
<td>71</td>
</tr>
</tbody>
</table>

The actual market price of the CMO tranche being valued is 70.17. The tranche’s option-adjusted spread (OAS) is closest to:
   A. 50 basis points.
   B. 60 basis points.
   C. 70 basis points.

3. An asset-backed security (ABS) backed by automobile loans is issued. This security should be evaluated using the:
   A. nominal spread.
   B. Z-spread.
   C. OAS from Monte Carlo simulation.

4. A Ginnie Mae security with no call or put features is issued against a pool of conventional mortgages. This security should be evaluated using the:
   A. Z-spread.
   B. OAS from the binomial model.
   C. OAS from Monte Carlo simulation.
5. Which of the following statements concerning the measurement of interest rate risk of a mortgage-backed security (MBS) is least accurate?
   A. Coupon curve duration looks at other MBS issues with different coupons to determine how one MBS price will change when interest rates change.
   B. Empirical duration uses regression analysis to determine how the MBS price has responded to interest rate changes in the past.
   C. Cash flow duration allows the prepayment speed of the pool to vary over time.

6. A model that incorporates backward induction methodology, like the binomial model, cannot be used to value a mortgage-backed security (MBS) because:
   A. the cash flows are assumed to occur with certainty.
   B. the cash flows for an MBS are dependent upon the path that interest rates follow.
   C. effective duration measures cannot be computed using backward induction.

7. Which of the following statements concerning the nominal spread, the Z-spread, and the option-adjusted spread (OAS) is least accurate?
   A. The Z-spread is the spread that must be added to Treasury spot rates that will cause the discounted value of the cash flows for an MBS or ABS to equal its price, assuming that the security is held until maturity.
   B. The OAS is equal to the Z-spread plus the option cost.
   C. The OAS from a Monte Carlo simulation model is the spread that must be added to all of the spot rates along each interest rate path that will force equality between the average present value of the path cash flows and the market price for the MBS being evaluated.

8. Suppose that ten equally weighted representative paths are used in the Monte Carlo simulation model. For the different spreads used, the present value of each representative path is shown in the following figure for a collateralized mortgage obligation (CMO) tranche.

<table>
<thead>
<tr>
<th>Representative Path</th>
<th>Present Value if Spread is:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75 bps</td>
</tr>
<tr>
<td>1</td>
<td>72</td>
</tr>
<tr>
<td>2</td>
<td>77</td>
</tr>
<tr>
<td>3</td>
<td>81</td>
</tr>
<tr>
<td>4</td>
<td>84</td>
</tr>
<tr>
<td>5</td>
<td>69</td>
</tr>
<tr>
<td>6</td>
<td>83</td>
</tr>
<tr>
<td>7</td>
<td>91</td>
</tr>
<tr>
<td>8</td>
<td>87</td>
</tr>
<tr>
<td>9</td>
<td>69</td>
</tr>
<tr>
<td>10</td>
<td>93</td>
</tr>
</tbody>
</table>
If the market price of Tranche X is 74, the option-adjusted spread (OAS) is closest to:
A. 75 bps.
B. 80 bps.
C. 85 bps.

9. Which of the following statements correctly identifies a limitation of the nominal spread for analyzing a mortgage-backed security (MBS)? The nominal spread does not:
A. account for differences in credit risk between the MBS and the Treasury security.
B. adjust for the prepayment risk inherent in the MBS.
C. result in an effective comparison because the effective life rather than the maturity of the MBS is measured against the maturity of the Treasury.

10. Which of the following is least likely to be considered an advantage of using the empirical duration approach?
A. The volatility of the spread to Treasury securities does not distort how the price of mortgage-backed securities reacts to yield changes.
B. Its calculation relies on few assumptions.
C. Required parameters are easy to estimate using regression analysis.

11. Which of the following statements is least accurate concerning valuation methodologies?
A. A callable corporate bond should be valued using the Monte Carlo OAS approach.
B. An auto loan-backed ABS should be valued using the Z-spread approach.
C. A high-quality home equity loan ABS should be valued using the Monte Carlo OAS approach.

12. Does the OAS of a PAC I tranche from a CMO backed by Ginnie Mae pass-throughs, calculated using Treasury yields as a benchmark, reflect credit risk and/or modeling risk?
A. Yes, it reflects both risks.
B. It reflects one risk, but not the other.
C. No, it does not reflect either risk.
13. Consider the collateralized mortgage obligation (CMO) tranches shown in the following figure.

<table>
<thead>
<tr>
<th>Tranche</th>
<th>OAS (bps)</th>
<th>Z-spread (bps)</th>
<th>Effective Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68</td>
<td>85</td>
<td>2.60</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
<td>91</td>
<td>2.90</td>
</tr>
<tr>
<td>3</td>
<td>73</td>
<td>136</td>
<td>8.25</td>
</tr>
</tbody>
</table>

Is Tranche 3 relatively more expensive than Tranche 1 and/or Tranche 2?
A. Yes, it is relatively more expensive than both tranches.
B. It is relatively more expensive than one tranche, but not the other.
C. No, it is relatively less expensive than both tranches.

14. Quantitative Duration Associates estimates the effective duration of a mortgage-backed passthrough security to be 14.7. Sundial Partners estimates the effective duration of the same passthrough to be 18.9. Which is the least likely reason for the difference in the effective duration estimates provided by the two firms? Each firm used a different:
A. proprietary prepayment model.
B. refinancing spread.
C. effective convexity estimate.

15. Which of the following variables is least likely to change when using a Monte Carlo simulation model to compute the effective duration of a mortgage-backed security (MBS)?
A. Option-adjusted spread.
B. Prepayment rates.
C. Expected cash flows.

16. An investor is comparing securities that differ only in effective duration and effective convexity. The security with the lowest interest rate risk will have:
A. the lowest duration and the highest convexity.
B. the lowest duration and the lowest convexity.
C. the highest duration and the lowest convexity.
1. C The term structure is not assumed to be normal in order to appropriately interpret the cash flow yield.

2. B The question tells us that the market price of the CMO tranche is 70.17. The OAS is the spread added to the interest rates along the interest rate path that makes the market price and the theoretical value equal. The theoretical value of the CMO will be the weighted average of the values of each interest path. Because we are told in the problem that the paths are equally weighted, we simply find the arithmetic average for each path and choose the theoretical value that equals the market price. In this case, the average of the 60 bps spread column is:

\[
\frac{68 + 70 + 66 + 69 + 75 + 73}{6} = 70.17
\]

Therefore, the OAS must be 60 basis points.

3. B Automobile loans have a prepayment option, but it is not typically exercised as a result of a change in interest rates. Therefore the Z-spread is appropriate for evaluating the security.

4. C The Z-spread is not appropriate because there is a prepayment option present. The binomial model is not appropriate because the future cash flows on the pool are dependent upon interest rates. The OAS based on Monte Carlo simulation is most appropriate.

5. C Cash flow duration, unlike effective duration based on Monte Carlo simulation, assumes a single prepayment speed over the life of the MBS.

6. B The cash flows for an MBS are dependent upon the path that interest rates follow and, therefore, cannot be properly valued with the binomial model or any other model that employs the backward induction methodology. The cash flows for pass-through securities are a function of prepayment rates, and prepayment rates in any given month are affected by interest rates in the past (e.g., refinancing burnout).

7. B The OAS is the Z-spread minus the option cost. All the other statements are correct.

8. B Because it is assumed that each path has the same weight, the theoretical value is simply the average of the present values of the interest rate paths. The average present value of each path using the four spreads is tabulated in the following figure.

<table>
<thead>
<tr>
<th>Present Value if Spread is:</th>
<th>75 bps</th>
<th>80 bps</th>
<th>85 bps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average PV</td>
<td>80.6</td>
<td>74.0</td>
<td>71.2</td>
</tr>
</tbody>
</table>

Therefore the OAS of Tranche X is 80 basis points.

9. B The nominal spread does not adjust for the prepayment risk of the MBS.

10. A A disadvantage of the empirical duration model is that the volatility of the spreads with reference to Treasuries can distort the price reaction to interest rate changes.
11. A A callable corporate bond should be valued using the binomial OAS model, not the Monte Carlo OAS model. Given that the cash flows are not interest rate path dependent, the callable corporate bond can be valued more easily with the simpler binomial OAS model.

12. B If we use Treasury securities as the benchmark, the OAS on a CMO backed by Ginnie Mae passthroughs does not reflect credit risk, because Ginnie Maes carry the full faith and credit of the U.S. government. The OAS does, however, reflect modeling risk.

ANSWERS – CHALLENGE PROBLEMS

13. A First calculate the option cost of each tranche:

Tranche 1 option cost = 85 – 68 = 17 basis points
Tranche 2 option cost = 91 – 71 = 20 basis points
Tranche 3 option cost = 136 – 73 = 63 basis points

Tranche 3, despite its longer effective duration, has a comparable OAS and a much higher option cost than Tranches 1 and 2. A risk-averse investor would demand a higher OAS on Tranche 3 than on Tranches 1 and 2 because of its longer duration. Therefore, on a relative basis, Tranche 3 is relatively more expensive than Tranches 1 and 2. If you saw two tranches with the same effective duration, the tranche with the largest OAS is cheaper. In this problem, Tranche #3 has a tiny advantage in OAS in exchange for a large difference in effective duration; this is not adequate compensation.

14. C Effective durations reported by vendors and dealers are most likely to differ as a result of differences in the (1) size of the interest rate shock ($\Delta y$), (2) prepayment model, (3) option-adjusted spread, and (4) refinancing spread. Effective convexity would be estimated using the same procedure as for effective duration, but the convexity estimate is not used in the estimation of the effective duration.

15. A Calculating effective duration using a Monte Carlo simulation model starts by changing interest rates by a small amount ($\Delta y$) and observing the changes in the price of the MBS. The price of the MBS changes because the change in interest rates causes prepayment rates to change, which causes expected cash flows to change. The option-adjusted spread (OAS) is assumed to remain constant in this procedure.

16. A Effective duration is a measure of interest rate risk: the larger the duration of a security, all else equal, the greater the interest rate risk. For a given duration, the greater the convexity, the lower the interest rate risk.
Use the following information for Questions 1 through 6.

Jonathan Song is a CFA candidate who recently took the Level 2 exam and is currently waiting to receive his test results. Song is also pursuing his MBA at a prestigious Ivy League university. He accepted a position as an intern at a large brokerage firm in New York for this year's summer break. Over the course of his internship, he will rotate among the different areas of the firm, spending two weeks in each. His current rotation is in the brokerage firm's Research department, where he will report to Bill Dixon, a managing director whose group is responsible for economic forecasting and analysis. Dixon is evaluating all of the interns that rotate through his department this Summer to identify possible candidates for permanent positions at the brokerage firm after graduation.

Song has successfully completed a course in the basic principles of finance, and Dixon seeks to assess Song's knowledge of various concepts that are of specific importance to his area. Dixon decides to focus first on the term structure of interest rates, because this area is directly applicable to economic forecasting. To this end, Dixon supplies Song with some fundamental market information, asks him to interpret the shape of the yield curve, and to forecast how a portfolio might react to various changes in interest rates. Dixon also wants to explore Song's knowledge concerning the various theories of the term structure of interest rates, including their similarities, differences, and appropriate usage.

Dixon has constructed a sample portfolio of Treasury bonds with different maturities to simulate an actual portfolio management situation. This portfolio is similar to a barbell portfolio that Dixon is considering for a client of the firm.

<table>
<thead>
<tr>
<th>Security</th>
<th>Weight</th>
<th>Current Yield</th>
<th>Key Rate Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-year</td>
<td>45%</td>
<td>4.50</td>
<td>0.91</td>
</tr>
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<td>10-year</td>
<td>15%</td>
<td>4.63</td>
<td>2.15</td>
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<tr>
<td>20-year</td>
<td>10%</td>
<td>4.82</td>
<td>3.89</td>
</tr>
<tr>
<td>25-year</td>
<td>30%</td>
<td>4.97</td>
<td>4.12</td>
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Dixon has asked Song to determine exactly how the sample portfolio value would react to several specific interest rate scenarios.

1. A recent change in interest rates has caused the Treasury yield curve to become significantly more curved. This change in shape of the yield curve is often called:
   A. butterfly shift.
   B. negative butterfly shift.
   C. positive yield curve twist.
2. Dixon has asked Song to construct a theoretical spot rate curve. Dixon will have the greatest degree of confidence in the accuracy of the spot curve if Song uses:
A. treasury strips.
B. on-the-run Treasury securities.
C. all available Treasury coupon securities and bills.

3. Dixon asks Song for his interpretation of the yield curve represented by the sample portfolio. Which of the following statements regarding term structure theories is most accurate?
A. According to the pure expectations theory, the bond yields imply that investors expect short-term rates to remain constant.
B. According to the liquidity theory, the bond yields imply that investors demand a premium for exposure to interest rate risk.
C. According to the preferred habitat theory, the risk premium of the 20-year bond is 32 basis points greater than that of the 2-year security.

4. The effective duration for the portfolio for a parallel shift in the yield curve is closest to:
A. 2.36.
B. 4.69.
C. 11.07.

5. Now Dixon wants Song to assume that the yield curve shifts in a nonparallel fashion. The anticipated change for the 2-year and 10-year rates is an increase of 50 basis points, while the 20-year and 25-year rates are expected to increase by 100 basis points. Song correctly calculates the effect of this yield shift as:
A. 0.70% decrease in value.
B. 2.00% decrease in value.
C. 2.73% decrease in value.

6. The swap rate curve (based upon LIBOR rates) is preferred by some market participants as a yield benchmark. Which of the following statements is least valid regarding why these participants may prefer the swap rate curve over the traditional treasury curve?
A. The swap market is highly regulated, which makes swap rates more consistent among markets.
B. Swap pricing depends only on supply and demand, and is not affected by technical market factors.
C. Unlike government bond yield curves, swap curves do not reflect sovereign risk unique to a particular country.
1. **B** Yield curve butterfly shifts describe changes in the degree of curvature in the yield curve. A negative butterfly shift means that there is more curvature to the yield curve.

2. **B** On-the-run issues are the most-recently issued Treasury securities. These have the largest trading volume and are the most accurately priced issues. The main drawback to relying only on on-the-run issues is that there are likely to be large gaps between maturities available. To fill in the gaps, the analyst could use some off-the-run securities. This has the potential to reduce the accuracy of the spot rate estimates.

3. **B** According to the liquidity theory of the term structure, investors require a liquidity premium to compensate them for exposure to interest rate risk. The longer the maturity of the issue, the greater the interest rate risk, and the greater the liquidity premium. The other answers are incorrect or cannot be substantiated.

4. **A** The key rate duration of a portfolio is simply the weighted average of the key rate durations of the individual securities.

\[
\begin{align*}
D_2 &= (0.45 \times 0.91) + (0.15 \times 0) + (0.30 \times 0) = 0.41 \\
D_{10} &= (0.45 \times 0) + (0.15 \times 2.15) + (0.30 \times 0) = 0.32 \\
D_{20} &= (0.45 \times 0) + (0.15 \times 0) + (0.30 \times 3.89) + (0.30 \times 0) = 0.39 \\
D_{25} &= (0.45 \times 0) + (0.15 \times 0) + (0.10 \times 0) + (0.30 \times 4.12) = 1.24 \\
&= 2.36
\end{align*}
\]

5. **B** Using the individual key rate durations calculated above, a change in portfolio value can be derived by computing the change in value associated with each bond. Remember that an increase in rates will cause a decrease in value.

Change in Portfolio Value:

- Change from 2-year: \(-0.50\% \times 0.41 = -0.21\%\)
- Change from 10-year: \(-0.50\% \times 0.32 = -0.16\%\)
- Change from 20-year: \(-1.00\% \times 0.39 = -0.39\%\)
- Change from 25-year: \(-1.00\% \times 1.24 = -1.24\%\) \(-2.00\%\)

6. **A** In fact, the swap market is lightly regulated (if at all) by any government, which makes swap rates in different countries more comparable.
STUDY SESSION 13: ALTERNATIVE ASSET VALUATION

real estate valuation:

\[ \text{income taxes payable} = (\text{NOI} - \text{depreciation} - \text{interest}) \times \text{tax rate} \]

\[ \text{CFAT} = \text{NOI} - \text{debt service} - \text{taxes payable} \]

\[ \text{ERAT} = \text{selling price} - \text{selling costs} - \text{mortgage balance} - \text{taxes on sale} \]

\[ MV_0 = \frac{\text{NOI}}{r - g} = \frac{\text{NOI}}{R_0} \]

\[ R_0(\text{ME}) = \frac{\text{NOI}}{MV} \]

\[ R_0(\text{BOI}) = (\text{mortgage weight} \times \text{mortgage cost}) + (\text{equity weight} \times \text{equity cost}) \]

\[ R_0(\text{Bu}) = \text{pure rate} + \text{liquidity premium} + \text{recapture premium} + \text{risk premium} \]

\[ \text{gross income multiplier (M)} = \frac{\text{sale price}}{\text{gross income}} \]

\[ MV = \text{gross income} \times M \]

exit value:

\[
\begin{align*}
\text{Investment Cost} & + \text{Earnings Growth} + \text{Increase in Price Multiple} + \text{Reduction in Debt} = \text{Exit Value}
\end{align*}
\]

NAV before distributions:

\[
\begin{align*}
= & \quad \text{NAV after distributions in prior year} + \text{Capital Called Down} - \text{Management Fees} + \text{Operating Results}
\end{align*}
\]

NAV after distributions:

\[
\begin{align*}
= & \quad \text{NAV before distributions} - \text{Carried Interest} - \text{Distributions}
\end{align*}
\]
venture capital method:

The post-money portion of a firm purchased by an investment is:

\[ f_1 = \frac{\text{investment}_1}{\text{PV}_1(\text{exit value})} \]

The new shares issued are:

\[ \text{shares}_{\text{VC}} = \text{shares}_{\text{EQUITY}} \left( \frac{f_1}{1-f_1} \right) \]

where \( \text{shares}_{\text{EQUITY}} \) is the pre-investment number of shares, and share price is:

\[ \text{price}_1 = \frac{\text{investment}_1}{\text{shares}_{\text{VC}}} \]

beta equity:

\[ \beta_{\text{Equity}} = \beta_{\text{Assets}} \left( \frac{D+E}{E} \right) \]

return for equity:

\[ E(R_{\text{Equity}}) = R_f + \beta_{\text{Equity}} \left( E(R_{\text{Market}}) - R_f \right) \]

Sharpe ratio:

\[ S_A = \frac{\overline{R_A} - R_f}{\sigma_A} \]

**STUDY SESSIONS 14 & 15: FIXED INCOME**

profitability ratios:

\[ \text{ROE} = \frac{\text{net income}}{\text{stockholders' equity}} = \left( \frac{\text{net income}}{\text{sales}} \right) \times \left( \frac{\text{sales}}{\text{total assets}} \right) \times \left( \frac{\text{total assets}}{\text{stockholders' equity}} \right) \]

short-term solvency ratios:

\[ \text{current ratio} = \frac{\text{current assets}}{\text{current liabilities}} \]

\[ \text{acid-test ratio} = \frac{\text{current assets} - \text{inventories}}{\text{current liabilities}} \]
capitalization (financial leverage) ratios:

long-term debt-to-capitalization ratio

\[
= \frac{\text{long-term debt}}{\text{long-term debt} + \text{minority shareholders' common debt} + \text{interest} + \text{and preferred equity}}
\]

total debt-to-capitalization ratio

\[
= \frac{\text{current liabilities} + \text{long-term debt}}{\text{current liabilities} + \text{long-term debt} + \text{interest} + \text{and preferred equity}}
\]

coverage ratios:

\[
\frac{\text{EBIT}}{\text{annual interest expense}}
\]

\[
\frac{\text{EBITDA}}{\text{annual interest expense}}
\]

S & P cash flow ratios:

\[
\frac{\text{funds from operations}}{\text{total debt}}
\]

\[
\frac{\text{funds from operations}}{\text{capital spending requirements}}
\]

\[
\frac{\text{free operating cash flow} + \text{interest}}{\text{interest}}
\]

\[
\frac{\text{free operating cash flow} + \text{interest}}{\text{interest} + \text{annual principal repayment}} = "\text{debt service coverage}"
\]

\[
\frac{\text{total debt}}{\text{discretionary cash flow}} = "\text{debt payback period}"
\]

standard deviation of daily yield changes:

\[
\sigma_{\text{annual}} = \sigma_{\text{daily}} \times (\text{number of trading days in the year})^{1/2}
\]

value of embedded call option: \( V_{\text{call}} = V_{\text{noncallable}} - V_{\text{callable}} \)

value of embedded put option: \( V_{\text{put}} = V_{\text{puttable}} - V_{\text{nonputable}} \)
effective duration: \( ED = \frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y} \)

effective convexity: \( EC = \frac{BV_{-\Delta y} + BV_{+\Delta y} - (2 \times BV_0)}{2 \times BV_0 \times \Delta y^2} \)

convertible bonds:

conversion value = market price of stock \times conversion ratio

market conversion price = \( \frac{\text{market price of convertible bond}}{\text{conversion ratio}} \)

market conversion premium per share = market conversion price – market price

market conversion premium ratio = \( \frac{\text{market conversion premium per share}}{\text{market price of common stock}} \)

premium payback period = \( \frac{\text{market conversion premium per share}}{\text{favorable income difference per share}} \)

favorable income difference per share = \( \frac{\text{coupon interest} - (\text{conversion ratio} \times \text{dividends per share})}{\text{conversion ratio}} \)

premium over straight value = \( \left( \frac{\text{market price of convertible bond}}{\text{straight value}} \right) - 1 \)

mortgage prepayment speed:

single monthly mortality rate = \( 1 - (1 - \text{conditional prepayment rate})^{\frac{1}{12}} \)

mortgage prepayment:

\( \text{prepayment}_m = SMM_m \times \left( \frac{\text{mortgage balance at \begin{align*} \text{beginning of month } m \quad \text{scheduled principal payment for month } m \end{align*}}}{\text{beginning of month } m - \text{scheduled principal payment for month } m} \right) \)

commercial MBS credit analysis:

debt-to-service coverage ratio = \( \frac{\text{net operating income}}{\text{debt service}} \)

loan-to-value ratio = \( \frac{\text{current mortgage amount}}{\text{current appraised value}} \)

bond-equivalent yield = \( 2[(1 + \text{monthly cash flow yield})^6 - 1] \)
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